

$$\text{In[17]:= } mc = \frac{d-2}{d};$$

$$a = 2d \frac{1-m}{m};$$

$$b = 2d(m-mc);$$

$$\text{In[20]:= } \text{delta} = 0.25;$$

$$\text{In[21]:= } \text{Xint} = \text{Simplify}\left[\left\{-\frac{a}{2\sqrt{b+4}} \frac{2m}{(d+2)m-d}, \frac{-a}{b+4} \frac{2m}{(d+2)m-d}\right\}\right];$$

$$\text{In[22]:= } F[m_, d_] :=$$

$$\text{Show}\left[\text{VectorPlot}\left[\left\{2d \frac{1-m}{m} y - 4x, -2d \left(m - \frac{d-2}{d}\right) y\right\}, \left\{x, -\frac{d(1-m)}{(d+2)m-d} - \text{delta}, \frac{d(1-m)}{(d+2)m-d}\right\}, \left\{y, -\frac{2m}{(d+2)m-d} - \text{delta}, \frac{2m}{(d+2)m-d}\right\}, \text{VectorStyle} \rightarrow \text{Brown}\right],$$

$$\text{Plot}\left[\left\{-\frac{2m}{(d+2)m-d} - 3\text{delta}, \frac{2m}{(d+2)m-d} + 2\text{delta}\right\}, \left\{x, -\frac{d(1-m)}{(d+2)m-d} - 3\text{delta}, -\frac{d(1-m)}{(d+2)m-d}\right\}, \text{Filling} \rightarrow \{1 \rightarrow \{2\}\}\right],$$

$$\text{Plot}\left[\left\{-\frac{2m}{(d+2)m-d} - 3\text{delta}, -\frac{2m}{(d+2)m-d}\right\}, \left\{x, -\frac{d(1-m)}{(d+2)m-d}, \frac{d(1-m)}{(d+2)m-d} + 3\text{delta}\right\}, \text{Filling} \rightarrow \{1 \rightarrow \{2\}\}\right],$$

$$\text{ListLinePlot}\left[\left\{\left\{-\frac{d(1-m)}{(d+2)m-d}, -\frac{2m}{(d+2)m-d}\right\}, \left\{\frac{d(1-m)}{(d+2)m-d} (1+\text{delta}), \frac{2m}{(d+2)m-d} (1+\text{delta})\right\}\right\}, \text{PlotStyle} \rightarrow \text{Red}\right],$$

$$\text{ListLinePlot}\left[\left\{\left\{-\frac{d(1-m)}{(d+2)m-d}, \frac{2m}{(d+2)m-d}\right\}, \left\{-\frac{d(1-m)}{(d+2)m-d}, -\frac{2m}{(d+2)m-d}\right\}, \left\{\frac{d(1-m)}{(d+2)m-d} + 3\text{delta}, -\frac{2m}{(d+2)m-d}\right\}\right\}, \text{PlotStyle} \rightarrow \text{Red}\right],$$

$$\text{ListLinePlot}\left[\left\{\left\{0, \frac{2m}{(d+2)m-d}\right\}, \left\{0, -\frac{2m}{(d+2)m-d}\right\}\right\}\right],$$

$$\text{ListLinePlot}\left[\left\{\left\{-\frac{d(1-m)}{(d+2)m-d} - 3\text{delta}, 0\right\}, \left\{\frac{d(1-m)}{(d+2)m-d} + 3\text{delta}, 0\right\}\right\}\right],$$

$$\text{Plot}\left[\left\{\frac{2m}{(d+2)m-d} \left(\left(1 + \frac{x}{\frac{2m}{(d+2)m-d}}\right)^m - 1\right), \frac{2m}{(d+2)m-d} + \text{delta}\right\},$$

$$\left\{x, -\frac{d(1-m)}{(d+2)m-d}, \frac{d(1-m)}{(d+2)m-d} + 3\text{delta}\right\}, \text{Filling} \rightarrow \{1 \rightarrow \{2\}\}\right],$$

$$\text{Plot}\left[\frac{2m}{(d+2)m-d} \left(\left(1 + \frac{x}{\frac{2m}{(d+2)m-d}}\right)^m - 1\right),$$

