Choice of the parameters

Based on the computations of N. Bacaër in: Un modèle mathématique des débuts de l’épidémie de coronavirus en France
The starting dates are T = March 15, 2020 and initial values are labelled ST, ET, etc.
Parameters of the SEIR model are denoted by a, b, c as in the above paper and correspond to β, a and γ with standard notations

```
In[•]:= NTot = 67.8 * 10^6;
a = 2.33;
b = 0.25;
c = 1;
R0 = a/c;
ET = 5970 / NTot;
IT = 1278 / NTot;
RT = (ET + IT)/(R0 - 1);
ST = 1 - R0 RT;
```

Basic reproduction ratio

```
In[•]:= Raverage[p_, q1_, q2_] := ((1 - p)/q1 + p/q2) R0
```

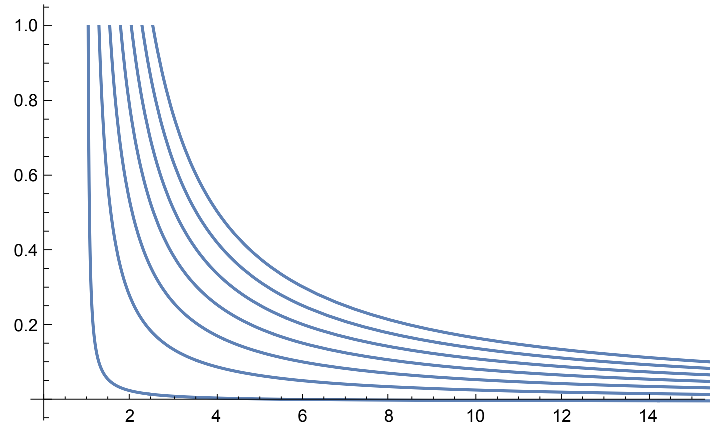
```
In[•]:= Rmin = R0/2.4;
Rmax = 10 R0;
```

Percentage for various global R0 (Rtarget) as a function of the R0,2 group with highest transmission coefficient

```
In[•]:= Ratio[Rtarget_, Rvar_] :=
  {p, q2} /. Solve[{Raverage[p, 2.4, q2] == Rtarget, R0/q2 == Rvar}, {p, q2}][[1]]
```

In[]:=
```
Show[Table[Plot[Ratio[Rt, Rvar][[1]], {Rvar, Rt, Rmax}, PlotRange → All],
   {Rt, 1, 2.5, 0.25}], PlotRange → {{0, 15}, {0, 1}}, AxesOrigin → {0, 0}]
```

Out[]=



Epidemic size 1

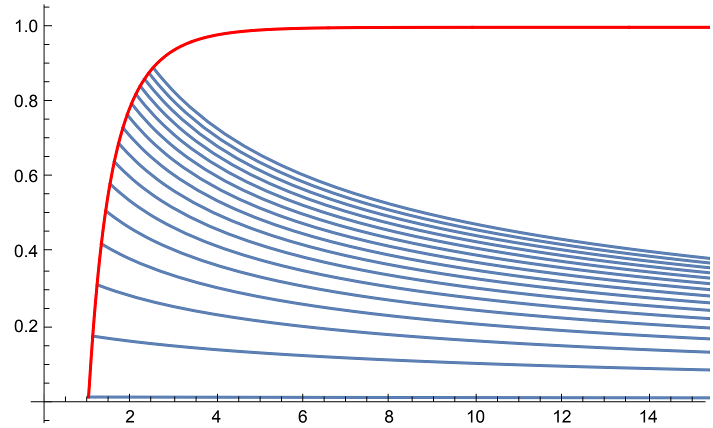
In[]:=
```
f0[Rvar_] := r /. FindRoot[ST e^(Rvar (RT-r)) + r == 1, {r, 0.5}]
```

