Projet ANR-08-BLAN-0333-01

CBDif-Fr : collective behaviour and diffusion

Programme 2009-2012

A IDENTIFICATION

Acronyme du projet	CBDif-Fr
Titre du projet	Collective behaviour and diffusion : mathematical models and simulations
Coordinateur du projet (société/organisme)	CNRS
Période du projet (date de début – date de fin)	Du 01/01/2009 au 31/12/2012
Site web du projet, le cas échéant	www.ceremade.dauphine.fr/~dolbeaul/CBDif/

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Date de rédaction	22/02/13

Si différent du rédacteur, indiquer un contact pour le projet	
Civilité, prénom, nom	
Téléphone	
Adresse électronique	

Liste des partenaires présents à la fin du projet (société/organisme et responsable scientifique)	Ceremade, Université Paris-Dauphine

B RESUME CONSOLIDE PUBLIC

B.1 RESUME CONSOLIDE PUBLIC EN FRANÇAIS

Comportement collectif et diffusion : modèles mathématiques et simulations

Aider et fédérer les travaux de modélisation et l'analyse mathématique du comportement collectif en sciences fondamentales et en sciences humaines. Le projet CBDif-Fr avait été proposé pour financer la participation française à un projet européen ESF intitulé CBDif : ce projet, présenté par un réseau de participants français et européens ayant un long passé de collaborations à travers des projets de recherche, contrats bilatéraux et réseaux, était tourné vers l'étude mathématique, analytique et numérique de phénomènes complexes en sociologie, économie, biologie, physique ou ingénierie. Il s'agissait en particulier d'étudier des systèmes composés d'un grand nombre d'individus développant un comportement collectif, et de rendre compte dans ce cadre du rôle de termes de transport de type champ moyen, de termes d'interaction de diverses natures et aussi de différentes sortes de diffusions. Le projet ESF n'a pas été financé, et le projet français CBDif-Fr a dû, à sa mesure, jouer un rôle fédérateur aux niveaux français et européen. Il a permis de structurer la recherche d'une large communauté scientifique et de favoriser l'intégration de jeunes chercheurs dans un domaine de recherche en plein développement.

Méthodes de travail. La compréhension des phénomènes collectifs observés en sciences fondamentales et en sciences humaines nécessite des phases de modélisation, d'analyse mathématique, de calcul scientifique et d'analyse numérique. Le projet CBDif-Fr a participé à

chacune de ces étapes en facilitant les rencontres entre chercheurs français et européens du projet, via

- le financement de séjours de jeunes chercheurs (doctorants, post-doctorants, chercheurs ayant un poste permanent depuis peu) européens dans les laboratoires français , et de jeunes chercheurs français dans des laboratoires européens (1 semaine à 6 mois),
- le financement de la participation de chercheurs français à des congrès internationaux ou programmes thématiques en lien avec le projet,
- la co-organisation de workshops et congrès internationaux, en France ou en Europe, sur les thèmes du projet.

Un site internet dédié

http://www.ceremade.dauphine.fr/~dolbeaul/CBDif/

référence les prépublications des bénéficiaires du projet, et les congrès et workshops organisés ou co-organisés dans le cadre du projet.

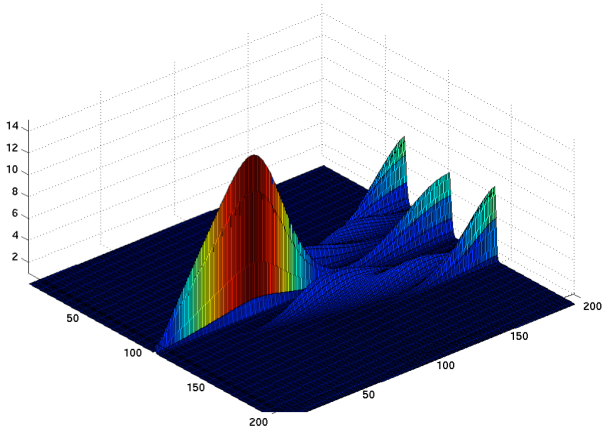
Résultats majeurs du projet

- l'obtention, l'analyse mathématique et numérique du comportement en temps grand de modèles de comportement collectif (J.A. Carrillo, P. Degond, A. Frouvelle, K. Fellner, G. Raoul, etc),
- le développement de méthodes analytiques d'hypocoercivité dans l'étude de la dynamique en temps grand de problèmes dégénérés (J. Dolbeault, S. Mischler, C. Mouhot, C. Schmeiser, C. Villani),
- le lien entre échelles microscopique, cinétique et macroscopique pour des systèmes d'étoiles, bactéries, molécules (limites de champ moyen et hydrodynamique, F. Bolley, S. Mischler, C. Mouhot),
- une approche fonctionnelle de la dynamique de Keller-Segel modélisant l'évolution d'amas de bactéries (A. Blanchet, E. A. Carlen, J.A. Carrillo, J. Dolbeault, J. Campos, etc),
- l'obtention d'inégalités de type Sobolev améliorées par des flots d'évolution non-linéaires (J. Dolbeault, M. Del Pino, G. Toscani, S. Mischler, C. Mouhot, etc).

Production scientifique. Les résultats obtenus ont été exposés lors de séminaires et congrès et ont fait l'objet de plus de 75 articles publiés ou à paraître dans des revues à comité de lecture, d'audience internationale. Le projet CBDif-Fr a permis à ses bénéficiaires (liste en ligne sur le serveur) de développer de nombreuses collaborations scientifiques, en France et en Europe.

Illustration

Dans leur article sur *Existence of nontrivial steady states for populations structured with respect to space and a continuous trait*, A. Arnold, L. Desvillettes, C. Prévost ont montré dans un modèle prenant en compte sélection, mutations, compétition et migration comment la prise en compte de la variable d'espace pouvait conduire à des états d'équilibre non-triviaux, ce qui illustre la complexité des processus de spéciation.



Informations factuelles. Le projet CBDif-Fr est un projet de recherche fondamentale coordonné par Jean Dolbeault, Ceremade, CNRS – Université Paris-Dauphine. Le projet a commencé en janvier 2009 et a duré 48 mois. Il a bénéficié d'une aide ANR de 160 000 € pour un coût complet de l'ordre de 177 602 €.

B.2 RESUME CONSOLIDE PUBLIC EN ANGLAIS

Collective behavior and diffusion : mathematical models and simulations

To help and federate modelling and mathematical analysis of collective behaviors in fundamental and human sciences. The CBDif-Fr project was initially intended to support the French participation to an ESF project entitled CBDif : this project, prepared by a network of French and European researchers with an important past experience of collaborations through research projects, bilateral contracts and networks, was oriented towards the mathematical study, both analytical and numerical, of complex phenomena in sociology, economics, biology, physics and engineering. The purpose was centered on the study of systems made of a large number of individuals and the emergence of a collective behavior, taking into account mean field transport terms, interactions of various natures, and various kinds of diffusion. The ESF project has not been funded and so, the French CBDif-Fr project has played a federative role, at its own scale. As a result, it has been structuring research objectives of a large scientific community and has been a very efficient tool to integrate young researchers in a rapidly developing field of research.

Methods. The understanding of collective phenomena in fundamental and human sciences requires various phases of modelling, mathematical analysis, scientific computing and numerical analysis. The CBDif-Fr project has been involved at each stage of this program by facilitating meeting of French and European collaborators to the project, through

- the support of visits for young European researchers (PhD students, post-docs, researchers recently recruited) in French labs, and the visit of young French researchers in European labs (1 week up to 6 months),
- the support of French researchers to international meetings and thematic programs linked with the objectives of the project,
- the co-organisation of workshops and international meetings, in France or in Europe, on the themes of the project.

A web site has been set up

http://www.ceremade.dauphine.fr/~dolbeaul/CBDif/

in order to collect prepublications of people supported by CBDif-Fr and announce workshops and international meetings organized or co-organized within the framework of the project.

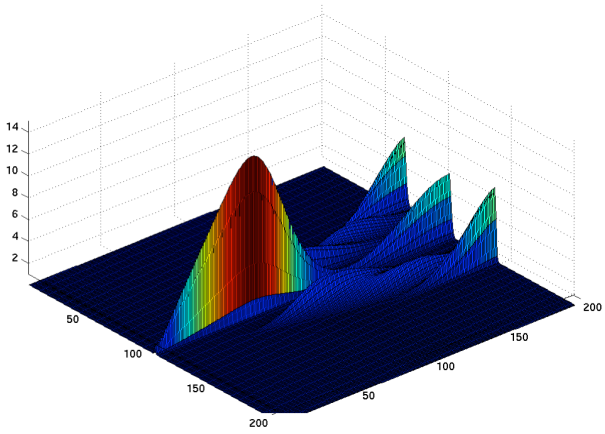
Main results

- mathematical and numerical analysis of large time behavior of models of collective behavior (J.A. Carrillo, P. Degond, A. Frouvelle, K. Fellner, G. Raoul, etc),
- new methods for hypercoercivity approaches in the study of large time dynamics of degenerate problems (J. Dolbeault, S. Mischler, C. Mouhot, C. Schmeiser, C. Villani),
- link betwen microscopi, kinetic and macroscopic scales for systems of gravitating stars, bacteriae, molecules (mean field and hydrodynamic limits, F. Bolley, S. Mischler, C. Mouhot),
- functional approaches of the Keller-Segel dynamics for the modelling of aggregates of bacteriae (A. Blanchet, E. A. Carlen, J.A. Carrillo, J. Dolbeault, J. Campos, etc),
- improved Sobolev type inequalities and nonlinear evolution flows (J. Dolbeault, M. Del Pino, G. Toscani, S. Mischler, C. Mouhot, etc).

Scientific production. Results have been exposed during meetings and seminar and have resulted in more than 75 research articles published or to appear in journals with editorial committees, of international audience. People supported by CBDif-Fr (on-line list on the web server) have been able to develop various scientific collaborations, in France and in Europe.

