The quadratic form estimated for our test function

In[1]:= $\left(\frac{\omega 2}{\alpha+1}+1\right)\xi^2 - 4\,\xi\,a + \left(\frac{p}{p-2}\right)^2 \frac{\omega 2}{\alpha+1} + \left(1+\lambda+a^2\right) - (p-1)\,\frac{p}{2}\left(\lambda+a^2\right)\frac{\alpha}{\alpha+1}$

$\text{Simplify}\left[\%\ /.\ \alpha \to \frac{2\,p}{p-2}\right]$

$\text{Res} = \text{Simplify}\left[\%\ /.\ \omega 2 \to \frac{(p-2)^2}{4}\left(\lambda+a^2\right)\right]$

Out[1]= $1 + a^2 - 4\,a\,\xi + \lambda - \frac{(-1+p)\,p\,\alpha\,\left(a^2+\lambda\right)}{2\,(1+\alpha)} + \frac{p^2\,\omega 2}{(-2+p)^2\,(1+\alpha)} + \xi^2\left(1+\frac{\omega 2}{1+\alpha}\right)$

Out[2]= $1 + a^2 - 4\,a\,\xi + \lambda - \frac{(-1+p)\,p^2\,\left(a^2+\lambda\right)}{-2+3\,p} + \frac{p^2\,\omega 2}{4-8\,p+3\,p^2} + \xi^2\left(1+\frac{(-2+p)\,\omega 2}{-2+3\,p}\right)$

Out[3]= $1 + a^2 - 4\,a\,\xi + \lambda - \frac{(-1+p)\,p^2\,\left(a^2+\lambda\right)}{-2+3\,p} + \frac{(-2+p)\,p^2\,\left(a^2+\lambda\right)}{-8+12\,p} + \xi^2\left(1+\frac{(-2+p)^3\,\left(a^2+\lambda\right)}{-8+12\,p}\right)$

For ξ=0, we recover the computation of Felli & Schneider

In[4]:= $\text{Res}\ /.\ \xi \to 0$

Simplify[λ /. Solve[% == 0, λ]][[1]]

Out[4]= $1 + a^2 + \lambda - \frac{(-1+p)\,p^2\,\left(a^2+\lambda\right)}{-2+3\,p} + \frac{(-2+p)\,p^2\,\left(a^2+\lambda\right)}{-8+12\,p}$

Out[5]= $-a^2 + \frac{4}{-4+p^2}$

Computation for the optimal value of ζ

In[6]:=
```
Simplify[Solve[D[Res, ζ] == 0, ζ][[1]]]
Simplify[Res /. %]
FullSimplify[Solve[% == 0, λ], Assumptions → a > 0 && a < 1/2 && p > 2]
Resλ = λ /. %
```

Out[6]= $\left\{\zeta \to \frac{2\,a}{1+\frac{(-2+p)^3\,(a^2+\lambda)}{-8+12\,p}}\right\}$

Out[7]= $1+\lambda-\frac{p^2\,\lambda}{4}+a^2\left(1+\frac{3\,p^3}{8-12\,p}+\frac{p^2}{-4+6\,p}-\frac{4}{1+\frac{(-2+p)^3\,(a^2+\lambda)}{-8+12\,p}}\right)$

