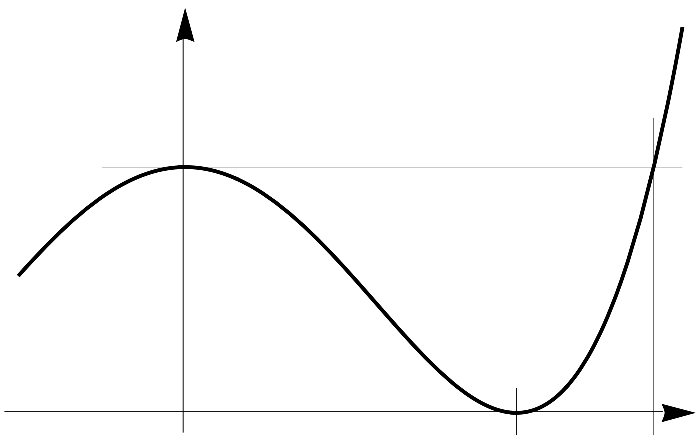
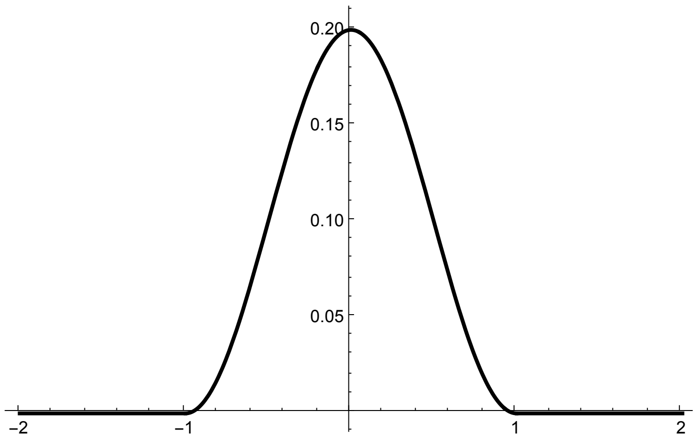
## Figure of Section 1

In[∘]:=
```
Graphics[Arrow[{{-.5, -.25}, {1.5, -.25}}, PlotStyle → {Black, Thin}]];
Show[Plot[1/4 - 1/2 v^2 + 1/4 v^4, {v, -0.5, 1.5}, PlotStyle → {Black, Thick}],
 ListLinePlot[{{-.25, 0.25}, {1.5, 0.25}}, PlotStyle → {Black, Thin}],
 ListLinePlot[{{1, -0.025}, {1, 0.025}}, PlotStyle → {Black, Thin}],
 ListLinePlot[{{√2, -0.025}, {√2, 0.3}}, PlotStyle → {Black, Thin}],
 AxesStyle → Arrowheads[{0.0, 0.05}], Ticks → None]
```



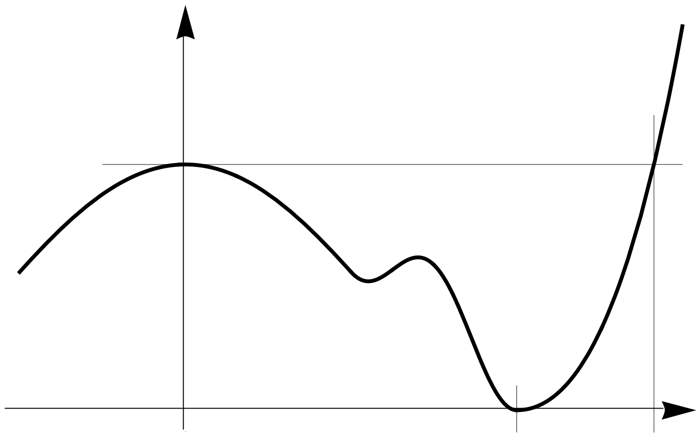
In[∘]:=
```
p[v_] := If[-1 < v < 1, (1 + Cos[π v]) 0.1, 0]
Plot[p[v], {v, -2, 2}, PlotStyle → {Black, Thick}, PlotRange → All]
```

