Magnetic Rings

Numerical resolutions of the ODEs

```
SolutionPlot[p_?NumericQ, l_?NumericQ, A_?NumericQ,
  k_?NumericQ, λ_?NumericQ] := Module[{M = Evaluate[u[s] /.
      NDSolve[{v'[r] - (A^2 π^2)/(k^2 u[r]^3) + Abs[u[r]]^(p-2) u[r] - λ u[r] == 0,
        u'[r] == v[r], v[0] == 0, u[0] == 1}, {u, v}, {r, 0, π}]]},
  Plot[M, {s, 0, π}, AxesOrigin → {0, 0}, PlotStyle → Black]]
```

```
SolutionDataatPi[p_?NumericQ, l_?NumericQ, A_?NumericQ,
  k_?NumericQ, λ_] := {A, l, u[π], v[π], w[π]/π, w0[π]/π, w2[π]/π,
   (w2[π]/π + (A^2 π)/wminus2[π] + λ w0[π]/π)/(w[π]/π)^(2/p), wminus2[π]/π, (A^2 π)/wminus2[π]} /.
  NDSolve[{v'[r] - (A^2 π^2)/(k^2 u[r]^3) + Abs[u[r]]^(p-2) u[r] - λ u[r] == 0,
     u'[r] == v[r], w'[r] == Abs[u[r]]^p, w2'[r] == Abs[v[r]]^2,
     w0'[r] == Abs[u[r]]^2, wminus2'[r] == Abs[u[r]]^-2, v[0] == 0,
     u[0] == 1, w[0] == 0, w2[0] == 0, w0[0] == 0, wminus2[0] == 0},
    {u, v, w, w0, w2, wminus2}, {r, 0, π}]
```

```
DerivativeatPi[p_?NumericQ, l_?NumericQ,
  A_?NumericQ, k_?NumericQ, λ_] := N[v[π]] /.
  NDSolve[{v'[r] - (A^2 π^2)/(k^2 u[r]^3) + Abs[u[r]]^(p-2) u[r] - λ u[r] == 0,
      u'[r] == v[r], w'[r] == Abs[u[r]]^p, w2'[r] == Abs[v[r]]^2,
      w0'[r] == Abs[u[r]]^2, wminus2'[r] == Abs[u[r]]^-2, u[0] == 1,
      v[0] == 0, w[0] == 0, w2[0] == 0, w0[0] == 0, wminus2[0] == 0},
     {u, v, w, w2, w0, wminus2}, {r, 0, π}][[1]]
```

```
Integraluminus2[p_?NumericQ, l_?NumericQ, A_?NumericQ,
```

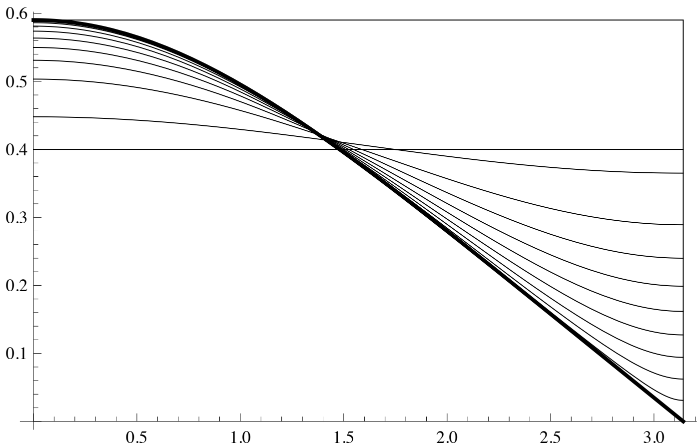
```
k_?NumericQ, λ_?NumericQ] := N[wminus2[π]] /.