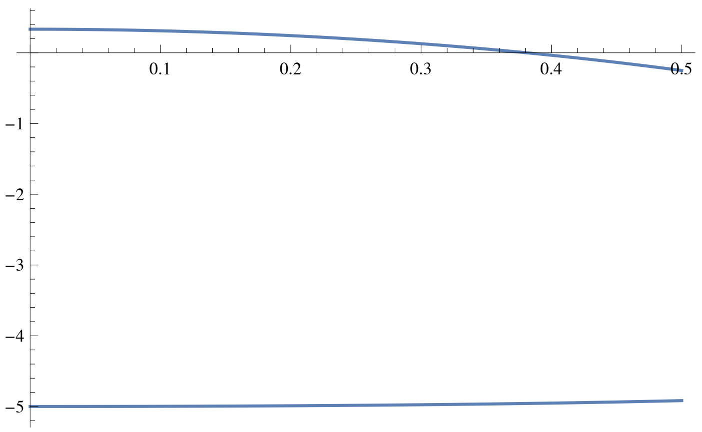
Out[8]= $\left\{\left\{\lambda \to -\left(-16+a^2\,(-2+p)^3\,(2+p)+4\,p\,(4+p)+8\sqrt{\left(p^4-a^2\,(-2+p)^2\,(2+p)\,(-2+3\,p)\right)}\right)\Big/\left((-2+p)^3\,(2+p)\right)\right\},\left\{\lambda \to \left(-a^2\,(-2+p)^3\,(2+p)-4\,p\,(4+p)+8\left(2+\sqrt{\left(p^4-a^2\,(-2+p)^2\,(2+p)\,(-2+3\,p)\right)}\right)\right)\Big/\left((-2+p)^3\,(2+p)\right)\right\}\right\}$

Out[9]= $\left\{-\left(-16+a^2\,(-2+p)^3\,(2+p)+4\,p\,(4+p)+8\sqrt{\left(p^4-a^2\,(-2+p)^2\,(2+p)\,(-2+3\,p)\right)}\right)\Big/\left((-2+p)^3\,(2+p)\right),\left(-a^2\,(-2+p)^3\,(2+p)-4\,p\,(4+p)+8\left(2+\sqrt{\left(p^4-a^2\,(-2+p)^2\,(2+p)\,(-2+3\,p)\right)}\right)\right)\Big/\left((-2+p)^3\,(2+p)\right)\right\}$

Selection of the threshold value for λ

In[1]:=
```
f[p_, a_] :=
  {-(-16 + a^2 (-2 + p)^3 (2 + p) + 4 p (4 + p) + 8 √(p^4 - a^2 (-2 + p)^2 (2 + p) (-2 + 3 p))) /
     ((-2 + p)^3 (2 + p)),
   (-a^2 (-2 + p)^3 (2 + p) - 4 p (4 + p) + 8 (2 + √(p^4 - a^2 (-2 + p)^2 (2 + p) (-2 + 3 p)))) /
    ((-2 + p)^3 (2 + p))}
F0[p_] := Plot[f[p, a], {a, 0, 1/2}]
F0[4]
```

Out[3]=

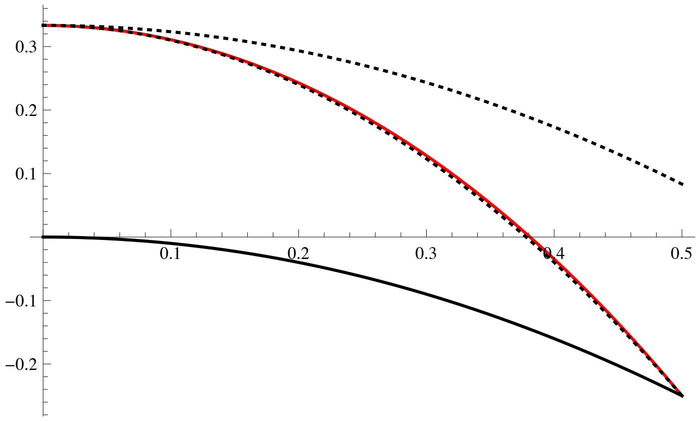


Plots: dotted values = the previous bounds for symmetry & symmetry breaking; thick-black = $-a^2$; red = the new upper bound for the symmetry breaking region