In[]:=
```
f[p_, q1_, q2_] := r /. FindRoot[(1 - p) ST e^(R0 (RT-r)/q1) + p ST e^(R0 (RT-r)/q2) + r == 1, {r, 0.5}]
```

In[]:=
```
g[Rtarget_] := Plot[f[Ratio[Rtarget, Rvar][[1]],
    2.4, Ratio[Rtarget, Rvar][[2]]], {Rvar, Rtarget, Rmax}]
```

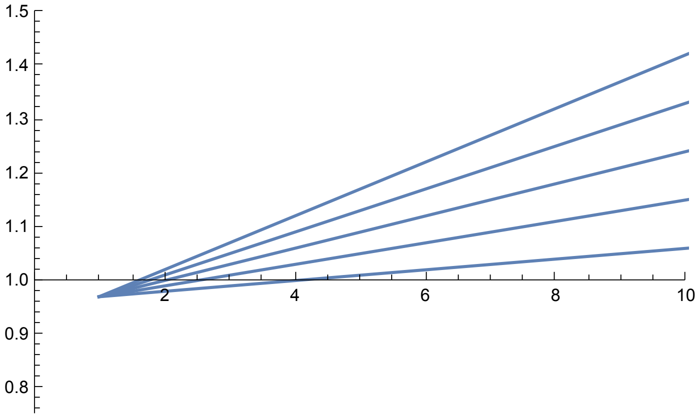
In[]:=
```
Show[Table[g[Rt], {Rt, 1, 2.5, 0.1}],
  Plot[f0[Rvar], {Rvar, 1, Rmax}, PlotRange → All, PlotStyle → Red],
  PlotRange → {{0, 15}, {0, 1}}, AxesOrigin → {0, 0}]
```

Out[]=



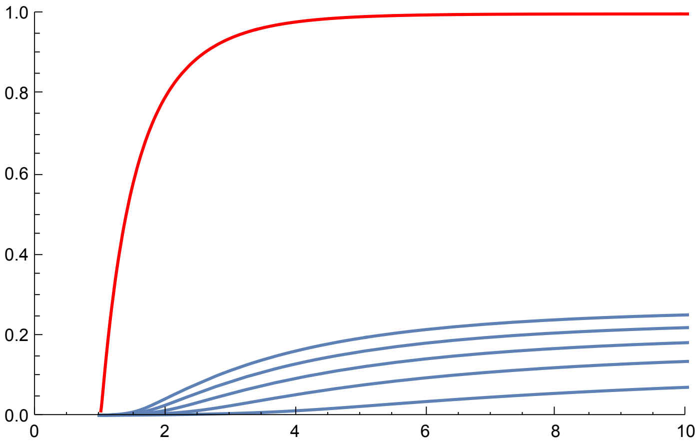
Epidemic size 2

In[∘]:=
```
Show[Table[Plot[Raverage[p, 2.4, R0/Rvar], {Rvar, Rmin, Rmax},
   PlotRange → {{0, 10}, {0.75, 1.5}}, AxesOrigin → {0, 1}], {p, 0.01, 0.05, 0.01}]]
```

Out[∘]=


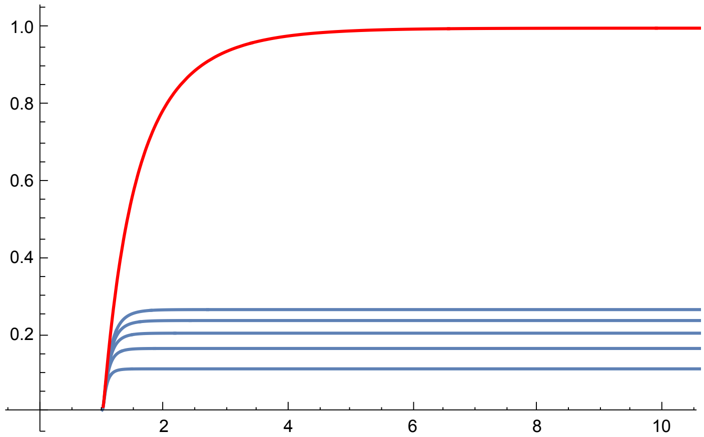
In[∘]:=
```
h[p_] := Plot[f[p, 2.4, R0/Rvar], {Rvar, Rmin, Rmax}, PlotRange → {{0, Rmax}, {0, 1}}]
```