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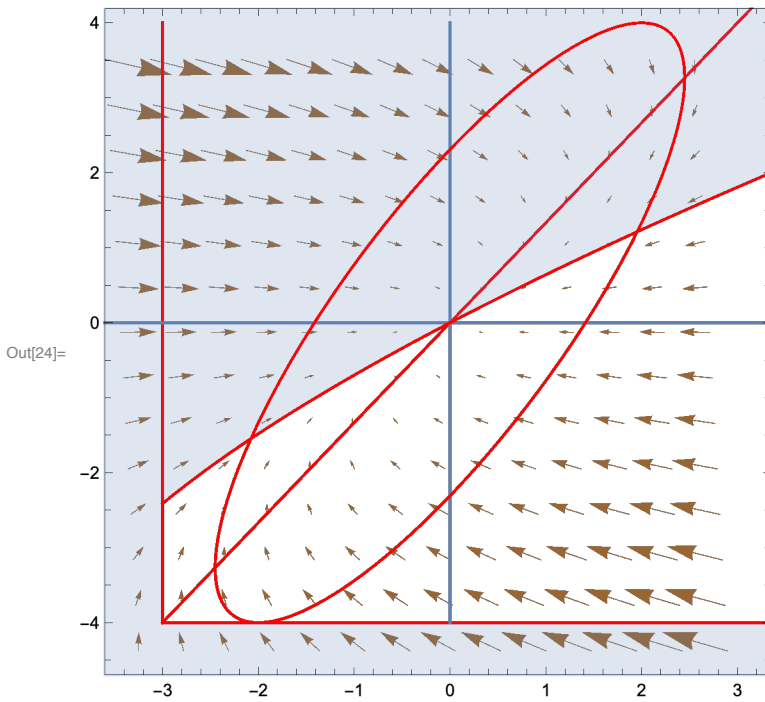
{ x, -  $\frac{d (1 - m)}{(d + 2) m - d}$ ,  $\frac{d (1 - m)}{(d + 2) m - d} + 3 \text{delta}$  }, PlotStyle -> Red],

Plot[ { -  $\frac{1}{d^2 (-1 + m)^2} m \left( 2 d (-1 + m) x + \sqrt{2} m \sqrt{ - \left( (d^2 (2 + d (-1 + m)) (-1 + m)^2 \right. \right.}$ 
 $\left. \left. (d^3 (-1 + m)^3 x^2 + 16 m^2 x^2 + 4 d m (-4 + 3 m + m^2) x^2 + 2 d^2 (-1 + m)^2 \right. \right.}$ 
 $\left. \left. (-1 + 2 (1 + m) x^2) \right) \right) / \left( (4 + d (-1 + m)) m^2 (d (-1 + m) + 2 m^2) \right) \right) \right\},$ 
 $\frac{1}{d^2 (-1 + m)^2} m \left( -2 d (-1 + m) x + \sqrt{2} m \sqrt{ - \left( (d^2 (2 + d (-1 + m)) (-1 + m)^2 \right. \right.}$ 
 $\left. \left. (d^3 (-1 + m)^3 x^2 + 16 m^2 x^2 + 4 d m (-4 + 3 m + m^2) x^2 + 2 d^2 (-1 + m)^2 \right. \right.}$ 
 $\left. \left. (-1 + 2 (1 + m) x^2) \right) \right) / \left( (4 + d (-1 + m)) m^2 (d (-1 + m) + 2 m^2) \right) \right) \right\},$ 
{ x, -  $\frac{d (1 - m)}{(d + 2) m - d}$ ,  $\frac{d (1 - m)}{(d + 2) m - d}$  }, PlotStyle -> Red]]

G[m_, d_] := Show[F[m, d], PlotRange -> { { -  $\frac{d (1 - m)}{(d + 2) m - d} - \text{delta}$ ,  $\frac{d (1 - m)}{(d + 2) m - d}$  },
{ -  $\frac{2 m}{(d + 2) m - d} - \text{delta}$ ,  $\frac{2 m}{(d + 2) m - d} - \text{delta}$  } } ]