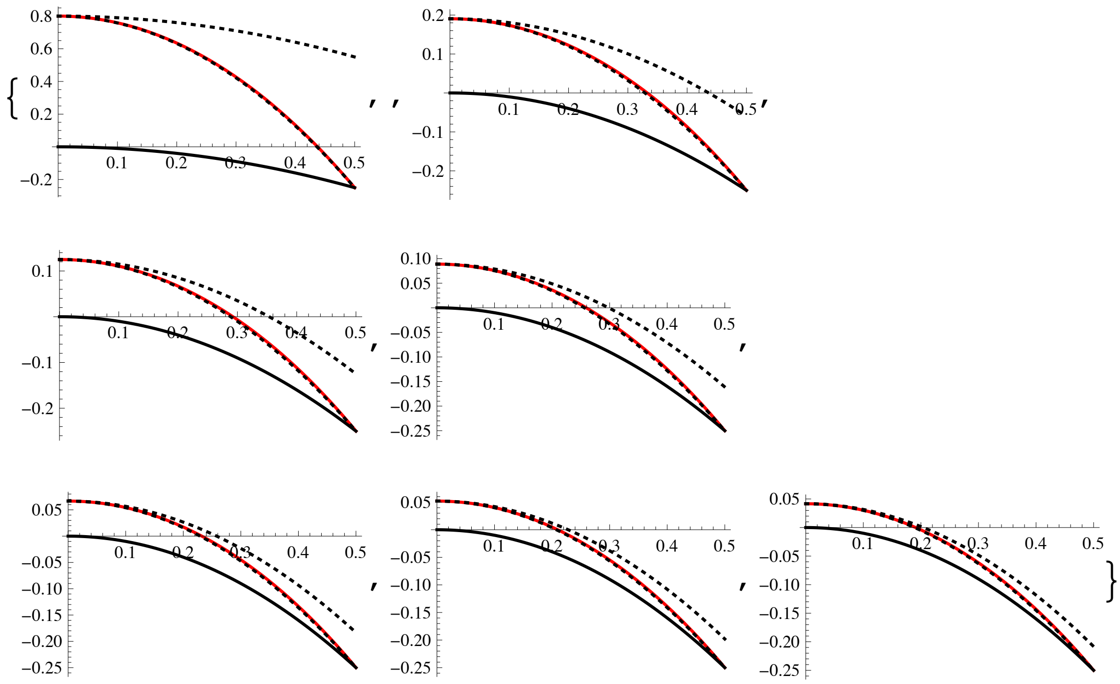
In[13]:=
```
F[p_] := Show[Plot[f[p, a][[2]], {a, 0, 1/2}, PlotStyle → Red],
  Plot[-a^2, {a, 0, 1/2}, PlotStyle → Black], Plot[{4/(p^2 - 4) - a^2, 4 (1 - 4 a^2)/(p^2 - 4) - a^2},
   {a, 0, 1/2}, PlotStyle → {{Black, Dotted}, {Black, Dotted}}],
  PlotRange → {All, {-0.25, 4/(p^2 - 4)}}]
```

In[14]:=
```
F[4]
```

Out[14]=



In[15]:=
```
Table[F[p], {p, 3, 10}]
```



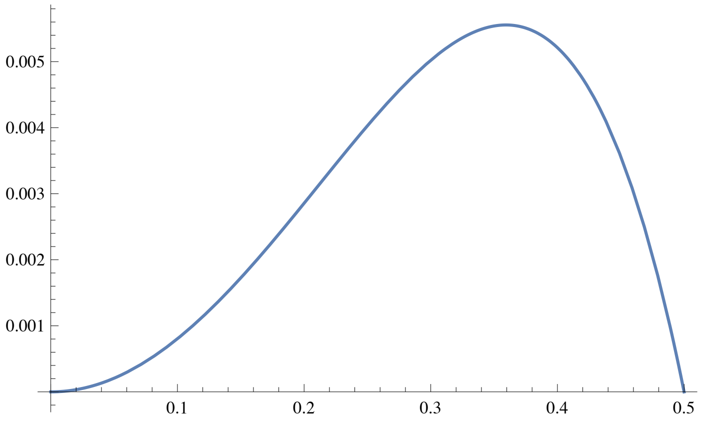
The width of the interval for which we ignore if there is symmetry or

symmetry breaking

In[139]:=
$$G[p\_, PS\_] := \text{Plot}\left[f[p, a][[2]] - \left(\frac{4\,(1 - 4\,a^2)}{p^2 - 4} - a^2\right), \left\{a, 0, \frac{1}{2}\right\}, \text{PlotStyle} \to PS\right]$$