Illustration

In their paper on *Existence of nontrivial steady states for populations structured with respect to space and a continuous trait*, A. Arnold, L. Desvillettes, C. Prévost have shown in a model taking into account selection, mutations, competition and migration how taking into account the space variable could lead to non-trivial steady states, which illustrates the complexity of speciation processes.



Facts. The CBDif-Fr project is a fundamental research project coordinated by Jean Dolbeault, Ceremade, CNRS – Université Paris-Dauphine. The project started in january 2009 has been 48 months long. It has received a support of 160 000 € from the ANR for a total cost of the order of 177 602 €.

C MEMOIRE SCIENTIFIQUE

Mémoire scientifique confidentiel : non

C.1 RESUME DU MEMOIRE

Le projet CBDif-Fr avait pour but d'aider et de fédérer les travaux de modélisation et l'analyse mathématique du comportement collectif en sciences fondamentales et en sciences humaines au travers d'un large réseau français et plus largement européen. Tourné vers l'étude mathématique, analytique et numérique de phénomènes complexes en sociologie,

économie, biologie, physique ou ingénierie, il avait en particulier l'objectif d'étudier des systèmes composés d'un grand nombre d'individus développant un comportement collectif, et de rendre compte dans ce cadre du rôle de termes de transport de type champ moyen, de termes d'interaction de diverses natures et aussi de différentes sortes de diffusions. Le projet CBDif-Fr a permis de structurer la recherche d'une large communauté scientifique et de favoriser l'intégration de jeunes chercheurs dans un domaine de recherche en plein développement.

La compréhension des phénomènes collectifs observés en sciences fondamentales et en sciences humaines nécessite des phases de modélisation, d'analyse mathématique, de calcul scientifique et d'analyse numériques. Le projet CBDif-Fr a participé à chacune de ces étapes en facilitant les rencontres entre chercheurs français et européens du projet, via

- le financement de séjours de jeunes chercheurs (doctorants, post-doctorants, chercheurs ayant un poste permanent depuis peu) européens dans les laboratoires français , et de jeunes chercheurs français dans des laboratoires européens (1 semaine à 6 mois),
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référence les prépublications des bénéficiaires du projet, et les congrès et workshops organisés ou co-organisés dans le cadre du projet.

Bien qu'il s'agisse d'un choix quelque peu arbitraire, on peut mettre en avant les résultats suivants :

- l'obtention, l'analyse mathématique et numérique du comportement en temps grand de modèles de comportement collectif (J.A. Carrillo, P. Degond, A. Frouvelle, K. Fellner, G. Raoul, etc),
- le développement de méthodes analytiques d'hypocoercivité dans l'étude de la dynamique en temps grand de problèmes dégénérés (J. Dolbeault, S. Mischler, C. Mouhot, C. Schmeiser, C. Villani),
- le lien entre échelles microscopique, cinétique et macroscopique pour des systèmes d'étoiles, bactéries, molécules (limites de champ moyen et hydrodynamique, F. Bolley, S. Mischler, C. Mouhot),
- une approche fonctionnelle de la dynamique de Keller-Segel modélisant l'évolution d'amas de bactéries (A. Blanchet, E. A. Carlen, J.A. Carrillo, J. Dolbeault, J. Campos, etc),
- l'obtention d'inégalités de type Sobolev améliorées par des flots d'évolution non-linéaires (J. Dolbeault, M. Del Pino, G. Toscani, S. Mischler, C. Mouhot, etc).

Deux annexes jointes à ce rapport regroupent (Annexe A) la liste des publications à laquelle renvoient les références ci-dessous, et (Annexe B) un document avec les résumés des articles. Une analyse des publications montre grossièrement comment se sont répartis les efforts.

1) En ce qui concerne les modèles de sciences du vivant, on retiendra une série de travaux sur les phénomènes d'agrégation, avec en particulier des études poussées sur les questions d'invasion, de répartition spatiale, de distribution de traits en théorie de l'évolution [2, 5] avec une étude de modèles à champ moyen attractifs-répulsifs [7, 8, 20] et aussi l'étude de l'agrégation spatiale (chemotactisme, relaxation) [10, 22, 23] ou

de l'agrégation en vitesse (swarming, flocking, etc) [12, 13, 18, 30, 31, 32, 33, 34, 54, 56, 57, 74], cette dernière thématique ayant été extrêmement active tant du point de vue de la modélisation que de l'étude analytique ou théorique. Bien qu'ils suscitent un fort intérêt chez les mathématiciens, les modèles de neurosciences pour la vision [26] ou pour les modèles de prise de décision [26, 27] n'ont pour l'instant débouché que sur un nombre beaucoup plus limité de publications.

2) Pour les modèles de transport avec applications en physique et en ingénierie, on retiendra plusieurs études numériques et théoriques sur les champs magnétiques intenses et les plasmas tels qu'on les rencontre dans les tokamaks [16, 17], quelques travaux sur les modèles quantiques [1, 55] et sur la gravitation [1, 21], liés en particulier aux propriétés communes entre gravité et chemotactisme. On notera aussi un résultat sur la conservation de la masse (absence de gélation) dans certains modèles de coagulation-fragmentation [24, 25]. Signalons enfin [76] qui porte sur l'amortissement Landau et a eu un très gros impact sur la communauté des chercheurs concernée par CBDif, même si ce travail n'a pas encore eu de suite.
3) Les autres travaux sont en général transverses ou relèvent plus des méthodes ou de la théorie mathématique des équations aux dérivées partielles, et c'est donc sous cet angle qu'ils seront présentés. Notons toutefois qu'ils sont très largement motivés par les diverses questions listées dans le projet initial, et qu'ils permettent de répondre à une large gamme de questions qui étaient ouvertes il y a quatre ans. Tout d'abord un effort important a été consenti pour déterminer des espaces fonctionnels optimaux en vue de l'utilisation de méthodes d'entropie et calculer les meilleures constantes dans les inégalités fonctionnelles correspondant à des structures d'entropie – dissipation d'entropie [9, 22, 23, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 51, 52, 53, 75] tout en soulevant au passage des questions nouvelles, comme les questions de brisure de symétrie. Dans le cas de modèles avec diffusions dégénérées, de nouvelles méthodes pour l'étude de l'hypo-coercivité ont été développées [48, 49] tandis que de nombreux cas de convergence exponentielle vers des profils stationnaires ou auto-similaires (ou d'explosion) ont été établis [6, 14, 15, 19, 37, 48, 49, 50, 66, 67, 68, 77] enrichissant d'autant la compréhension de ces phénomènes fondamentaux. Dans cette direction, on peut aussi insister sur les avancées sur le théorème H et la propagation du chaos [66, 71, 72, 73] et mentionner aussi [28, 29] qui relèvent des méthodes générales de l'étude des équations cinétiques. Particulièrement importantes pour la modélisation, les méthodes de passage à la limite hydrodynamique ou à la limite de diffusion, en lien avec les méthodes micro-macro ou le couplage entre modèles fluides et cinétiques ont fait l'objet d'un certain nombre de travaux [3, 30, 31, 64, 65, 70]. Enfin, une part importante de l'effort de recherche a été consacrée à des travaux numériques, essentiels pour l'interaction avec les domaines d'application mais dont il serait difficile de rendre compte de manière exhaustive dans ce rapport. En se limitant aux articles portant sur les schémas numériques pour les équations cinétiques, on peut en particulier repérer [58, 59, 60, 61, 62, 63] sans que cette liste soit complète pour autant, du fait de très nombreuses contributions dispersées dans des articles dont ce n'est pas l'objet principal.

C.2 ENJEUX ET PROBLEMATIQUE, ETAT DE L'ART

Le projet Collective Behaviour & Diffusion: mathematical models and simulations (CBDif) a concerné une large communauté scientifique dont le dénominateur commun est de s'intéresser à des modèles basés sur des équations aux dérivées partielles non-linéaires,

d'utiliser des approches multi-échelles, et d'être fortement motivée par des applications dans des domaines aussi divers que la sociologie, l'économie, les sciences du vivant, la physique et les sciences de l'ingénieur. Cette apparente diversité recouvre en fait un ensemble de méthodes communes; pour que la communauté scientifique concernée ait la possibilité d'améliorer ses méthodes ou d'en mettre au point de nouvelles, il est essentiel qu'elle puisse se réunir régulièrement pour partager ses avancées. C'est le rôle qui a été tenu par les workshops et congrès mentionnés plus loin dans ce rapport. Du fait de son orientation vers la modélisation, la communauté concernée est particulièrement attractive pour les jeunes qui sont nombreux à faire des thèses sur des sujets liés aux thèmes mis en avant dans CBDif. Pour faciliter leur insertion et surtout pour leur permettre d'acquérir une compétence de mathématicien dans les divers domaines vers lesquels ils appliquent les mathématiques, il est absolument crucial qu'ils soient exposés à une variété d'outils qu'ils ne trouveront pas nécessairement dans le champ d'applications sur lequel ils ont travaillé, au moins dans la phase initiale de leur formation. En les confrontant régulièrement à des démarches analogues à la leur, au contact de chercheurs plus avancés et maîtrisant des techniques difficiles comme celles qui ont été développées pour les équations cinétiques au cours des trente dernières années (par exemple), par des collaborations avec d'autres mathématiciens appliqués ou par des séjours dans des groupes de recherche ou des laboratoires partenaires de CBDif, ils ont eu l'opportunité de renforcer ce qui fait leur spécificité de mathématiciens appliqués et de se confronter à des recherches menées au meilleur niveau mondial.