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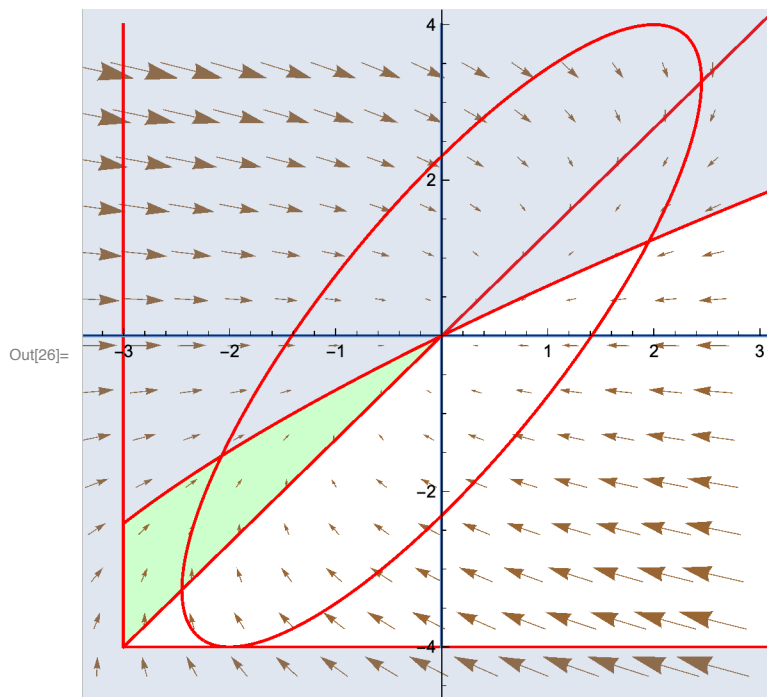
In[24]:= G[2 / 3, 3]



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In[25]:= HA[m_, d_] := Show[Plot[{ $\frac{2m}{(d+2)m-d} \left( \left( 1 + \frac{x}{\frac{2m}{(d+2)m-d}} \right)^m - 1 \right), \frac{2m}{d(1-m)} x$ },
 $\left\{ x, -\frac{d(1-m)}{(d+2)m-d}, 0 \right\}$ , Filling → {1 → {2}}, PlotStyle → Green],
F[m, d], PlotRange → {{ $-\frac{d(1-m)}{(d+2)m-d} - \text{delta}, \frac{d(1-m)}{(d+2)m-d}$ },
 $\left\{ -\frac{2m}{(d+2)m-d} - \text{delta}, \frac{2m}{(d+2)m-d} - \text{delta} \right\}}$ , AspectRatio → 1]
HA[
2 /
3,
3]

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In[27]:= HB[m_, d_, x1_, x2_] :=

Show[Plot[$\left\{-\frac{1}{d^2 (-1+m)^2} m \left(2 d (-1+m) x + \sqrt{2} m \sqrt{-\left(d^2 (2+d (-1+m)) (-1+m)^2\right.}\right.\right.$

$\left.\left.\left(d^3 (-1+m)^3 x^2 + 16 m^2 x^2 + 4 d m (-4+3 m+m^2) x^2 + 2 d^2 (-1+m)^2\right.\right.\right.$

$\left.\left.\left.(-1+2 (1+m) x^2)\right)\right) / \left((4+d (-1+m)) m^2 (d (-1+m)+2 m)^2\right)\right)\right\},$

$\text{Min}\left[\frac{2 m}{(d+2) m-d} \left(\left(1+\frac{x}{\frac{2 m}{(d+2) m-d}}\right)^m - 1\right), \frac{2 m}{d (1-m)} x\right], \{x, x1, x2\},$

Filling → {1 → {2}}, PlotStyle → Green],

Plot[$\left\{-\frac{2 m}{(d+2) m-d}, \text{Min}\left[\frac{2 m}{(d+2) m-d} \left(\left(1+\frac{x}{\frac{2 m}{(d+2) m-d}}\right)^m - 1\right), \frac{2 m}{d (1-m)} x\right]\right\},$