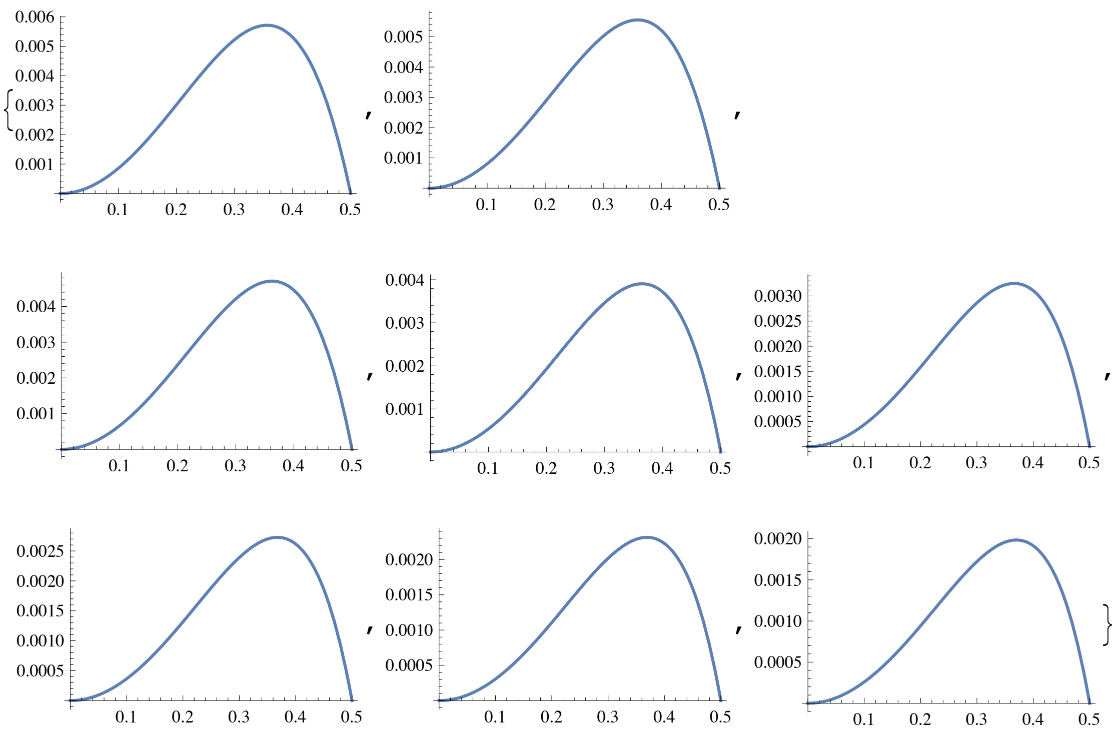
In[140]:=
```
G[4, Automatic]
```

Out[140]=



In[141]:=
```
Table[G[p, Automatic], {p, 3, 10}]
```

Out[141]=



Computation in the z variable

In[285]:=
$$G[z\_] := \left(1 - z^2\right)^{\frac{1}{2}\,\frac{p}{p-2}}$$
$$H[z\_] := \left(1 - z^2\right)^{\frac{1}{2}\,\frac{p}{p-2}}$$

In[322]:=
$$\text{Simplify}\left[\zeta^2\left(\omega 2\,(1-z^2)\,G'[z]^2+\left(1+\frac{4\,\omega 2}{(p-2)^2}\right)\frac{G[z]^2}{1-z^2}-\frac{2\,p\,\omega 2}{(p-2)^2}G[z]^2\right)-\frac{4\,a\,\zeta}{1-z^2}G[z]\,H[z]+\omega 2\,(1-z^2)\,H'[z]^2+\left(1+\frac{4\,\omega 2}{(p-2)^2}\right)\frac{H[z]^2}{1-z^2}-(p-1)\,\frac{2\,p\,\omega 2}{(p-2)^2}H[z]^2\right];$$

$$\text{Resx}=\text{FullSimplify}\left[\text{PowerExpand}\left[\%\ /.\ z^2\to 1-x^2\right]\right]$$

Out[323]=
$$\frac{1}{(-2+p)^2}x^{\frac{4}{-2+p}}\left(-(-2+p)^2\left(-1+4\,a\,\zeta-\zeta^2\right)+\left(4+p^2+2\,p\,x^2-3\,p^2\,x^2+\left(4+p^2-p\,(2+p)\,x^2\right)\zeta^2\right)\omega 2\right)$$