Les échanges d'informations (workshops) et la formation des jeunes chercheurs ne doivent pas pour autant occulter le fait que le projet a été constitué autour d'un certain nombre de thèmes qui ont drainé un effort de recherche considérable au cours des dernières années. Il convient tout d'abord de rappeler que des calculs multidimensionnels de phénomènes complexes multi-échelles sont désormais à portée sans nécessairement recourir à des moyens de calcul exceptionnels; les outils sophistiqués d'analyse non-linéaire permettent d'approfondir notre connaissance de modèles d'une complexité grandissante; les calculs numériques sont réintroduits dans le processus de modélisation et permettent de saisir des mécanismes fins qui souvent ne peuvent être étudiés par l'expérience. Dans ce contexte, la modélisation des systèmes composés d'un grand nombre d'individus se comportant de manière collective, qu'ils soient représentés par des EDO stochastiques ou des EDP cinétiques, est aujourd'hui encore un domaine en pleine extension; cette approche permet aussi de construire des modèles de dynamique moyenne, décrits par des modèles continus de diffusion ou hydrodynamiques, et qui ont été abondamment étudiés. Des phénomènes important ont été observés au cours des quatre années écoulées et ont conduit à des études plus théoriques, par exemple sur la notion de symétrie et de brisure de symétrie ou dans la compréhension fine des modes de relaxation et des comportements en temps grand des systèmes. La combinaison du comportement collectif, de phénomènes de transport linéaires ou non-linéaires et de diffusion non-linéaire est la difficulté principale de l'analyse asymptotique des modèles dont l'étude a été proposée dans le cadre de CBDif. Sans entrer dans le détail, on peut néanmoins insister sur le fait que les thèmes concernés par CBDif ont connu un certain nombre d'avancées dont les principales ont été listées dans la section C.1.

C.3 APPROCHE SCIENTIFIQUE ET TECHNIQUE

Le but de ce projet a été d'aider et de fédérer les travaux de recherche sur la modélisation, puis l'analyse mathématique et les simulations numériques, de phénomènes complexes, multi-échelles, présents en économie, sociologie, biologie, physique et ingénierie. Ceci a pu être mené à bien par

1. l'organisation de trois workshops annuels de 2 ou 3 jours, centrés sur des questions très spécifiques, qui ont permis la rencontre des spécialistes mondiaux de ces questions.

- 6-7/10/2009, Institut Henri Poincaré, Paris

Titre : Concentration en vitesse et en espace dans les modèles cinétiques et diffusifs, chemotaxis, gravitation, swarming.

Organisation : J. A. Carrillo et J. Dolbeault
Le programme et les présentations ont été mis en ligne à l'adresse:
http://www.ceremade.dauphine.fr/~dolbeaul/CBDif/workshops/IHP2009/

- 1-3/9/2010, Université de Toulouse 1

Titre : Optimal transport and Kinetics Applied to Socio-Economics

Comité scientifique: G. Buttazzo (Pise), P. Degond et J.-C. Rochet (Toulouse).

Organisation : A. Blanchet, F. Bolley et S. Cordier, en coordination avec le projet EVaMEF (étude de modèles variationnels appliqués en économie et en finance).

Le programme a été mis en ligne à l'adresse:

http://math.univ-tlse1.fr/OKASE/

- 12-13/1/2012, Université Paris-Dauphine Titre : Functional Inequalities and PDE in the Life Sciences Organisation : J. Dolbeault et P. Markowich
  Le programme et les présentations ont été mis en ligne à l'adresse:

http://www.ceremade.dauphine.fr/~dolbeaul/CBDif/workshops/Paris2012/

2. la participation à l'organisation de congrès sur (au moins en partie) les thèmes du projet

- 31/5-4/6/2010 : Emerging Topics in Dynamical Systems and Partial Differential Equations

DSPDEs'10, Barcelone

http://www.siam.org/meetings/dspdes/index.php

- 5-7/1/2011: Colloque « Equations cinétiques, géométrie, probabilités, transport: dialogues et perspectives », Congrès en l'honneur de Cédric Villani, lauréat de la médaille Fields 2010, et participant au projet CBDif-Fr, École normale supérieure, Paris

http://www.math.ens.fr/VillaniFields2010/

Organisation : F. Bolley, A. Figalli (Austin), C. Mouhot

- 9-11/2/2011: Colloque « Boltzmann equation: mathematics, modeling and simulations, in memory of Carlo Cercignani », Institut Henri Poincaré, Paris

http://www.ceremade.dauphine.fr/~bolley/Conference_C.Cercignani.html

Organisation: F. Bolley, L. Desvillettes, S. Lorenzani (Milan) et C. Mouhot.

3. le soutien aux participants de divers congrès et programmes thématiques. Rappelons que l'un des objectifs du projet est de faire participer les jeunes chercheurs affiliés au projet à un certain nombre de workshops organisés au niveau européen en relation avec CBDif-Fr.

- 16/08-22/12/2010 : Special semester of the Isaac Newton Institute for Mathematical Sciences, Cambridge, « Partial Differential Equations in Kinetic Theories »

Organisation: J. A. Carrillo, S. Jin et P. A. Markowich,

http://www.newton.ac.uk/programmes/KIT/index.html

- 21-25/2/2011 : Conference Applied Mathematics from waves to fluids, in honor of Claude Bardos, Université de Nice.

http://math.unice.fr/~iooss/Colloque%20AMWF%20-%20Bardos.html

Claude Bardos est un membre actif de la communauté scientifique concernée par CBDif.

- 14-18/3/2011: Kinetic models of classical and quantum particle systems, a conference in the memory of Naoufel Ben Abdallah

http://www.math.univ-toulouse.fr/CongresNBA/

Naoufel Ben Abdallah est décédé accidentellement durant l'été 2010 ; il a eu une influence considérable sur le projet CBDif, directement et au travers de ses nombreux élèves.

- 6-8/6/2011: BIOMAT 2011, Perspectives in Mathematics and Life Sciences, Grenade, http://www.ugr.es/~kinetic/biomat/
- 20-24/6/2011: Fronts et EDP non linéaires, colloque en l'honneur de Henri Berestycki, Ecole normale supérieure, Paris

http://www.math.univ-toulouse.fr/berestycki2011/

- 17-22/7/2011 : BIRS workshop, Banff, Canada: Geometric properties of solutions of nonlinear PDEs and their applications

http://www.birs.ca/events/2011/5-day-workshops/11w5019

- 18-22/6/2012 : Optimal Transport (to) Orsay, Paris

http://www.math.u-psud.fr/~otto/

- 2-7/9/2012 : ERC conference, Applied Partial Differential Equations in Physics, Biology and Social Sciences: Classical and Modern Perspectives, Centre de Recerca Matemàtica, Barcelone

http://www.esf.org/index.php?id=9153

4. les échanges et en particulier les séjours de longue durée pour de jeunes chercheurs.

Le site internet

http://www.ceremade.dauphine.fr/~dolbeaul/CBDif/

du projet recense les prépublications des membres du projet, sur les thèmes du projet, et les congrès organisés par les membres du projet. Comme c'est l'usage en mathématiques, les résultats obtenus se mesurent principalement aux publications dont la liste est donnée en annexe (voir aussi le document concernant les résumés des travaux).

C.4 EXPLOITATION DES RESULTATS

Publications.

C.5 DISCUSSION

Le programme initial présentait un très large éventail de sujets, et déterminer un pourcentage de réussite n'a pas beaucoup de sens. Un des écueils consiste en la multiplication des modèles. Le fait de pouvoir structurer des classes de modèles par leurs

propriétés mathématiques est un atout essentiel des mathématiciens appliqués par rapport à leurs collègues d'autres disciplines. Un projet comme CBDif est essentiel pour maintenir le lien entre acteurs et insérer les jeunes chercheurs dans une démarche avec un haut niveau d'exigence scientifique et de rigueur.

C.6 Conclusions

Pour conclure ce rapport essayons de tracer quelques perspectives en termes de problématiques scientifiques, du point de vue des mathématiques appliquées. Les quelques idées ci-dessous ressortent de nombreuses discussions entre membres du projet mais constituent un choix en grande partie arbitraire et qui engage bien entendu plus le rédacteur du rapport que les autres participants. En particulier, axer la discussion sur les méthodes d'entropie, leurs limites et leurs extensions, ne rend pas compte de la richesse et de la diversité des problèmes qui ont été étudiés dans les quatre années écoulées.

Les méthodes d'entropie ont connu un succès important dans les dernières années dans l'étude des équations aux dérivées partielles appliquées, du fait qu'il s'agit souvent de méthodes robustes, nécessitant peu de régularité, et bien adaptées aux problèmes multi-échelles. Le fait qu'un certain nombre d'équations d'évolution puissent être vues comme flot gradient de l'entropie pour une distance appropriée a constitué un guide précieux dans l'étude de bien des problèmes et fournit un cadre en passe de devenir classique pour la démonstration de résultats d'existence. Dans les dernières années, on a de plus acquis une compréhension détaillée des asymptotiques en temps grand et du cadre fonctionnel dans lequel il est possible d'approximer un certain nombre de problèmes non-linéaires autour de profils stationnaires ou asymptotiques.

Toutefois les méthodes d'entropie ne s'appliquent pas à tous les modèles, loin s'en faut et comprendre comment aller au-delà pour certains des modèles étudiés dans le cadre de CBDif-Fr devrait constituer une priorité dans les années à venir. Dans cet état d'esprit, voici quelques éléments prospectifs de thèmes de recherche qui pourraient se révéler important dans les années à venir:

- généralisation des méthodes d'entropie à une certaine classe de systèmes d'équations d'évolution, en particulier pour des modèles à diffusion croisée en biologie,
- méthodes d'entropie pour la capture de phénomènes d'explosion (blow-up) dans des équations de champ moyen avec interactions non-locales,
- équations d'évolution permettant de relier plusieurs inégalités fonctionnelles ou possédant plusieurs fonctionnelles de Lyapunov, comme dans le cas récent du modèle de Keller-Segel critique,
- approaches unificatrices liant structures de flot gradient et théorie des grandes déviations ou de la concentration de la mesure, susceptibles d'avoir un impact important sur les fondements des méthodes d'entropie elles-même, et versions raffinées des méthodes d'entropie permettant de caractériser des régimes intermédiaires ou de dégager des régimes de convergence accélérée vers des états asymptotiques,
- méthodes d'entropie discrètes et application en analyse numérique.

C.7 References

Les informations relatives au projet CBDif-Fr sont disponibles sur le site internet dédié

http://www.ceremade.dauphine.fr/~dolbeaul/CBDif/

D LISTE DES LIVRABLES

Publications.