In[∘]:=
```
Show[Table[h[p], {p, 0.01, 0.05, 0.01}],
 Plot[f0[Rvar], {Rvar, 1, Rmax}, PlotRange → All, PlotStyle → Red],
 PlotRange → {{0, 10}, {0, 1}}, AxesOrigin → {0, 0}]
```

Out[∘]=


In[*]:=
```
Show[Table[ParametricPlot[{Raverage[p, 2.4, q], f[p, 2.4, q]},
    {q, 0, 1}, PlotRange → All], {p, 0.01, 0.05, 0.01}],
 Plot[f0[Rvar], {Rvar, 1, Rmax}, PlotRange → All, PlotStyle → Red],
 PlotRange → {{0, 10}, {0, 1}}, AxesOrigin → {0, 0}, AspectRatio → 1 / GoldenRatio]
```

Out[*]=



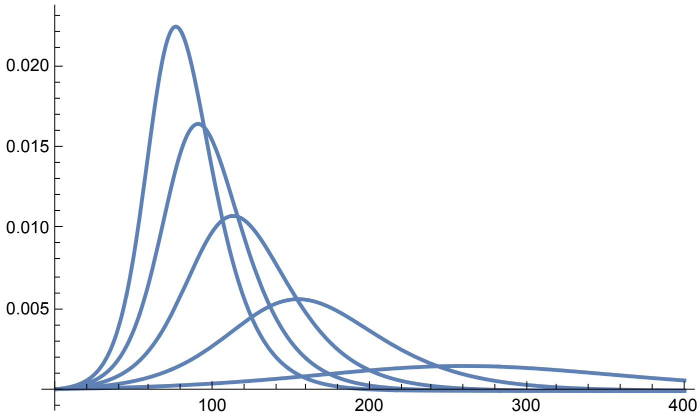
Epidemic peak

In[*]:=
```
FEIRmodif[p_, q1_, q2_, T_, PS_] :=
 Plot[Ex1[s] + Ex2[s] + Inf1[s] + Inf2[s] /. NDSolve[
    {S1'[t] == -a (Inf1[t] + Inf2[t])/q1 S1[t], S2'[t] == -a (Inf1[t] + Inf2[t])/q2 S2[t],
     Ex1'[t] == a (Inf1[t] + Inf2[t])/q1 S1[t] - b Ex1[t],
     Ex2'[t] == a (Inf1[t] + Inf2[t])/q2 S2[t] - b Ex2[t], Inf1'[t] == b Ex1[t] - c Inf1[t],
     Inf2'[t] == b Ex2[t] - c Inf2[t], R1'[t] == c Inf1[t], R2'[t] == c Inf2[t],
     S1[0] == (1 - p) ST, S2[0] == p ST, Ex1[0] == (1 - p) ET, Ex2[0] == p ET,
     Inf1[0] == (1 - p) IT, Inf2[0] == p IT, R1[0] == (1 - p) RT, R2[0] == p RT},
    {S1, S2, Ex1, Ex2, Inf1, Inf2, R1, R2}, {t, 0, T}],
  {s, 0, T}, PlotRange → All, PlotStyle → PS]
```