In[342]:=
$$\text{I1}=\text{Integrate}\left[(1-z^2)^{\frac{2}{-2+p}},\{z,-1,1\},\text{Assumptions}\to p>2\right];$$
$$\text{I2}=\text{Integrate}\left[(1-z^2)^{\frac{p}{-2+p}},\{z,-1,1\},\text{Assumptions}\to p>2\right];$$
$$\text{FullSimplify}\left[\frac{\text{I2}}{\text{I1}}\right]$$
$$\text{Resx}\ /.\ x^2\to\%;$$
$$\%\ /.\ x\to 1;$$
$$\text{Res}=\text{Simplify}\left[\%\ /.\ \omega 2\to\frac{(p-2)^2}{4}\left(\lambda+a^2\right)\right]$$

Out[344]=
$$\frac{2\,p}{-2+3\,p}$$

Out[347]=
$$\frac{1}{-8+12\,p}\left(16\,a\,(2-3\,p)\,\zeta+a^2\,(-2+p)\left(p^2\left(-3+\zeta^2\right)+4\left(1+\zeta^2\right)-4\,p\left(1+\zeta^2\right)\right)+p^2\left(2-6\,\zeta^2\right)\lambda+p^3\left(-3+\zeta^2\right)\lambda-8\left(1+\zeta^2\right)(1+\lambda)+12\,p\left(1+\zeta^2\right)(1+\lambda)\right)$$

In[372]:=
$$\text{Res}\zeta=\text{Simplify}[\text{Solve}[\text{D}[\text{Res},\zeta]==0,\zeta][[1]]]$$
$$\text{Simplify}[\text{Res}\ /.\ \%]$$
$$\text{Res}\lambda=\text{Simplify}\left[\text{FullSimplify}\left[\text{Solve}[\%==0,\lambda],\text{Assumptions}\to a>0\ \&\&\ a<\frac{1}{2}\ \&\&\ p>2\right]\right]$$

Out[372]=
$$\left\{\zeta\to(8\,a\,(-2+3\,p))\Big/\left(a^2\,(-2+p)^3-6\,p^2\,\lambda+p^3\,\lambda-8\,(1+\lambda)+12\,p\,(1+\lambda)\right)\right\}$$

Out[373]=
$$-\left(\left(a^4\,(-2+p)^4\,(2+p)-6\,p^4\,\lambda^2+p^5\,\lambda^2+16\,p^2\,\lambda\,(1+\lambda)+8\,p^3\,\lambda\,(1+\lambda)+32\,(1+\lambda)^2-48\,p\,(1+\lambda)^2+2\,a^2\left(32\,(-1+\lambda)-48\,p\,(-1+\lambda)-6\,p^4\,\lambda+p^5\,\lambda+8\,p^2\,(1+2\,\lambda)+p^3\,(4+8\,\lambda)\right)\right)\Big/\left(4\left(a^2\,(-2+p)^3-6\,p^2\,\lambda+p^3\,\lambda-8\,(1+\lambda)+12\,p\,(1+\lambda)\right)\right)\right)$$

Out[374]=
$$\left\{\left\{\lambda\to-\left(-16+a^2\,(-2+p)^3\,(2+p)+4\,p\,(4+p)+8\sqrt{\left(p^4-a^2\,(-2+p)^2\,(2+p)\,(-2+3\,p)\right)}\right)\Big/\left((-2+p)^3\,(2+p)\right)\right\},\left\{\lambda\to\left(-a^2\,(-2+p)^3\,(2+p)-4\,p\,(4+p)+8\left(2+\sqrt{\left(p^4-a^2\,(-2+p)^2\,(2+p)\,(-2+3\,p)\right)}\right)\right)\Big/\left((-2+p)^3\,(2+p)\right)\right\}\right\}$$

In[385]:=
$$\text{Simplify}\left[\text{FullSimplify}\left[\text{Res}\zeta\ /.\ \text{Res}\lambda[[2]],\text{Assumptions}\to a>0\ \&\&\ a<\frac{1}{2}\ \&\&\ p>2\right]\right]$$