E IMPACT DU PROJET

E.1 INDICATEURS D'IMPACT

Nombre de publications et de communications (à détailler en E.2)

Les publications référencées ci-dessous (uniquement destinées à des journaux internationaux à comité de lecture) ont été soit mises en ligne sur le serveur du projet par l'un de leurs auteurs, soit explicitement renseignées dans le serveur HAL. Le projet était destiné à une large communauté scientifique, ce qui rend le comptage des publications difficile. Rappelons que le projet a été structuré de la manière suivante:

Organizing team : J. Dolbeault (CEREMADE, CNRS & Université Paris-Dauphine), dolbeaul@ceremade.dauphine.fr (25 %), F. Bolley (CEREMADE, CNRS & Université Paris-Dauphine), bolley@ceremade.dauphine.fr (20 %), C. Mouhot (DAMTP, University of Cambridge), C.Mouhot@dpmms.cam.ac.uk (20 %)

Scientific committee : Y. Brenier, J.A. Carrillo, F. Castella, P. Degond, M. Esteban, F. Filbet, T. Goudon

Participants : dans la mesure où le projet s'insérait dans une large collaboration européenne et concernait prioritairement des jeunes chercheurs, établir une liste de participants fermée au début du projet aurait été une absurdité. Outre la liste initiale jointe au projet, ont donc été considérés comme participants les chercheurs qui ont participé et contribué aux workshops ou qui ont réalisé des visites de longue durée dans les laboratoires partenaires dont la liste avait été définie lors du dépôt du projet. La liste des participants (à peu près 80 personnes) est tenue à jour sur le serveur, à l'adresse :

http://www.ceremade.dauphine.fr/~dolbeaul/CBDif/participants/index.php

Pour le comptage des publications, nous avons retenu la règle suivante : sont comptabilisés les publications dont l'un au moins des auteurs a été « participant » (au sens ci-dessus) ; les publications multi-partenaires sont celles qui font intervenir au moins deux participants.

		Publications multipartenaires	Publications monopartenaires
International	Revues à comité de lecture	55	20

Autres valorisations scientifiques (à détailler en E.3)

Non applicable.

E.2 LISTE DES PUBLICATIONS ET COMMUNICATIONS

Voir liste en Annexe A et les résumés des articles en Annexe B.

E.3 LISTE DES ELEMENTS DE VALORISATION

Non applicable.

ANNNEXE A
COLLECTIVE BEHAVIOUR & DIFFUSION: MATHEMATICAL MODELS AND SIMULATIONS

LISTE DES PUBLICATIONS

WARNING.– The list below may slightly differ from the one which is on the web server at

http://www.ceremade.dauphine.fr/∼dolbeaul/CBDif/publi/index.php

because it corresponds to published versions or paper which acknowledge the support of CBDif but whose preprint have not been made available on the server of the project.

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[3] T. Allemand. The incompressible Euler limit of the Boltzmann equation for a gas of fermions. Prépublication.

[4] A. Athanassoulis, T. Paul, F. Pezzotti, M. Pulvirenti. Strong Semiclassical Approximation of Wigner Functions for the Hartree Dynamics. *Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei* 22, 4, 525-552, 2011.

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[8] D. Balagué, J. A. Carrillo, T. Laurent, G. Raoul. Dimensionality of Local Minimizers of the Interaction Energy. Prépublication.

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ANNNEXE B
COLLECTIVE BEHAVIOUR & DIFFUSION: MATHEMATICAL MODELS AND SIMULATIONS

PUBLICATIONS (AVEC RÉSUMÉS)

WARNING.– The list below may slightly differ from the one which is on the web server at

http://www.ceremade.dauphine.fr/∼dolbeaul/CBDif/publi/index.php

because it corresponds to published versions or paper which acknowledge the support of CBDif but whose preprint have not been made available on the server of the project.

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[1] G. L. Aki, J. Dolbeault, C. Sparber. Thermal effects in gravitational Hartree systems. *Ann. H. Poincaré* 12, 6, 2011, 1055-1079.

We consider the non-relativistic Hartree model in the gravitational case, i.e. with attractive Coulomb-Newton interaction. For a given mass, we construct stationary states with non-zero temperature by minimizing the corresponding free energy functional. It is proved that minimizers exist if and only if the temperature of the system is below a certain threshold(possibly infinite), which itself depends on the specific choice of the entropy functional. We also investigate whether the corresponding minimizers are mixed or pure quantum states and characterize a positive critical temperature above which mixed states appear.

[2] M. Alfaro, J. Coville, G. Raoul. Travelling waves in a nonlocal equation as a model for a population structured by a space variable and a phenotypical trait. Prépublication.

We consider a nonlocal reaction-diffusion equation as a model for a population structured by a space variable and a phenotypical trait. To sustain the possibility of invasion in the case where an underlying principal eigenvalue is negative, we investigate the existence of travelling wave solutions. We identify a minimal speed $c^* > 0$, and prove the existence of waves when $c \geq c^*$ and the non existence when $0 \leq c$.

[3] T. Allemand. The incompressible Euler limit of the Boltzmann equation for a gas of fermions. Prépublication.

We are interested in the hydrodynamic limit of the Boltzmann equation for a gas of fermions in the incompressible Euler regime. We use the relative entropy method as improved by Saint-Raymond. Our result is analogous to what is obtained in the classical case, but the treatment is slightly complicated by the cubic nonlinearity of the collision operator.

[4] A. Athanassoulis, T. Paul, F. Pezzotti, M. Pulvirenti. Strong Semiclassical Approximation of Wigner Functions for the Hartree Dynamics. *Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei* 22, 4, 525-552, 2011.

We consider the Wigner equation corresponding to a nonlinear Schroedinger evolution of the Hartree type in the semiclassical limit $\hbar \to 0$. Under appropriate assumptions on the initial data and the interaction potential, we show that the Wigner function is close in L^2 to its weak limit, the solution of the corresponding Vlasov equation. The strong approximation allows the construction of semiclassical operator-valued observables, approximating their quantum counterparts in Hilbert-Schmidt topology. The proof makes use of a pointwise-positivity manipulation, which seems necessary in working with the L^2 norm and the precise form of the

nonlinearity. We employ the Husimi function as a pivot between the classical probability density and the Wigner function, which – as it is well known – is not pointwise positive in general.

[5] A. Arnold, L. Desvillettes, C. Prévost. Existence of nontrivial steady states for populations structured with respect to space and a continuous trait. *Comm. Pure Appl. Anal.* 11, 1, 83-96, 2012.

We prove the existence of nontrivial steady states to reaction-diffusion equations with a continuous parameter appearing in selection/mutation/competition/migration models for populations, which are structured both with respect to space and a continuous trait.

[6] A. Arnold, I. M. Gamba, M. P. Gualdani, S. Mischler, C.Mouhot, C. Sparber. The Wigner-Fokker-Planck equation: Stationary states and large time behavior. *Math. Mod. Meth. Appl. Sci.* 22, 11, 2012.

We consider the linear Wigner-Fokker-Planck equation subject to confining potentials which are smooth perturbations of the harmonic oscillator potential. For a certain class of perturbations we prove that the equation admits a unique stationary solution in a weighted Sobolev space. A key ingredient of the proof is a new result on the existence of spectral gaps for Fokker-Planck type operators in certain weighted L^2-spaces. In addition we show that the steady state corresponds to a positive density matrix operator with unit trace and that the solutions of the time-dependent problem converge towards the steady state with an exponential rate.

[7] D. Balagué, J. A. Carrillo, T. Laurent, G. Raoul. Nonlocal interactions by repulsive-attractive potentials: radial in/stability. A paraître dans *Physica D.*

In this paper, we investigate nonlocal interaction equations with repulsive-attractive radial potentials. Such equations describe the evolution of a continuum density of particles in which they repulse each other in the short range and attract each other in the long range. We prove that under some conditions on the potential, radially symmetric solutions converge exponentially fast in some transport distance toward a spherical shell stationary state. Otherwise we prove that it is not possible for a radially symmetric solution to converge weakly toward the spherical shell stationary state. We also investigate under which condition it is possible for a non-radially symmetric solution to converge toward a singular stationary state supported on a general hypersurface. Finally we provide a detailed analysis of the specific case of the repulsive-attractive power law potential as well as numerical results. We point out the the conditions of radial ins/stability are sharp.

[8] D. Balagué, J. A. Carrillo, T. Laurent, G. Raoul. Dimensionality of Local Minimizers of the Interaction Energy. Prépublication.

In this work we consider local minimizers (in the topology of transport distances) of the interaction energy associated to a repulsive-attractive potential. We show how the imensionality of the support of local minimizers is related to the repulsive strength of the potential at the origin.

[9] J.-P. Bartier, A. Blanchet, J. Dolbeault, M. Escobedo. Improved intermediate asymptotics for the heat equation. *Appl. Math. Lett.* 24, 1, 76-81, 2011.

This letter is devoted to results on intermediate asymptotics for the heat equation. We study the convergence towards a stationary solution in self-similar variables. By assuming the equality of some moments of the initial data and of the stationary solution, we get improved convergence rates using entropy / entropy-production methods. We establish the equivalence of the exponential decay of the entropies with new, improved functional inequalities in restricted classes of functions. This letter is the counterpart in a linear framework of a recent work on fast diffusion equations, see [Bonforte-Dolbeault-Grillo-Vazquez]. Results extend to the case of a Fokker-Planck equation with a general confining potential.

[10] P. Biler, L. Corrias, J. Dolbeault. Large mass self-similar solutions of the parabolic-parabolic Keller-Segel model of chemotaxis. *J. Math. Biol.* 63, 1, 1-32, 2011.

In two space dimensions, the parabolic-parabolic Keller–Segel system shares many properties with the parabolic-elliptic Keller–Segel system. In particular, solutions globally exist in both cases as long as their mass is less than 8π. However, this threshold is not as clear in the parabolic-parabolic case as it is in the parabolic-elliptic case, in which solutions with mass above 8π always blow up. Here we study forward self-similar solutions of the parabolic-parabolic Keller–Segel system and prove that, in some cases, such solutions globally exist even if their total mass is above 8π, which is forbidden in the parabolic-elliptic case.