Out[∘]=



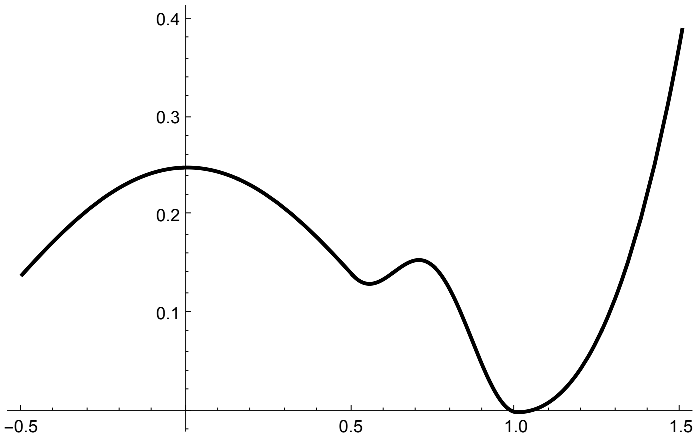
In[∘]:=
```
Graphics[Arrow[{{-.5, -.25}, {1.5, -.25}}, PlotStyle → {Black, Thin}]];
Show[Plot[1/4 - 1/2 v^2 + 1/4 v^4 + 0.5 p[4 (v - 0.75)], {v, -0.5, 1.5}, PlotStyle → {Black,
    Thick}], ListLinePlot[{{-.25, 0.25}, {1.5, 0.25}}, PlotStyle → {Black, Thin}],
 ListLinePlot[{{1, -0.025}, {1, 0.025}}, PlotStyle → {Black, Thin}],
 ListLinePlot[{{√2, -0.025}, {√2, 0.3}}, PlotStyle → {Black, Thin}],
 AxesStyle → Arrowheads[{0.0, 0.05}], Ticks → None]
```

Out[∘]=



In[∘]:=
```
Plot[1/4 - 1/2 v^2 + 1/4 v^4 + 0.5 p[4 (v - 0.75)], {v, -0.5, 1.5}, PlotStyle → {Black, Thick}]
```

Out[∘]=




# On the computations of Section 5.4

## A preliminary computation

In[∘]:=
```
Simplify[Φ'[x]^4 D[Φ[x]/Φ'[x]^2, {x, 2}]]
```

Out[∘]= $-3\,\Phi'[x]^2\,\Phi''[x] + 6\,\Phi[x]\,\Phi''[x]^2 - 2\,\Phi[x]\,\Phi'[x]\,\Phi^{(3)}[x]$

## Computation of K’

In[∘]:=
```
Φ[x_] := x^(m p) - m x^p + m - 1
W[y_] := y^m - m y + m - 1
h[y_] := 1/Sqrt[q] Sqrt[y^m - m y + m - 1]
Res1 = FullSimplify[PowerExpand[(Sqrt[q] h'[y])^2 /. y → x^p]];
Res2 = FullSimplify[PowerExpand[(Φ'[x] / (2 p y^((p-1)/p) Sqrt[Φ[x]]))^2 /. y → x^p]]
FullSimplify[PowerExpand[Res2 - Res1]]
```

Out[∘]= $-\frac{m^2 x^{-2p} \left(x^p - x^{m p}\right)^2}{4 \left(1 - x^{m p} + m \left(-1 + x^p\right)\right)}$

Out[∘]= $0$

## Discussion of the sign of K’

In[∘]:=
```
f[a_, m_, y_, z_] := Simplify[a^2 Simplify[
    2 (-1 + m - m y + y z) ((q - 1) (2 q - 1) z^2 + (q^2 - 3 (p - 1) q + p^2 - 3 (q - 1) p - 2) z +
        (p - 1) (2 p - 1)) - 3 q (q - 1) y (z - 1)^2 (z - (p - 1)/(q - 1)) /. q → m p] /. p → 1/a]
```

In[∘]:=
```
x^4/(m^2 p^2) Simplify[Φ'[x]^4 D[Φ[x]/Φ'[x]^2, {x, 2}]] /.
   {x^(m p) → y^m, x^(2 m p) → y^(2 m), x^p → y, x^(2 p) → y^2, x^((1+m) p) → y^(m+1)};
Simplify[%/y^2 /. {y^m → y z, y^(1+m) → y^2 z, y^(2 m) → y^2 z^2}]
Simplify[% - p^2 f[1/p, m, y, z] /. q → m p]
```

Out[∘]= $-3 m p y (-1+z)^2 (1 - z + p (-1 + m z)) + 2 (-1 + m - m y + y z) \left((-1+z)^2 - 3 p (-1+z) (-1 + m z) + p^2 \left(2 + \left(1 - 6 m + m^2\right) z + 2 m^2 z^2\right)\right)$