$\{x, x2, \frac{d (1-m)}{(d+2) m-d} + 3 \text{delta}\},$

Filling → {1 → {2}},

PlotStyle → Green], F[m, d],

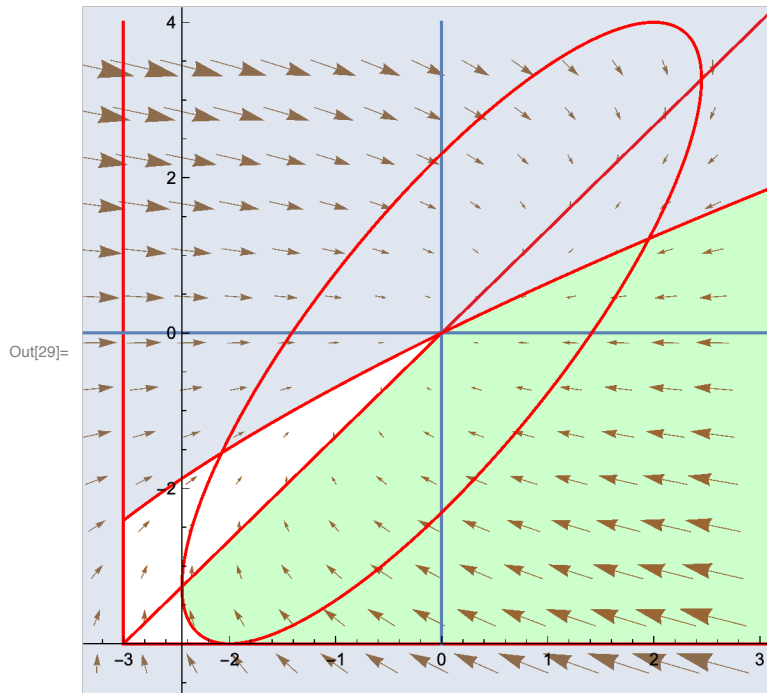
PlotRange → $\left\{\left\{-\frac{d (1-m)}{(d+2) m-d} - \text{delta}, \frac{d (1-m)}{(d+2) m-d}\right\},$

$\left\{-\frac{2 m}{(d+2) m-d} - \text{delta}, \frac{2 m}{(d+2) m-d} - \text{delta}\right\}\right\}, \text{AspectRatio} \rightarrow 1]$

Simplify[Xint /. {m → 2/3, d → 3}]

HB[2/3, 3, %[[1]], %[[2]]]

Out[28]= $\{-\sqrt{6}, -2\}$



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In[30]:= HC[m_, d_, x1_, x2_] := Show[
  Plot[{-  $\frac{2 m}{(d+2) m-d}$ ,  $\frac{2 m}{d (1-m)} x$ },
    {x, -  $\frac{d (1-m)}{(d+2) m-d}$ , x1}, Filling -> {1 -> {2}}, PlotStyle -> Green],
  Plot[{-  $\frac{2 m}{(d+2) m-d}$ , -  $\frac{1}{d^2 (-1+m)^2} m$ 
    ( $2 d (-1+m) x + \sqrt{2} m \sqrt{-((d^2 (2+d (-1+m)) (-1+m)^2 (d^3 (-1+m)^3 x^2 + 16 m^2 x^2 +$ 
       $4 d m (-4+3 m+m^2) x^2 + 2 d^2 (-1+m)^2 (-1+2 (1+m) x^2)) /$ 
       $((4+d (-1+m)) m^2 (d (-1+m) + 2 m^2))}$ ),
    {x, x1, x2}, Filling -> {1 -> {2}}, PlotStyle -> Green], F[
  m,
  d],
  PlotRange -> {{-  $\frac{d (1-m)}{(d+2) m-d} - \text{delta}$ ,  $\frac{d (1-m)}{(d+2) m-d}$ },
    {-  $\frac{2 m}{(d+2) m-d} - \text{delta}$ ,  $\frac{2 m}{(d+2) m-d} - \text{delta}$ }}, AspectRatio -> 1]
Simplify[Xint /. {m -> 2/3, d -> 3}]
HC[2/3, 3, %[[1]], %[[2]]]

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Out[31]= $\{-\sqrt{6}, -2\}$

Out[32]=