[11] R. Blossey, J.-F. Bodart, A. Devys, T. Goudon, P. Lafitte. Signal propagation in the MAPK cascade: role of bistability and ultrasensitivity for a mixed problem. *J. Math. Biol.* 64, 1-2, 1-39, 2012.

The MAPK signaling cascade is nowadays understood as a network module highly conserved across species. Its main function is to transfer a signal arriving at the plasma membrane to the cellular interior, the nucleus. Current understanding of 'how' this is achieved involve the notions of ultrasensitivity and bistability which relate to the nonlinear dynamics of the biochemical network, ignoring spatial aspects. Much less, indeed, is so far known about the propagation of the signal through the cytoplasm. In this work we formulate, starting from a Michaelis-Menten model for the MAPK cascade in *Xenopus* oocytes, a reaction-diffusion model of the cascade. We study this model in one space dimension. Basing ourselves on previous general results on reaction diffusion models, we particularly study for our model the conditions for signal propagation. We show that the existence of a propagating front depends sensitively on the initial and boundary conditions at the plasma membrane. Possible biological consequences of this finding are discussed.

[12] F. Bolley, J. A. Cañizo, J. A. Carrillo. Stochastic mean-field limit: non-Lipschitz forces and swarming. *Math. Mod. Meth. Appl. Sci.* 21, 2179-2210, 2011.

We consider general stochastic systems of interacting particles with noise which are relevant as models for the collective behavior of animals, and rigorously prove that in the mean-field limit the system is close to the solution of a kinetic PDE. Our aim is to include models widely studied in the literature such as the Cucker-Smale model, adding noise to the behavior of individuals. The difficulty, as compared to the classical case of globally Lipschitz potentials, is that in several models the interaction potential between particles is only locally Lipschitz, the local Lipschitz constant growing to infinity with the size of the region considered. With this in mind, we present an extension of the classical theory for globally Lipschitz interactions, which works for only locally Lipschitz ones.

[13] F. Bolley, J. A. Cañizo, J. A. Carrillo. Mean-field limit for the stochastic Vicsek model. *Appl. Math. Letters* 25, 339-343, 2012.

We consider the continuous version of the Vicsek model with noise, proposed as a model for collective behavior of individuals with a fixed speed. We rigorously derive the kinetic mean-field partial differential equation satisfied when the number N of particles tends to infinity, quantifying the convergence of the law of one particle to the solution of the PDE. For this we adapt a classical coupling argument to the present case in which both the particle system and the PDE are defined on a surface rather than on the whole space. As part of the study we give existence and uniqueness results for both the particle system and the PDE.

[14] F. Bolley, A. Guillin, F. Malrieu. Trend to equilibrium and particle approximation for a weakly selfconsistent Vlasov-Fokker-Planck equation. *Math. Model. Numer. Anal.* 44, 5, 867-884, 2010.

We consider a Vlasov-Fokker-Planck equation governing the evolution of the density of interacting and diffusive matter in the space of positions and velocities. We use a probabilistic interpretation to obtain convergence towards equilibrium in Wasserstein distance with an explicit exponential rate. We also prove a propagation of chaos property for an associated particle system, and give rates on the approximation of the solution by the particle system. Finally, a transportation inequality for the distribution of the particle system leads to quantitative deviation bounds on the approximation of the equilibrium solution of the equation by an empirical mean of the particles at given time.

[15] M. Bonforte, J. Dolbeault, G. Grillo, J.-L. Vázquez. Sharp rates of decay of solutions to the nonlinear fast diffusion equation via functional inequalities. *Proc. Natl. Acad. Sci. USA* 107, 38, 16459-16464, 2010.

The goal of this note is to state the optimal decay rate for solutions of the nonlinear fast diffusion equation and, in self-similar variables, the optimal convergence rates to Barenblatt self-similar profiles and their generalizations. It relies on the identification of the optimal constants in some related Hardy-Poincaré inequalities and concludes a long series of papers devoted to generalized entropies, functional inequalities and rates for nonlinear diffusion equations.

[16] M. Bostan, C. Caldini-Queiros. Finite Larmor radius approximation for collisional magnetic confinement. Part II: the Fokker-Planck-Landau equation. Prépublication.

The subject matter of this paper concerns the derivation of the finite Larmor radius approximation, when collisions are taken into account. Several studies are performed, corresponding to different collision kernels. The main motivation consists in computing the gyroaverage of the Fokker-Planck-Landau operator, which plays a major role in plasma physics. We show that the new collision operator enjoys the usual physical properties; the averaged kernel balances the mass, momentum, kinetic energy and dissipates the entropy.

[17] M. Bostan, C. Caldini-Queiros. Finite Larmor radius approximation for collisional magnetic confinement. Part I: the linear Boltzmann equation. Prépublication.

The subject matter of this paper concerns the derivation of the finite Larmor radius approximation, when collisions are taken into account. Several studies are performed, corresponding to different collision kernels. The main motivation consists in computing the gyroaverage of the Fokker-Planck-Landau operator, which plays a major role in plasma physics. We show that the new collision operator enjoys the usual physical properties; the averaged kernel balances the mass, momentum, kinetic energy and dissipates the entropy.

[18] M. Bostan, J. A. Carrillo. Asymptotic fixed-speed reduced dynamics for kinetic equations in swarming. A paraître dans *Math. Mod. Meth. Appl. Sci.*

We perform an asymptotic analysis of general particle systems arising in collective behavior in the limit of large self-propulsion and friction forces. These asymptotics impose a fixed speed in the limit, and thus a reduction of the dynamics to a sphere in the velocity variables. The limit models are obtained by averaging with respect to the fast dynamics. We can include all typical effects in the applications: short-range repulsion, long-range attraction, and alignment. For instance, we can rigorously show that the Cucker-Smale model is reduced to the Vicsek model without noise in this asymptotic limit. Finally, a formal expansion based on the reduced dynamics allows us to treat the case of diffusion. This technique follows closely the gyroaverage method used when studying the magnetic confinement of charged particles. The main new mathematical difficulty is to deal with measure solutions in this expansion procedure.

[19] M. Bostan, C. Negulescu. Mathematical models for strongly magnetized plasmas with mass disparate particles. *Disc. Cont. Dyn. Syst. B* 15, 3, 513-544, 2011.

The controlled fusion is achieved by magnetic confinement : the plasma is confined into toroidal devices called tokamaks, under the action of strong magnetic fields. The particle motion reduces to advection along the magnetic lines combined to rotation around the magnetic lines. The rotation around the magnetic lines is much faster than the parallel motion and efficient numerical resolution requires homogenization procedures. Moreover the rotation period, being proportional to the particle mass, introduces very different time scales in the case when the plasma contains disparate particles; the electrons turn much faster than the ions, the ratio between their cyclotronic periods being the mass ratio of the electrons with respect to the ions. The subject matter of this paper concerns the mathematical study of such plasmas, under the action of strong magnetic fields. In particular, we are interested in the limit models when the small parameter, representing the mass ratio as well as the fast cyclotronic motion, tends to zero.

[20] A. Calsina, S. Cuadrado, L. Desvillettes, G. Raoul. Asymptotics of steady states of a selection mutation equation for small mutation rate. Prépublication.

We consider a selection mutation equation for the density of individuals with respect to a continuous phenotypic evolutionary trait in which the competition term for an individual of a given trait depends on the traits of all the other individuals giving then an infinite dimensional nonlinearity. Mutation is modelled by means of an integral operator. We prove existence of steady states and we show that their asymptotic profile, when the mutation rate goes to zero, is a Cauchy distribution.

[21] J. Campos, M. Del Pino, J. Dolbeault. Relative equilibria in continuous stellar dynamics. *Comm. Math. Phys.* 300, 3, 765-788, 2010.

We study a three dimensional continuous model of gravitating matter rotating at constant angular velocity. In the rotating reference frame, by a finite dimensional reduction, we prove the existence of non radial stationary solutions whose supports are made of an arbitrarily large number of disjoint compact sets, in the low angular velocity and large scale limit. At first order, the solutions behave like point particles, thus making the link with the relative equilibria in N-body dynamics.

[22] J. Campos, J. Dolbeault. A functional framework for the Keller-Segel system: Logarithmic Hardy-Littlewood-Sobolev and related spectral gap inequalities. *C. R. Math. Acad. Sci. Paris* 350, 21-22, 949-954, 2012.

This note is devoted to several inequalities deduced from a special form of the logarithmic Hardy-Littlewood-Sobolev, which is well adapted to the characterization of stationary solutions of a Keller-Segel system written in self-similar variables, in case of a subcritical mass. For the corresponding evolution problem, such functional inequalities play an important role for identifying the rate of convergence of the solutions towards the stationary solution with same mass.

[23] J. Campos, J. Dolbeault. Asymptotic estimates for the parabolic-elliptic Keller-Segel model in the plane. Prépublication.

We investigate the large-time behavior of the solutions of the two-dimensional Keller-Segel system in self-similar variables, when the total mass is subcritical, that is less than 8π after a proper adimensionalization. It was known from previous works that all solutions converge to stationary solutions, with exponential rate when the mass is small. Here we remove this restriction and show that the rate of convergence measured in relative entropy is exponential for any mass in the subcritical range, and independent of the mass. The proof relies on symmetrization techniques, which are adapted from a paper of J.I. Diaz, T. Nagai, and J.-M. Rakotoson, and allow us to establish uniform estimates for L^p norms of the solution. Exponential convergence is obtained by the mean of a linearization in a space which is defined consistently with relative entropy estimates and in which the linearized evolution operator is self-adjoint. The core of proof relies on several new spectral gap estimates which are of independent interest.

[24] J. A. Cañizo, L. Desvillettes, K. Fellner. Regularity and mass conservation for discrete coagulation-fragmentation equations with diffusion. *Ann. Inst. H. Poincaré, Anal. non-lin.* 27, 2, 639-654, 2010.