In[∘]:=
```
CharacteristicsF[p_, q1_, q2_, T_] :=
 Module[{M = Ex1[s] + Ex2[s] + Inf1[s] + Inf2[s] /. NDSolve[
     {S1'[t] == -a (Inf1[t] + Inf2[t])/q1 S1[t], S2'[t] == -a (Inf1[t] + Inf2[t])/q2 S2[t],
      Ex1'[t] == a (Inf1[t] + Inf2[t])/q1 S1[t] - b Ex1[t], Ex2'[t] ==
       a (Inf1[t] + Inf2[t])/q2 S2[t] - b Ex2[t], Inf1'[t] == b Ex1[t] - c Inf1[t],
      Inf2'[t] == b Ex2[t] - c Inf2[t], R1'[t] == c Inf1[t], R2'[t] == c Inf2[t],
      S1[0] == (1 - p) ST, S2[0] == p ST, Ex1[0] == (1 - p) ET, Ex2[0] == p ET,
      Inf1[0] == (1 - p) IT, Inf2[0] == p IT, R1[0] == (1 - p) RT, R2[0] == p RT},
     {S1, S2, Ex1, Ex2, Inf1, Inf2, R1, R2}, {t, 0, T}]}, FindMaximum[M, {s, 0, T}]]
```

In[∘]:=
```
Show[Table[FEIRmodif[p, 2.4, 0.2, 400, Thick], {p, 0.01, 0.05, 0.01}],
 PlotRange → All]
Table[Print[CharacteristicsF[p, 2.4, 0.2, 400]], {p, 0.01, 0.05, 0.01}];
```

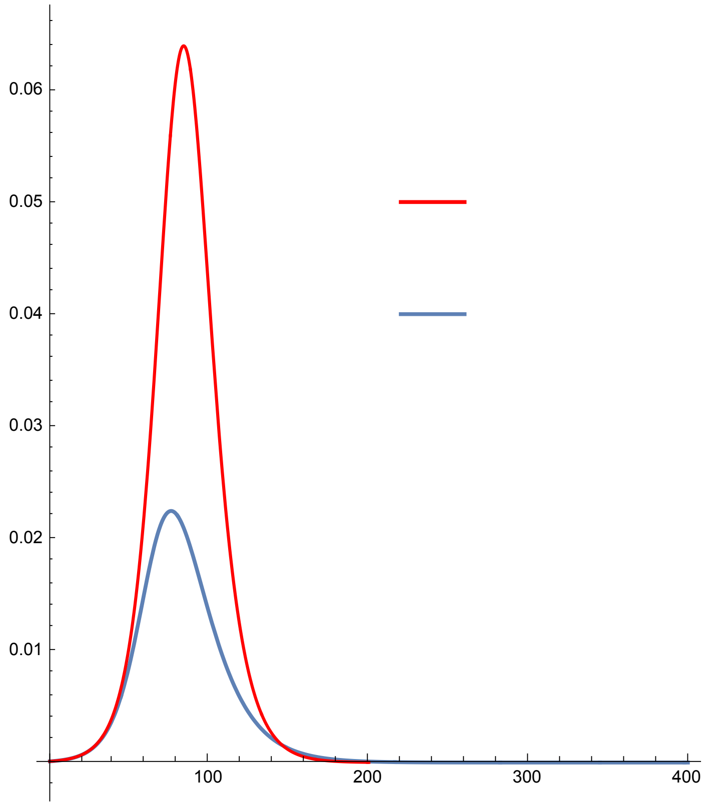
Out[∘]=




```
{0.00153404, {s → 259.331}}
{0.00564703, {s → 153.918}}
{0.0107782, {s → 112.576}}
{0.0164432, {s → 90.2026}}
{0.0224408, {s → 76.0206}}
```