  NDSolve[{v'[r] - A^2 π^2/(k^2 u[r]^3) + Abs[u[r]]^(p-2) u[r] - λ u[r] == 0,
     u'[r] == v[r], w'[r] == Abs[u[r]]^p, w2'[r] == Abs[v[r]]^2 ,
     w0'[r] == Abs[u[r]]^2, wminus2'[r] == Abs[u[r]]^-2 , u[0] == 1,
     v[0] == 0, w[0] == 0, w2[0] == 0, w0[0] == 0, wminus2[0] == 0},
    {u, v, w, w2, w0, wminus2}, {r, 0, π}][[1]]

ksol[kstart_, l_, A_, λ_, acc_: 4] :=
 k /. FindRoot[Evaluate[Integraluminus2[4, l, A, k, λ] - k] == 0,
   {k, kstart, 1.01 kstart},
   AccuracyGoal :→ acc, MaxIterations → 1000]

SolutionPlotDirichlet[p_, l_, λ_] := Module[{M =
     Evaluate[u[s] /. NDSolve[{v'[r] +Abs[u[r]]^(p-2) u[r] -λ u[r]  == 0,
         u'[r] == v[r], v[0] == 0, u[0] == 1}, {u, v}, {r, 0, π}]]},
  Plot[M, {s, 0, π}, AxesOrigin → {0, 0},
   PlotStyle → { Thick, Black}]]

ValueatPiDirichlet[p_, l_, λ_] :=
 N[u[π]] /. NDSolve[{v'[r] + Abs[u[r]]^(p-2) u[r] - λ u[r] == 0,
     u'[r] == v[r], w'[r] == Abs[u[r]]^p, w2'[r] == Abs[v[r]]^2 ,
     w0'[r] == Abs[u[r]]^2, v[0] == 0, u[0] == 1, w[0] == 0,
     w2[0] == 0, w0[0] == 0}, {u, v, w, w0, w2}, {r, 0, π}]
```

Solutions with p = 4 and 0.4 ≤ a ≤ 0.49 and α = 0; solutions with a < 0.4 are constant

```
Off[NDSolve::mxst]
Off[InterpolatingFunction::dmval]
Off[FindRoot::nlnum]
Off[ReplaceAll::reps]
Off[FindRoot::cvmit]
Off[FindRoot::lstol]
Off[NDSolve::ndinnt]
```

```
FindRoot[DerivativeatPi[4, l, 0.4, ksol[20, l, 0.4, 0], 0],
    {l, 0.5}][[1]][[2]];
PLLLOTSOL4040 = SolutionPlot[4, %, 0.4, ksol[20, %, 0.4, 0], 0];
FindRoot[DerivativeatPi[4, l, 0.41, ksol[20, l, 0.41, 0], 0],
    {l, 0.5}][[1]][[2]];
PLLLOTSOL4041 = SolutionPlot[4, %, 0.41, ksol[20, %, 0.41, 0], 0];
FindRoot[DerivativeatPi[4, l, 0.42, ksol[20, l, 0.42, 0], 0],
    {l, 0.5}][[1]][[2]];
PLLLOTSOL4042 = SolutionPlot[4, %, 0.42, ksol[20, %, 0.42, 0], 0];
FindRoot[DerivativeatPi[4, l, 0.43, ksol[20, l, 0.43, 0], 0],
    {l, 0.53}][[1]][[2]];
PLLLOTSOL4043 = SolutionPlot[4, %, 0.43, ksol[20, %, 0.43, 0], 0];
FindRoot[DerivativeatPi[4, l, 0.44, ksol[20, l, 0.44, 0], 0],
    {l, 0.56}][[1]][[2]];
PLLLOTSOL4044 = SolutionPlot[4, %, 0.44, ksol[20, %, 0.44, 0], 0];
FindRoot[DerivativeatPi[4, l, 0.45, ksol[20, l, 0.45, 0], 0],
    {l, 0.55}][[1]][[2]];
PLLLOTSOL4045 = SolutionPlot[4, %, 0.45, ksol[20, %, 0.45, 0], 0];
FindRoot[DerivativeatPi[4, l, 0.46, ksol[60, l, 0.46, 0], 0],
    {l, 0.56}][[1]][[2]];
PLLLOTSOL4046 = SolutionPlot[4, %, 0.46, ksol[60, %, 0.46, 0], 0];
FindRoot[DerivativeatPi[4, l, 0.47, ksol[30, l, 0.47, 0], 0],
    {l, 0.57}][[1]][[2]];