We present a new a-priori estimate for discrete coagulation-fragmentation systems with size-dependent diffusion within a bounded, regular domain confined by homogeneous Neumann boundary conditions. Following from a duality argument, this a-priori estimate provides a global L^2 bound on the mass density and was previously used, for instance, in the context of reaction-diffusion equations. In this paper we demonstrate two lines of applications for such an estimate: On the one hand, it enables to simplify parts of the known existence theory and allows to show existence of solutions for generalised models involving collision-induced, quadratic fragmentation terms for which the previous existence theory seems difficult to apply. On the other hand and most prominently, it proves mass conservation (and thus the absence of gelation) for almost all the coagulation coefficients for which mass conservation is known to hold true in the space homogeneous case.

[25] J. A. Cañizo, L. Desvillettes, K. Fellner. Absence of Gelation for Models of Coagulation-Fragmentation with Degenerate Diffusion. *Nuovo Cimento C*, 33, 1, 79-86, 2010.

We show in this work that gelation does not occur for a class of discrete coagulation-fragmentation models with size-dependent diffusion. With respect to a previous work, we do not assume here that the diffusion rates of clusters are bounded below. The proof uses a duality argument first devised for reaction-diffusion systems with a finite number of equations.

[26] J. A. Carrillo, S. Cordier, S. Mancini. A decision-making Fokker-Planck model in computational neuroscience. *J. Math. Biology* 63, 801-830, 2011.

Minimal models for the explanation of decision-making in computational neuroscience are based on the analysis of the evolution for the average firing rates of two interacting neuron populations. While these models typically lead to multi-stable scenario for the basic derived dynamical systems, noise is an important feature of the model taking into account finite-size effects and robustness of the decisions. These stochastic dynamical systems can be analyzed by studying carefully their associated Fokker-Planck partial differential equation. In particular, we discuss the existence, positivity and uniqueness for the solution of the stationary equation, as well as for the time evolving problem. Moreover, we prove convergence of the solution to the the stationary state representing the probability distribution of finding the neuron families in each of the decision states characterized by their average firing rates. Finally, we propose a numerical scheme allowing for simulations performed on the Fokker-Planck equation which are in agreement with those obtained recently by a moment method applied to the stochastic differential system. Our approach leads to a more detailed analytical and numerical study of this decision-making model in computational neuroscience.

[27] J. A. Carrillo, S. Cordier, S. Mancini. One dimensional Fokker-Planck reduced dynamics of decision making models in Computational Neuroscience. *Comm. Math. Sci.* 11, 523-540, 2013.

We study a Fokker-Planck equation modelling the firing rates of two interacting populations of neurons. This model arises in computational neuroscience when considering, for example, bistable visual perception problems and is based on a stochastic Wilson-Cowan system of differential equations. In a previous work, the slow-fast behavior of the solution of the Fokker-Planck equation has been highlighted. Our aim is to demonstrate that the complexity of the model can be drastically reduced using this slow-fast structure. In fact, we can derive a one-dimensional Fokker-Planck equation that describes the evolution of the solution along the so-called slow manifold. This permits to have a direct efficient determination of the equilibrium state and its effective potential, and thus to investigate its dependencies with respect to various parameters of the model. It also allows to obtain information about the time escaping behavior. The results obtained for the reduced 1D equation are validated with those of the original 2D equation both for equilibrium and transient behavior.

[28] J. A. Carrillo, R.-J. Duan, A. Moussa. Global Classical Solutions Close to Equilibrium to the Vlasov-Euler-Fokker-Planck System. *Kin. Rel. Mod.* 4, 227-258, 2011.

We are concerned with the global well-posedness of a two-phase flow system arising in the modelling of fluid-particle interactions. This system consists of the Vlasov-Fokker-Planck equation for the dispersed phase particles coupled to the incompressible Euler equations modelling a dense phase uid through the friction forcing. Global existence of classical solutions to the Cauchy problem in the whole space is established when initial data is a small smooth perturbation of a constant equilibrium state, and moreover an algebraic rate of convergence of solutions toward equilibrium is obtained under additional conditions on initial data. The proof is based on the macro-micro decomposition and Kawashima's hyperbolic-parabolic dissipation argument. This result is generalized to the periodic case, when particles are in the torus, improving the rate of convergence to exponential.

[29] A. Champmartin, L. Desvillettes, J. Mathiaud. A BGK-type model for inelastic Boltzmann equations with internal energy. *Riv. Mat. Univ. Parma (N. S)* 1, 2, 271-305, 2010.

We introduce a model of inelastic collisions for droplets in a spray, leading to a specific Boltzmann kernel. Then, we build caricatures of this kernel of BGK type, in which the behavior of the first moments of the solution of the Boltzmann equation (that is, mass, momentum, directional temperatures, variance of the internal energy) are mimicked. The quality of these caricatures is tested numerically at the end of the paper.

[30] P. Degond, J.-G. Liu. Hydrodynamics of self-alignment interactions with precession and derivation of the Landau-Lifschitz-Gilbert equation. *Math. Mod. Meth. Appl. Sci.* 22, 1, 2012.

We consider a kinetic model of self-propelled particles with alignment interaction and with precession about the alignment direction. We derive a hydrodynamic system for the local density and velocity orientation of the particles. The system consists of the conservative equation for the local density and a non-conservative equation for the orientation. First, we assume that the alignment interaction is purely local and derive a first order system. However, we show that this system may lose its hyperbolicity. Under the assumption of weakly non-local interaction, we derive diffusive corrections to the first order system which lead to the combination of a heat flow of the harmonic map and Landau-Lifschitz-Gilbert dynamics. In the particular case of zero self-propelling speed, the resulting model reduces to the phenomenological Landau-Lifschitz-Gilbert equations. Therefore the present theory provides a kinetic formulation of classical micromagnetization models and spin dynamics.

[31] P. Degond, J.-G. Liu, S. Motsch, V. Panferov. Hydrodynamic models of self-organized dynamics: derivation and existence theory. A paraître dans *Meth. Anal. Appl.*

This paper is concerned with the derivation and analysis of hydrodynamic models for systems of self-propelled particles subject to alignment interaction and attraction-repulsion. The starting point is the kinetic model considered in earlier work of Degond & Motsch with the addition of an attraction-repulsion interaction potential. Introducing different scalings than in Degond & Motsch, the non-local effects of the alignment and attraction-repulsion interactions can be kept in the hydrodynamic limit and result in extra pressure, viscosity terms and capillary force. The systems are shown to be symmetrizable hyperbolic systems with viscosity terms. A local-in-time existence result is proved in the 2D case for the viscous model and in the 3D case for the inviscid model. The proof relies on the energy method.

[32] P. Degond, S. Motsch. A macroscopic model for a system of swarming agents using curvature control. *J. Stat. Phys.* 143, 685-714, 2011.

In this paper, we study the macroscopic limit of a new model of collective displacement. The model, called PTWA, is a combination of the Vicsek alignment model (Vicsek et al. in Phys. Rev. Lett. 75(6):1226-1229, 1995) and the Persistent Turning Walker (PTW) model of motion by curvature control (Degond and Motsch in J. Stat. Phys. 131(6):989-1021, 2008; Gautrais et al. in J. Math. Biol. 58(3):429-445, 2009). The PTW model was designed to fit measured trajectories of individual fish (Gautrais et al. in J. Math. Biol. 58(3):429-445, 2009). The PTWA model (Persistent Turning Walker with Alignment) describes the displacements of agents which modify their curvature in order to align with their neighbors. The derivation of its macroscopic limit uses the non-classical notion of generalized collisional invariant introduced in (Degond and Motsch in Math. Models Methods Appl. Sci. 18(1):1193-1215, 2008). The macroscopic limit of the PTWA model involves two physical quantities, the density and the mean velocity of individuals. It is a system of hyperbolic type but is non-conservative due to a geometric constraint on the velocity. This system has the same form as the macroscopic limit of the Vicsek model (Degond and Motsch in Math. Models Methods Appl. Sci. 18(1):1193-1215, 2008) (the "Vicsek hydrodynamics") but for the expression of the model coefficients. The numerical computations show that the numerical values of the coefficients are very close. The "Vicsek Hydrodynamic model" appears in this way as a more generic macroscopic model of swarming behavior as originally anticipated.

[33] P. Degond, L. Navoret, R. Bon, D. Sanchez. Congestion in a macroscopic model of self-driven particles modeling gregariousness. *J. Stat. Phys.* 138, 85-125, 2010.

We analyze a macroscopic model with a maximal density constraint which describes short range repulsion in biological systems. This system aims at modeling finite-size particles which cannot overlap and repel each

other when they are too close. The parts of the fluid where the maximal density is reached behave like incompressible fluids while lower density regions are compressible. This paper investigates the transition between the compressible and incompressible regions. To capture this transition, we study a one-dimensional Riemann problem and introduce a perturbation problem which regularizes the compressible-incompressible transition. Specific difficulties related to the non-conservativity of the problem are discussed.

[34] P. Degond, T. Yang. Diffusion in a continuum model of self-propelled particles with alignment interaction. *Math. Mod. Meth. Appl. Sci.* 20, 1459-1490, 2010.

In this paper, we provide the $O(\epsilon)$ corrections to the hydrodynamic model derived by Degond and Motsch from a kinetic version of the model by Vicsek & coauthors describing flocking biological agents. The parameter ϵ stands for the ratio of the microscopic to the macroscopic scales. The $O(\epsilon)$ corrected model involves diffusion terms in both the mass and velocity equations as well as terms which are quadratic functions of the first order derivatives of the density and velocity. The derivation method is based on the standard Chapman-Enskog theory, but is significantly more complex than usual due to both the non-isotropy of the fluid and the lack of momentum conservation.

[35] M. del Pino, J. Dolbeault. The Euclidean Onofri inequality in higher dimensions. A paraître dans *Int. Math. Res. Notices*.

The classical Onofri inequality in the two-dimensional sphere assumes a natural form in the plane when transformed via stereographic projection. We establish an optimal version of a generalization of this inequality in the d-dimensional Euclidean space for any $d \geq 2$, by considering the endpoint of a family of optimal Gagliardo-Nirenberg interpolation inequalities. Unlike the two-dimensional case, this extension involves a rather unexpected Sobolev-Orlicz norm, as well as a probability measure no longer related to stereographic projection.