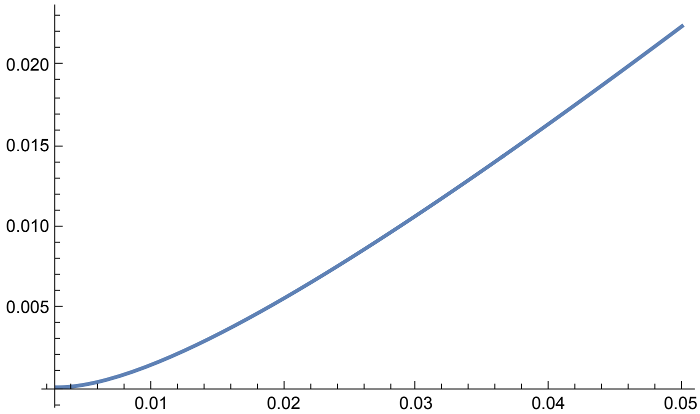
In[∘]:=
```
Show[FEIRmodif[0.05, 2.4, 0.2, 400, Thick], FEIRmodif[1, 2.4,
   q2 /. Solve[Raverage[1, 2.4, q2] == Raverage[0.05, 2.4, 0.2], q2][[1]], 200, Red],
  ListLinePlot[{{220, 0.04}, {260, 0.04}}, PlotStyle → Thick],
  ListLinePlot[{{220, 0.05}, {260, 0.05}}, PlotStyle → {Red, Thick}],
  PlotRange → All, AspectRatio → 1.2]
```

Out[∘]=

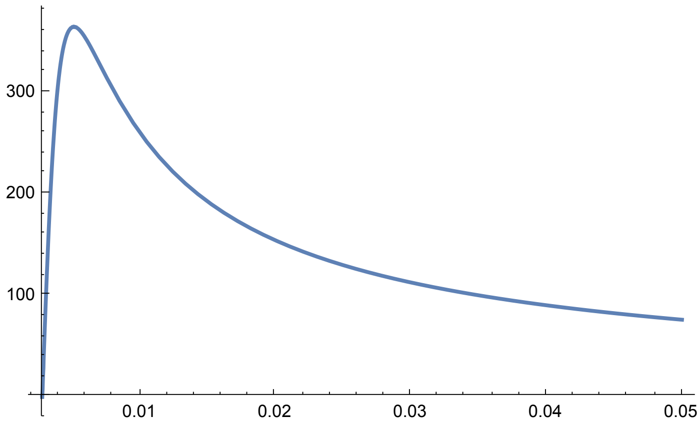


In[∘]:=
```
pmin[q1_, q2_] := p /. Solve[Raverage[p, q1, q2] == 1, p][[1]]
Plot[CharacteristicsF[p, 2.4, 0.2, 400][[1]],
 {p, pmin[2.4, 0.2], 0.05}, PlotStyle → Thick]
Plot[s /. CharacteristicsF[p, 2.4, 0.2, 400][[2]],
 {p, pmin[2.4, 0.2], 0.05}, PlotStyle → Thick]
```

Out[∘]=



Out[∘]=

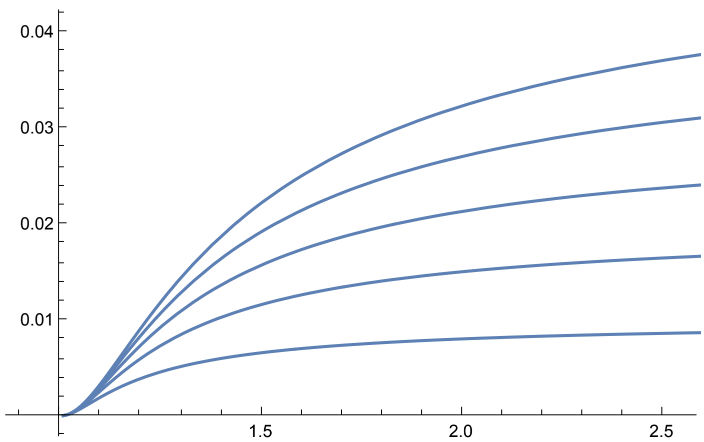


In[∘]:=
```
qmax[p_, q1_] := q2 /. Solve[Raverage[p, q1, q2] == 1, q2][[1]]
```

In[∘]:=
```
Peak[p_] :=
 ParametricPlot[{Raverage[p, 2.4, q], CharacteristicsF[p, 2.4, q, 1000][[1]]},
  {q, 0.001, qmax[p, 2.4] - 0.1}, PlotRange → All, AspectRatio → 1 / GoldenRatio]
```