Out[∘]= $0$

## The computation of *f*

In[∘]:=
```
f[a, m, y, z];
Resf = Simplify[{%, D[%, a], D[%, {a, 2}] / 2} /. a → 0, Assumptions → m > 1];
MatrixForm[Resf]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -3 m y (-1+z)^2 (-1 + m z) - 2 (1 + m (-1 + y) - y z) \left(2 + \left(1 - 6 m + m^2\right) z + 2 m^2 z^2\right) \\ 3 m y (-1+z)^3 - 6 (-1+z) (-1 + m z) (-1 + m - m y + y z) \\ 2 (-1+z)^2 (-1 + m - m y + y z) \end{pmatrix}$$

## Remarkable points

### *(y,z)=(0,0)*

In[∘]:=
```
Simplify[f[a, m, 0, 0]]
```

Out[∘]= $2\,(-2+a)\,(-1+a)\,(-1+m)$

### *(y,z)=(1,1)*

In[∘]:=
```
Series[f[a, m, 1 + t, (1 + t)^(m-1)], {t, 0, 4}]
Normal[Simplify[Series[Normal[12/((-1 + m)^3 t^4) %], {a, 0, 2}]]]
Simplify[% /. a -> 1/2]
```

Out[∘]= $\frac{1}{12}\left(-2\,m+12\,a\,m-12\,a^2\,m-m^2-24\,a\,m^2+36\,a^2\,m^2+13\,m^3-36\,a^2\,m^3-13\,m^4+24\,a\,m^4+12\,a^2\,m^4+m^5-12\,a\,m^5+2\,m^6\right)t^4+O[t]^5$

Out[∘]= $12\,a^2\,m-12\,a\,m\,(1+m)+m\left(2+7\,m+2\,m^2\right)$

Out[∘]= $m\left(-1+m+2\,m^2\right)$

### *(y,z)=($\gamma_m$,m)*

In[∘]:=
```
Simplify[f[a, m, m^(1/(m-1)), m]/(-1 + m)^3] /. {m^(2 + 1/(-1+m)) -> m^2 γ, m^(m/(-1+m)) -> m γ}
Simplify[% /. a -> 1/2]
```

Out[∘]= $4+2\,a^2+10\,m+4\,m^2-3\,m\,\gamma-3\,m^2\,\gamma+3\,a\,(-2-2\,m+m\,\gamma)$

Out[∘]= $\frac{3}{2}+m^2\,(4-3\,\gamma)+m\left(7-\frac{3\,\gamma}{2}\right)$

## The elimination problem

In[∘]:=
```
Simplify[Solve[f[a, m, y, z] == 0, y], Assumptions → 0 < a < 1/2 && 1 < m < 2][[1]]
Simplify[y D[f[a, m, y, z], y] + (m - 1) z D[f[a, m, y, z], z] /. %,
 Assumptions → 0 < a < 1/2 && 1 < m < 2]
(2 + 2 a^4 (-1 + z)^2 + 3 m z - 10 m^2 z + 3 m^3 z + 2 m^4 z^2 - 9 a^3 (-1 + z) (-1 + m z) +
   2 a^2 (7 - (1 + 12 m + m^2) z + 7 m^2 z^2) - 3 a (3 + (1 - 4 m - 4 m^2 + m^3) z + 3 m^3 z^2));
Simplify[{%, D[%, z], D[%, {z, 2}] / 2} /. z → 0];
Simplify[%[[2]]^2 - 4 %[[1]] × %[[3]]]
TeXForm[%]
```

Out[∘]=
```
{y → (2 (-1 + m) (2 + a^2 (-1 + z)^2 + (1 - 6 m + m^2) z + 2 m^2 z^2 - 3 a (-1 + z) (-1 + m z))) /
   (-2 z (2 + 3 a (-1 + z) + a^2 (-1 + z)^2 + z) + 2 m^3 z (1 + 2 z) - m^2 z
      (9 + 6 a (-1 + z) + 8 z + z^2) + m (1 + 2 a^2 (-1 + z)^2 + 8 z + 9 z^2 + 3 a (-1 + z) (1 + z)^2))}
```

Out[∘]=
```
(2 (-1 + m) m (-1 + z)^3
   (2 + 2 a^4 (-1 + z)^2 + 3 m z - 10 m^2 z + 3 m^3 z + 2 m^4 z^2 - 9 a^3 (-1 + z) (-1 + m z) +
     2 a^2 (7 - (1 + 12 m + m^2) z + 7 m^2 z^2) - 3 a (3 + (1 - 4 m - 4 m^2 + m^3) z + 3 m^3 z^2))) /
 (-2 z (2 + 3 a (-1 + z) + a^2 (-1 + z)^2 + z) + 2 m^3 z (1 + 2 z) - m^2 z (9 + 6 a (-1 + z) + 8 z + z^2) +
   m (1 + 2 a^2 (-1 + z)^2 + 8 z + 9 z^2 + 3 a (-1 + z) (1 + z)^2))
```

Out[∘]=
```
-3 (-1 + a)^2 (a - m)^2 (-1 + m)^2 (-3 + 5 a^2 + 14 m - 3 m^2 - 10 a (1 + m))
```

Out[∘]//TeXForm=
```
-3 (a-1)^2 (m-1)^2 (a-m)^2 \left(5 a^2-10 a (m+1)-3 m^2+14 m-3\right)
```