[36] M. Del Pino, J. Dolbeault, S. Filippas, A. Tertikas. A logarithmic Hardy inequality. *J. Funct. Anal.* 259, 8, 2045-2072, 2010.

We prove a new inequality which improves on the classical Hardy inequality in the sense that a nonlinear integral quantity with super-quadratic growth, which is computed with respect to an inverse square weight, is controlled by the energy. This inequality differs from standard logarithmic Sobolev inequalities in the sense that the measure is neither Lebesgue's measure nor a probability measure. All terms are scale invariant. After an Emden-Fowler transformation, the inequality can be rewritten as an optimal inequality of logarithmic Sobolev type on the cylinder. Explicit expressions of the sharp constant, as well as minimizers, are established in the radial case. However, when no symmetry is imposed, the sharp constants are not achieved among radial functions, in some range of the parameters.

[37] L. Desvillettes, C. Mouhot, C. Villani. Celebrating Cercignani's conjecture for the Boltzmann equation. *Kin. Rel. Mod.* 4, 1, 277-294, 2011.

Cercignani's conjecture assumes a linear inequality between the entropy and entropy production functionals for Boltzmann's nonlinear integral operator in rarefied gas dynamics. Related to the field of logarithmic Sobolev inequalities and spectral gap inequalities, this issue has been at the core of the renewal of the mathematical theory of convergence to thermodynamical equilibrium for rarefied gases over the past decade. In this review paper, we survey the various positive and negative results which were obtained since the conjecture was proposed in the 1980s.

[38] J. Dolbeault. Sobolev and Hardy-Littlewood-Sobolev inequalities: duality and fast diffusion. *Math. Res. Lett.* 18, 6, 1037-1050, 2011.

In the euclidean space, Sobolev and Hardy-Littlewood-Sobolev inequalities can be related by duality. In this paper, we investigate how to relate these inequalities using the flow of a fast diffusion equation in dimension $d \geq 3$. The main consequence is an improvement of Sobolev's inequality when $d \geq 5$, which involves the

various terms of the dual Hardy-Littlewood-Sobolev inequality. In dimension $d = 2$, Onofri's inequality plays the role of Sobolev's inequality and can also be related to its dual inequality, the logarithmic Hardy-Littlewood-Sobolev inequality, by a super-fast diffusion equation.

[39] J. Dolbeault, M. J. Esteban. A scenario for symmetry breaking in Caffarelli-Kohn-Nirenberg inequalities. A paraître dans *J. Num. Math.*

The purpose of this paper is to explain the phenomenon of symmetry breaking for optimal functions in functional inequalities by the numerical computations of some well chosen solutions of the corresponding Euler-Lagrange equations. For many of those inequalities it was believed that the only source of symmetry breaking would be the instability of the symmetric optimizer in the class of all admissible functions. But recently, it was shown by an indirect argument that for some Caffarelli-Kohn-Nirenberg inequalities this conjecture was not true. In order to understand this new symmetry breaking mechanism we have computed the branch of minimal solutions for a simple problem. A reparametrization of this branch allows us to build a scenario for the new phenomenon of symmetry breaking. The computations have been performed using *Freefem++*.

[40] J. Dolbeault, M. J. Esteban. About existence, symmetry and symmetry breaking for extremal functions of some interpolation functional inequalities. In Nonlinear partial Differential Equations. The Abel Symposium 2010. Oslo.

This article is devoted to a review of some recent results on existence, symmetry and symmetry breaking of optimal functions for Caffarelli-Kohn-Nirenberg (CKN) and weighted logarithmic Hardy (WLH) inequalities. These results have been obtained in a series of papers in collaboration with M. del Pino, S. Filippas, M. Loss, G. Tarantello and A. Tertikas. Here we put the highlights on a symmetry breaking result: extremals of some inequalities are not radially symmetric in regions where the symmetric extremals are linearly stable. Special attention is paid to the study of the critical cases for (CKN) and (WLH).

[41] J. Dolbeault, M. J. Esteban. Extremal functions in some interpolation inequalities: Symmetry, symmetry breaking and estimates of the best constants. In Mathematical results in quantum physics, 178-182, World Sci., Hackensack, 2011.

This contribution is devoted to a review of some recent results on existence, symmetry and symmetry breaking of optimal functions for Caffarelli-Kohn-Nirenberg and weighted logarithmic Hardy inequalities. These results have been obtained in a series of papers in collaboration with M. del Pino, S. Filippas, M. Loss, G. Tarantello and A. Tertikas and are presented from a new viewpoint.

[42] J. Dolbeault, M.J. Esteban. Extremal functions for Caffarelli-Kohn-Nirenberg and logarithmic Hardy inequalities. *Proc. Roy. Soc. Edinburgh Sect. A* 142, 4, 745-767, 2012.

We consider a family of Caffarelli-Kohn-Nirenberg interpolation inequalities and weighted logarithmic Hardy inequalities which have been obtained recently as a limit case of the first ones. We discuss the ranges of the parameters for which the optimal constants are achieved by extremal functions. The comparison of these optimal constants with the optimal constants of Gagliardo-Nirenberg interpolation inequalities and Gross' logarithmic Sobolev inequality, both without weights, gives a general criterion for such an existence result in some particular cases.

[43] J. Dolbeault, M. J. Esteban, M. Kowalczyk, M. Loss. Sharp interpolation inequalities on the sphere: New methods and consequences. *Chin. Ann. Math. Ser. B* 34, 1, 99-112, 2013.

These notes are devoted to various considerations on a family of sharp interpolation inequalities on the sphere, which in dimension two and higher interpolate between Poincaré, logarithmic Sobolev and critical Sobolev (Onofri in dimension two) inequalities. We emphasize the connexion between optimal constants and spectral properties of the Laplace-Beltrami operator on the sphere. We shall address a series of related observations and give proofs based on symmetrization and the ultraspherical setting.

[44] J. Dolbeault, M. Esteban, A. Laptev. Spectral estimates on the sphere. Prépublication.

In this article we establish optimal estimates for the first eigenvalue of Schrödinger operators on the d-dimensional unit sphere. These estimates depend on L^p norms of the potential, or of its inverse, and are equivalent to interpolation inequalities on the sphere. We also characterize a semi-classical asymptotic regime and discuss how our estimates on the sphere differ from those on the Euclidean space.

[45] J. Dolbeault, M. J. Esteban, M. Loss. Symmetry of extremals of functional inequalities via spectral estimates for linear operators. *J. Math. Phys.* 53 (P), 2012.

We prove new symmetry results for the extremals of the Caffarelli-Kohn-Nirenberg inequalities in any dimension larger or equal than 2 , in a range of parameters for which no explicit results of symmetry were previously known.

[46] J. Dolbeault, M. J. Esteban, M. Loss. Nonlinear flows and rigidity results on compact manifolds. Prépublication.

This paper is devoted to *rigidity* results for some elliptic PDEs and related interpolation inequalities of Sobolev type on smooth compact connected Riemannian manifolds without boundaries. Rigidity means that the PDE has no other solution than the constant one at least when a parameter is in a certain range. This parameter can be used as an estimate for the best constant in the corresponding interpolation inequality. Our approach relies in a nonlinear flow of porous medium / fast diffusion type which gives a clear-cut interpretation of technical choices of exponents done in earlier works. We also establish two integral criteria for rigidity that improve upon known, pointwise conditions, and hold for general manifolds without positivity conditions on the curvature. Using the flow, we are also able to discuss the optimality of the corresponding constant in the interpolation inequalities.

[47] J. Dolbeault, M. J. Esteban, G. Tarantello, A. Tertikas. Radial symmetry and symmetry breaking for some interpolation inequalities. *Calc. Var. Part. Diff. Eq.* 42, 3-4, 461-485, 2011.

We analyze the radial symmetry of extremals for a class of interpolation inequalities known as Caffarelli-Kohn-Nirenberg inequalities, and for a class of weighted logarithmic Hardy inequalities which appear as limiting cases of the first ones. In both classes we show that there exists a continuous surface that splits the set of admissible parameters into a region where extremals are symmetric and a region where symmetry breaking occurs. In previous results, the symmetry breaking region was identified by showing the linear instability of the radial extremals. Here we prove that symmetry can be broken even within the set of parameters where radial extremals correspond to local minima for the variational problem associated with the inequality. For interpolation inequalities, such a symmetry breaking phenomenon is entirely new.

[48] J. Dolbeault, A. Klar, C. Mouhot, C. Schmeiser. Exponential rate of convergence to equilibrium for a model describing fiber lay-down processes. A paraître dans *Appl. Math. Res. Express.*

This paper is devoted to the adaptation of the hypocoercivity method to a Fokker-Planck equation for fiber lay-down. Exponential convergence towards a unique stationary state is proved in a norm which is equivalent to a weighted L^2 norm. The method is based on a micro / macro decomposition which is well adapted to the diffusion limit regime.

[49] J. Dolbeault, C. Mouhot, C. Schmeiser. Hypocoercivity for linear kinetic equations conserving mass. A paraître dans *Trans. Amer. Math. Soc.*

We develop a new method for proving hypocoercivity for a large class of linear kinetic equations with only one conservation law. Local mass conservation is assumed at the level of the collision kernel, while transport involves a confining potential, so that the solution relaxes towards a unique equilibrium state. Our goal is to evaluate in an appropriately weighted L^2 norm the exponential rate of convergence to the equilibrium. The method covers various models, ranging from diffusive kinetic equations like Vlasov-Fokker-Planck equations,

to scattering models like the linear Boltzmann equation or models with time relaxation collision kernels corresponding to polytropic Gibbs equilibria, including the case of the linear Boltzmann model. In this last case and in the case of Vlasov-Fokker-Planck equations, any linear or superlinear growth of the potential is allowed.