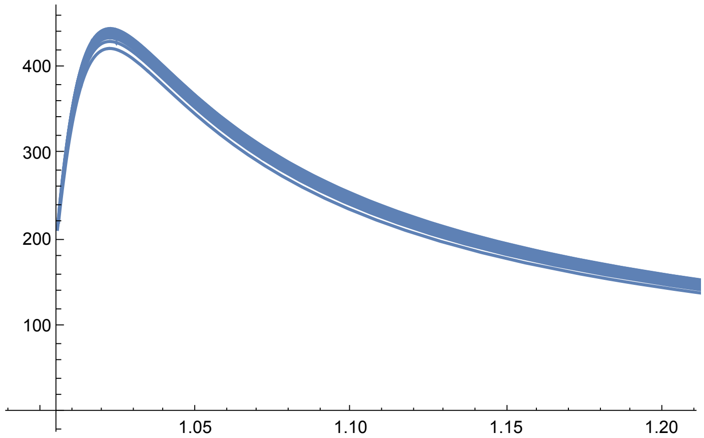
In[∘]:=
```
Show[Table[Peak[p], {p, 0.01, 0.05, 0.01}],
 PlotRange → {{0.95, 2.5}, {0, 0.04}}, AxesOrigin → {1, 0}]
```

Out[∘]=



```
In[*]:= DatePeak[p_] := ParametricPlot[
          {Raverage[p, 2.4, q], s /. CharacteristicsF[p, 2.4, q, 1000][[2]]},
          {q, 0.001, qmax[p, 2.4] - 0.1}, PlotRange → All, AspectRatio → 1 / GoldenRatio]

In[*]:= Show[Table[DatePeak[p], {p, 0.02, 0.2, 0.01}], PlotRange → {{1, 1.5}, Automatic}];
        Show[%, PlotRange → {{1, 1.2}, Automatic}]
```

Out[*]=



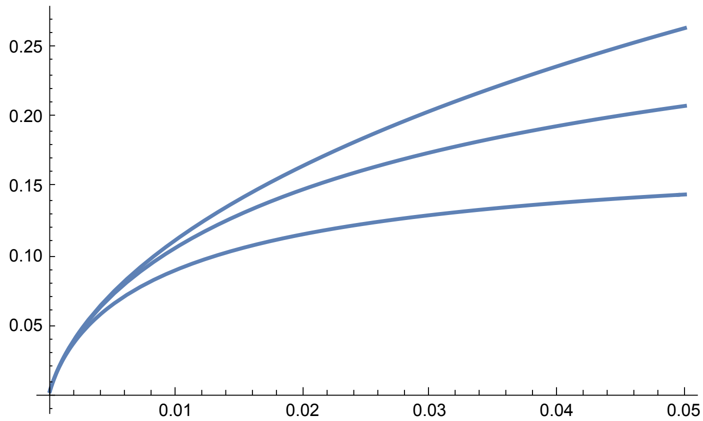
Some numerical values

```
In[*]:= {ST, ET, IT, RT}

Out[*]= {0.999813, 0.0000880531, 0.0000188496, 0.0000803779}

In[*]:= Solve[Raverage[0.02, 2.4, q] == 1.37, q][[1]]
        Solve[Raverage[0.5, q, q] == 1.37, q][[1]]
        f0[1.37]
        Solve[Raverage[0.01, 2.4, q] == 1.37, q][[1]]
        f[0.01, 2.4, q /. %]
        R0
        ───
        0.2
        Solve[Raverage[p, 2.4, 0.2] == 1, p][[1]]

Out[*]= {q → 0.111328}

Out[*]= {q → 1.70073}

Out[*]= 0.486633

Out[*]= {q → 0.0569856}

Out[*]= 0.111957

Out[*]= 11.65

Out[*]= {p → 0.00273117}

In[*]:= Raverage[0.05, 2.4, 0.2]
        R0 / %

Out[*]= 1.50479

Out[*]= 1.54839
```