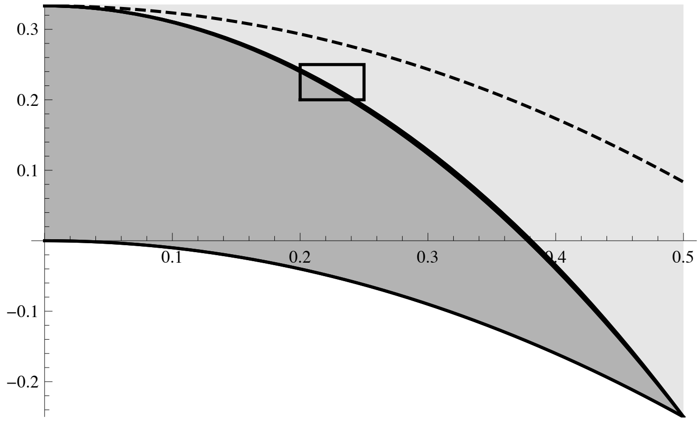
Out[385]=
$$\left\{\zeta\to(a\,(2+p)\,(-2+3\,p))\Big/\left(p^2+\sqrt{\left(p^4-a^2\,(-2+p)^2\,(2+p)\,(-2+3\,p)\right)}\right)\right\}$$

Figures of the paper

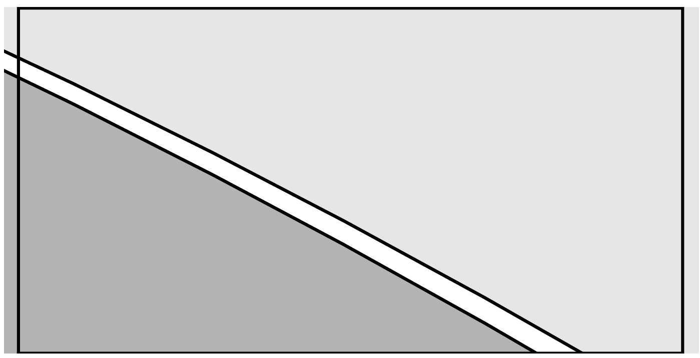
In[126]:=
```
f2[p_, a_] :=
  (-a^2 (-2 + p)^3 (2 + p) - 4 p (4 + p) + 8 (2 + √(p^4 - a^2 (-2 + p)^2 (2 + p) (-2 + 3 p)))) /
   ((-2 + p)^3 (2 + p))
F4[p_] := Show[Plot[{-a^2, (4 (1 - 4 a^2))/(p^2 - 4) - a^2}, {a, 0, 1/2},
    PlotStyle → {Black, Black}, PlotRange → {All, {-0.25, 0.4}},
    Filling → {1 → {2}}, FillingStyle → GrayLevel[0.7]],
   Plot[{f2[p, a], 0.5}, {a, 0, 1/2}, PlotStyle → {Black, Black},
    PlotRange → {All, {-0.25, 0.4}}, Filling → {1 → {2}},
    FillingStyle → GrayLevel[0.9]], Plot[f2[p, a], {a, 0, 1/2}, PlotStyle → Black],
   Plot[-a^2, {a, 0, 1/2}, PlotStyle → Black], Plot[{4/(p^2 - 4) - a^2, (4 (1 - 4 a^2))/(p^2 - 4) - a^2},
    {a, 0, 1/2}, PlotStyle → {{Dashed, Black}, Black}],
   ListLinePlot[{{a1, b1}, {a2, b1}, {a2, b2}, {a1, b2}, {a1, b1}},
    PlotStyle → Black], PlotRange → {All, {-0.25, 4/(p^2 - 4)}}]

a1 = 0.2; a2 = 0.25; b1 = 0.2; b2 = 0.25;
F4[4]
Show[%, PlotRange → {{a1, a2}, {b1, b2}}, AspectRatio → 0.5]
```

Out[129]=



Out[130]=



In[143]:= **G[4, Black]**

Out[143]=