PLLLOTSOL4047 = SolutionPlot[4, %, 0.47, ksol[30, %, 0.47, 0], 0];
FindRoot[DerivativeatPi[4, l, 0.48, ksol[90, l, 0.48, 0], 0],
    {l, 0.57}][[1]][[2]];
PLLLOTSOL4048 = SolutionPlot[4, %, 0.48, ksol[90, %, 0.48, 0], 0];
FindRoot[DerivativeatPi[4, l, 0.49, ksol[80, l, 0.49, 0], 0],
    {l, 0.58}][[1]][[2]];
PLLLOTSOL4049 = SolutionPlot[4, %, 0.49, ksol[80, %, 0.49, 0], 0];
FindRoot[DerivativeatPi[4, l, 0.495, ksol[200, l, 0.495, 0], 0],
    {l, 0.58}][[1]][[2]];
PLLLOTSOL40495 = SolutionPlot[4, %, 0.495,
   ksol[200, %, 0.495, 0], 0];

LimitValueAtZero =
  FindRoot[ValueatPiDirichlet[4, x, 0], {x, 0.58}][[1]][[2]];
PPPDir = SolutionPlotDirichlet[4, %, 0];
```

```
Show[PPPDir, PLLLOTSOL4049, PLLLOTSOL4048,
 PLLLOTSOL4047, PLLLOTSOL4046, PLLLOTSOL4045,
 PLLLOTSOL4044, PLLLOTSOL4043, PLLLOTSOL4042,
 PLLLOTSOL4041, PLLLOTSOL4040,
 ListLinePlot[{{π, 0}, {π, LimitValueAtZero},
   {0, LimitValueAtZero}}, PlotStyle → Black]]
```

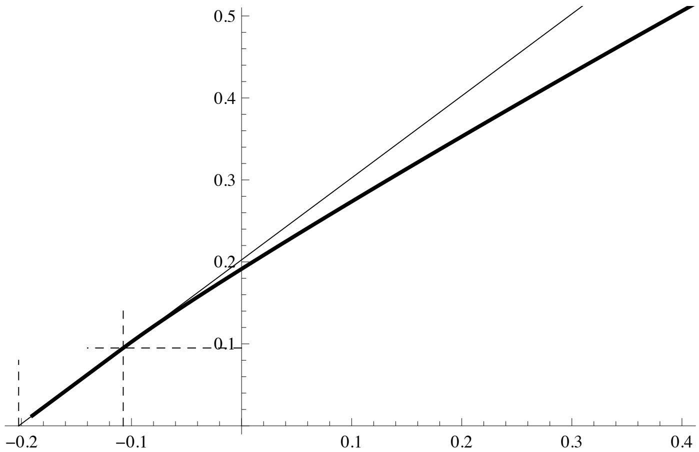


Values of the quotient as a function of α, for a = 0.45 and p = 4

```
BifurMuAlpha[a_, p_] := α /. Solve[a^2 (p + 2) + α (p - 2) == 1, α][[1]]
```

```
LLLLprov = FindRoot[DerivativeatPi[4, l, 0.45,
     ksol[45, l, 0.45, 0.1], 0.1], {l, 0.2}][[1]][[2]];
kkkkprov = ksol[45, LLLLprov, 0.45, 0.1];
energyprov =
  SolutionDataatPi[4, LLLLprov, 0.45, kkkkprov, 0.1][[1]][[8]];
LIST045 = {{0.1, energyprov}};

For [j = 1, j < 170, j++,
 LLLLprov =
  FindRoot[DerivativeatPi[4, l, 0.45, ksol[kkkkprov, l, 0.45,
      0.1 + 0.01 j], 0.1 + 0.01 j], {l, LLLLprov}][[1]][[2]];
 kkkkprov = ksol[kkkkprov, LLLLprov, 0.45, 0.1 + 0.01 j];
 energyprov = SolutionDataatPi[4,
    LLLLprov, 0.45, kkkkprov, 0.1 + 0.01 j][[1]][[8]];
 LIST045 = Append[LIST045, {0.1 + 0.01 j, energyprov}]]

LLLLprov = FindRoot[DerivativeatPi[4, l, 0.45,
     ksol[45, l, 0.45, 0.1], 0.1], {l, 0.2}][[1]][[2]];
kkkkprov = ksol[45, LLLLprov, 0.45, 0.1];
energyprov =
  SolutionDataatPi[4, LLLLprov, 0.45, kkkkprov, 0.1][[1]][[8]];
LIST045minus = {{0.1, energyprov}};

For [j = 1, j < 30, j++,
 LLLLprov =
  FindRoot[DerivativeatPi[4, l, 0.45, ksol[kkkkprov, l, 0.45,
      0.1 - 0.01 j], 0.1 - 0.01 j], {l, LLLLprov}][[1]][[2]];
 kkkkprov = ksol[kkkkprov, LLLLprov, 0.45, 0.1 - 0.01 j];
 energyprov =
  SolutionDataatPi[4, LLLLprov, 0.45, kkkkprov, 0.1 - 0.01 j][[1]][[
   8]];
 LIST045minus = Append[LIST045minus, {0.1 - 0.01 j, energyprov}]]

LIST045 = Join[Table[LIST045minus[[Length[LIST045minus] + 1 - j]],
    {j, 1, Length[LIST045minus] - 1}], LIST045];
```

```
Show[Plot[0.2025` + x, {x, -0.2025`, 1.5}, PlotStyle → Black],