Epidemic size for a given R_0

In[∘]:=
```
hh[Rzero_] := Plot[f[p, 2.4, q2 /. Solve[Raverage[p, 2.4, q2] == Rzero, q2][[1]]],
   {p, 0, 0.05}, PlotStyle → Thick]
```

In[∘]:=
```
Show[hh[1.1], hh[1.2], hh[1.7], PlotRange → All]
```

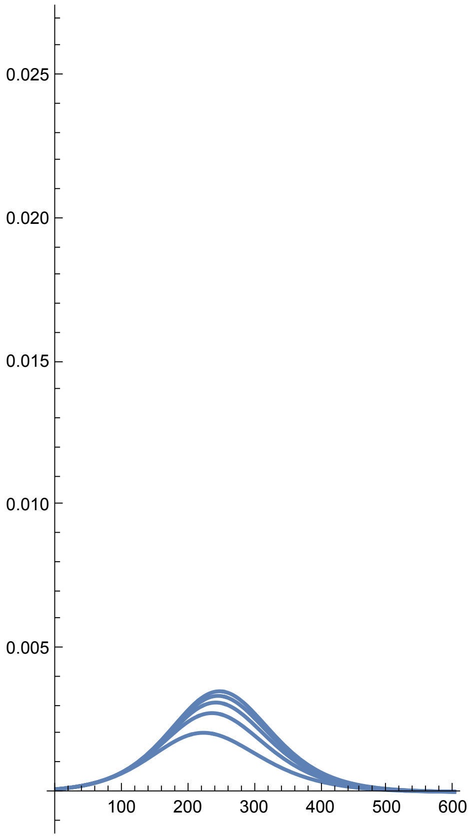
Out[∘]=


Epidemic peak for a given R_0

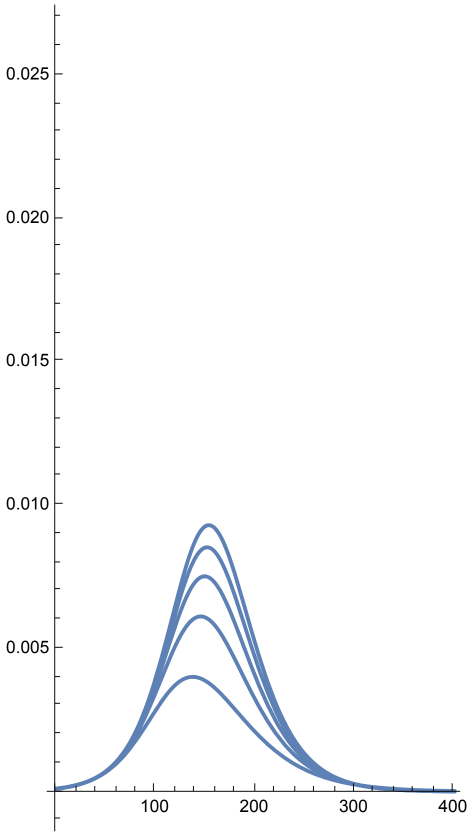
In[∘]:=
```
FixedR0[Rzero_, Tmax_] :=
 Show[Table[FEIRmodif[p, 2.4, q2 /. Solve[Raverage[p, 2.4, q2] == Rzero, q2][[1]],
    Tmax, Thick], {p, 0.01, 0.05, 0.01}],
  AspectRatio → 2, PlotRange → {All, {0, 0.026}}]
```

In[∘]:=
```
FixedR0[1.1, 600]
FixedR0[1.2, 400]
FixedR0[1.7, 200]
```

Out[∘]=


0.025
0.020
0.015
0.010
0.005
100
200
300
400
500
600

Out[∘]=


0.025
0.020
0.015
0.010
0.005
100
200
300
400

Out[ ]=