 ListLinePlot[LIST045, PlotStyle → {Black, Thick}],
 ListLinePlot[{{BifurMuAlpha[0.45, 4], 0},
   {BifurMuAlpha[0.45, 4], 0.14}}, PlotStyle → {Dashed, Black}],
 ListLinePlot[{{0, BifurMuAlpha[0.45, 4] + 0.45^2},
   {-0.14, BifurMuAlpha[0.45, 4] + 0.45^2}},
  PlotStyle → {Dashed, Black}], ListLinePlot[
  {{-0.45^2, 0}, {-0.45^2, 0.08}}, PlotStyle → {Dashed, Black}],
 PlotRange → {{-0.45^2, 0.4}, {0, 0.5}}]
```



Values of the quotient as a function of α, for a = 0.2 and p = 4

```
Off[InterpolatingFunction]
Off[$RecursionLimit::reclim]
```

```
LLLLprov = FindRoot[DerivativeatPi[4, l, 0.2,
      ksol[10, l, 0.2, 0.60], 0.60], {l, 1}][[1]][[2]];
kkkkprov = ksol[10, LLLLprov, 0.2, 0.60];
energyprov =
  SolutionDataatPi[4, LLLLprov, 0.2, kkkkprov, 0.60][[1]][[8]];
LIST020 = {{0.60, energyprov}};

For [j = 1, j < 91, j ++,
 LLLLprov =
  FindRoot[DerivativeatPi[4, l, 0.2, ksol[kkkkprov, l, 0.2,
       0.60 + 0.01 j], 0.60 + 0.01 j], {l, LLLLprov}][[1]][[2]];
 kkkkprov = ksol[kkkkprov, LLLLprov, 0.2, 0.60 + 0.01 j];
 energyprov =
  SolutionDataatPi[4, LLLLprov, 0.2, kkkkprov, 0.60 + 0.01 j][[1]][[
   8]];
 LIST020 = Append[LIST020, {0.60 + 0.01 j, energyprov}]]

LLLLprov =
  FindRoot[DerivativeatPi[4, l, 0.2, ksol[10, l, 0.2, 0.60], 0.60],
    {l, 1}][[1]][[2]];
kkkkprov = ksol[10, LLLLprov, 0.2, 0.60];
energyprov =
  SolutionDataatPi[4, LLLLprov, 0.2, kkkkprov, 0.60][[1]][[8]];
LIST020minus = {{0.60, energyprov}};

For[j = 1, j < 25, j ++,
 LLLLprov = FindRoot[DerivativeatPi[4, l, 0.2, ksol[kkkkprov, l,
       0.2, 0.60 - 0.01 j], 0.60 - 0.01 j], {l, LLLLprov}][[1]][[2]];
 kkkkprov = ksol[kkkkprov, LLLLprov, 0.2, 0.60 - 0.01 j];
 energyprov =
  SolutionDataatPi[4, LLLLprov, 0.2, kkkkprov, 0.60 - 0.01 j][[1]][[
   8]];
 LIST020minus = Append[LIST020minus, {0.60 - 0.01 j, energyprov}]]

LIST020 = Join[Table[LIST020minus[[Length[LIST020minus] + 1 - j]],
    {j, 1, Length[LIST020minus] - 1}], LIST020];
```

```
Show[ListLinePlot[LIST020, PlotStyle → {Black, Thick}],
 Plot[0.04 + x, {x, -0.04, 1.5}, PlotStyle → Black],
 Plot[0.04 + x, {x, -0.04, BifurMuAlpha[0.2, 4]},
  PlotStyle → {Black, Thick}], ListLinePlot[
  {{BifurMuAlpha[0.2, 4], 0}, {BifurMuAlpha[0.2, 4], 0.5}},
  PlotStyle → {Dashed, Black}],
 ListLinePlot[{{0, 0.04 + BifurMuAlpha[0.2, 4]},
   {0.45, 0.04 + BifurMuAlpha[0.2, 4]}},
  PlotStyle → {Dashed, Black}], ListLinePlot[
  {{-0.2^2, 0}, {-0.2^2, 0.2}}, PlotStyle → {Dashed, Black}],
 PlotRange → {{-0.04, 1.5}, {0, 1.3}}, AxesOrigin → {0, 0}]
```