[50] J. Dolbeault, B. Nazaret, G. Savaré. From Poincaré to logarithmic Sobolev inequalities: a gradient flow approach. A paraître dans *SIAM J. Math. Anal.*

We use the distances introduced in a previous joint paper to exhibit the gradient flow structure of some drift-diffusion equations for a wide class of entropy functionals. Functional inequalities obtained by the comparison of the entropy with the entropy production functional reflect the contraction properties of the flow. Our approach provides a unified framework for the study of the Kolmogorov-Fokker-Planck (KFP) equation.

[51] J. Dolbeault, G. Toscani. Fast diffusion equations: matching large time asymptotics by relative entropy methods. *Kin. Rel. Mod.* 4, 3, 701-716, 2011.

A non self-similar change of coordinates provides improved matching asymptotics of the solutions of the fast diffusion equation for large times, compared to already known results, in the range for which Barenblatt solutions have a finite second moment. The method is based on relative entropy estimates and a time-dependent change of variables which is determined by second moments, and not by the scaling corresponding to the self-similar Barenblatt solutions, as it is usually done.

[52] J. Dolbeault, G. Toscani. Improved interpolation inequalities, relative entropy and fast diffusion equations. A paraître dans *Ann. Inst. H. Poincaré, Anal. non-lin.*

We consider a family of Gagliardo-Nirenberg-Sobolev interpolation inequalities which interpolate between Sobolev's inequality and the logarithmic Sobolev inequality, with optimal constants. The difference of the two terms in the interpolation inequalities (written with optimal constant) measures a distance to the manifold of the optimal functions. We give an explicit estimate of the remainder term and establish an improved inequality, with explicit norms and fully detailed constants. Our approach is based on nonlinear evolution equations and improved entropy - entropy production estimates along the associated flow. Optimizing a relative entropy functional with respect to a scaling parameter, or handling properly second moment estimates, turns out to be the central technical issue. This is a new method in the theory of nonlinear evolution equations, which can be interpreted as the best fit of the solution in the asymptotic regime among all asymptotic profiles.

[53] J. Dolbeault, B. Volzone. Improved Poincaré inequalities. *Nonlinear Anal.* 75, 16, 5985-6001, 2012.

Although the Hardy inequality corresponding to one quadratic singularity, with optimal constant, does not admit any extremal function, it is well known that such a potential can be improved, in the sense that a positive term can be added to the quadratic singularity without violating the inequality, and even a whole asymptotic expansion can be built, with optimal constants for each term. This phenomenon has not been much studied for other inequalities. Our purpose is to prove that it also holds for the gaussian Poincaré inequality. The method is based on a recursion formula, which allows to identify the optimal constants in the asymptotic expansion, order by order. We also apply the same strategy to a family of Hardy-Poincaré inequalities which interpolate between Hardy and gaussian Poincaré inequalities.

[54] R.-J. Duan, M. Fornasier, G. Toscani. A Kinetic Flocking Model with Diffusion. *Comm. Math. Phys.* 300, 1, 95-145, 2010.

We study the stability of the equilibrium states and the rate of convergence of solutions towards them for the continuous kinetic version of the Cucker-Smale flocking in presence of diffusion whose strength depends on the density. This kinetic equation describes the collective behavior of an ensemble of organisms, animals or devices which are forced to adapt their velocities according to a certain rule implying a final configuration in which the ensemble flies at the mean velocity of the initial configuration. Our analysis takes advantage both

from the fact that the global equilibrium is a Maxwellian distribution function, and, on the contrary to what happens in the Cucker-Smale model [4], the interaction potential is an integrable function. Precise conditions which guarantee polynomial rates of convergence towards the global equilibrium are found.

[55] M. Escobedo, F. Pezzotti, M. Valle. Analytical approach to relaxation dynamics of condensed Bose gases. *Ann. Phys.* 326, 4, 808-827, 2011.

The temporal evolution of a perturbation of the equilibrium distribution of a condensed Bose gas is investigated using the kinetic equation which describes collision between condensate and noncondensate atoms. The dynamics is studied in the low momentum limit where an analytical treatment is feasible. Explicit results are given for the behavior at large times in different temperature regimes.

[56] K. Fellner, G. Raoul. Stability of stationary states of non-local equations with singular interaction potentials. *Math. Comp. Mod.* 53, 2011.

We study the large-time behaviour of a non-local evolution equation for the density of particles or individuals subject to an external and an interaction potential. In particular, we consider interaction potentials which are singular in the sense that their first derivative is discontinuous at the origin.

For locally attractive singular interaction potentials we prove under a linear stability condition local non-linear stability of stationary states consisting of a finite sums of Dirac masses. For singular repulsive interaction potentials we show stability of stationary states of uniformly bounded solutions under a convexity condition.

Finally, we present numerical simulations to illustrate our results.

[57] K. Fellner, G. Raoul. Stable stationary states of non-local interaction equations. *Math. Mod. Meth. Appl. Sci.* 20, 2011.

In this article, we are interested in the large-time behaviour of a solution to a non-local interaction equation, where a density of particles/individuals evolves subject to an interaction potential and an external potential. It is known that for regular interaction potentials, stable stationary states of this equations are generically finite sums of Dirac masses. For a finite sum of Dirac masses, we give i) a condition to be a stationary state, ii) two necessary conditions of linear stability w.r.t. shifts and reallocations of individual Dirac masses, and iii) show that these linear stability conditions implies local non-linear stability. Finally, we show that for regular repulsive interaction potential W_ε converging to a singular repulsive interaction potential W , the Dirac-type stationary states $\bar{\rho}_\varepsilon$ approximate weakly a unique stationary state $\bar{\rho} \in L^\infty$. We illustrate our results with numerical examples.

[58] F. Filbet, J. Hu, S. Jin. A Numerical Scheme for the Quantum Boltzmann Equation Efficient in the Fluid Regime. *Math. Mod. Num. Anal.* 42, 443-463, 2012.

Numerically solving the Boltzmann kinetic equations with the small Knudsen number is challenging due to the stiff nonlinear collision term. A class of asymptotic preserving schemes was introduced in [F. Filbet and S. Jin, A class of asymptotic-preserving schemes for kinetic equations and related problems with stiff sources, J. Comput. Phys., 229, (2010), 7625-7648] to handle this kind of problems. The idea is to penalize the stiff collision term by a BGK type operator. This method, however, encounters its own difficulty when applied to the quantum Boltzmann equation. To define the quantum Maxwellian (Bose-Einstein or Fermi-Dirac distribution) at each time step and every mesh point, one has to invert a nonlinear equation that connects the macroscopic quantity fugacity with density and internal energy. Setting a good initial guess for the iterative method is troublesome in most cases because of the complexity of the quantum functions (Bose-Einstein or Fermi-Dirac function). In this paper, we propose to penalize the quantum collision term by a 'classical' BGK operator instead of the quantum one. This is based on the observation that the classical Maxwellian, with the temperature replaced by the internal energy, has the same first five moments as the quantum Maxwellian. The scheme so designed avoids the aforementioned difficulty, and one can show that the density distribution is still driven toward the quantum equilibrium. Numerical results are present to illustrate the efficiency of

the new scheme in both the hydrodynamic and kinetic regimes. We also develop a spectral method for the quantum collision operator.

[59] F. Filbet, S. Jin. A class of asymptotic preserving schemes for kinetic equations and related problems with stiff sources. *J. Comp. Phys.* 229, 20, 7625-7648, 2010.

In this paper, we propose a general framework to design asymptotic preserving schemes for the Boltzmann kinetic and related equations. Numerically solving these equations are challenging due to the nonlinear stiff collision (source) terms induced by small mean free or relaxation time. We propose to penalize the nonlinear collision term by a BGK-type relaxation term, which can be solved explicitly even if discretized implicitly in time. Moreover, the BGK-type relaxation operator helps to drive the density distribution toward the local Maxwellian, thus natually imposes an asymptotic-preserving scheme in the Euler limit. The scheme so designed does not need any nonlinear iterative solver or the use of Wild Sum. It is uniformly stable in terms of the (possibly small) Knudsen number, and can capture the macroscopic fluid dynamic (Euler) limit even if the small scale determined by the Knudsen number is not numerically resolved. It is also consistent to the compressible Navier-Stokes equations if the viscosity and heat conductivity are numerically resolved. The method is applicable to many other related problems, such as hyperbolic systems with stiff relaxation, and high order parabolic equations.

[60] F. Filbet, S. Jin. An Asymptotic Preserving Scheme for the ES-BGK model. *J. Sci. Comp.* 46, 2, 204-224, 2011.

In this paper, we study a time discrete scheme for the initial value problem of the ES-BGK kinetic equation. Numerically solving these equations are challenging due to the nonlinear stiff collision (source) terms induced by small mean free or relaxation time. We study an implicit-explicit (IMEX) time discretization in which the convection is explicit while the relaxation term is implicit to overcome the stiffness. We first show how the implicit relaxation can be solved explicitly, and then prove asymptotically that this time discretization drives the density distribution toward the local Maxwellian when the mean free time goes to zero while the numerical time step is held fixed. This naturally imposes an asymptotic-preserving scheme in the Euler limit. The scheme so designed does not need any nonlinear iterative solver for the implicit relaxation term. Moreover, it can capture the macroscopic fluid dynamic (Euler) limit even if the small scale determined by the Knudsen number is not numerically resolved. We also show that it is consistent to the compressible Navier-Stokes equations if the viscosity and heat conductivity are numerically resolved. Several numerical examples, in both one and two space dimensions, are used to demonstrate the desired behavior of this scheme.

[61] F. Filbet, C. Mouhot. Analysis of spectral methods for the homogeneous Boltzmann equation. *Trans. Amer. Math. Soc.* 363, 4, 1947-1980, 2011.

The development of accurate and fast algorithms for the Boltzmann collision integral and their analysis represent a challenging problem in scientific computing and numerical analysis. Recently, several works were devoted to the derivation of spectrally accurate schemes for the Boltzmann equation, but very few of them were concerned with the stability analysis of the method. In particular there was no result of stability except when the method is modified in order to enforce the posivity preservation, which destroys the spectral accuracy. In this paper we propose a new method to study the stability of homogeneous Boltzmann equations perturbed by smoothed balanced operators which do not preserve positivity of the distribution. This method takes advantage of the "spreading" property of the collision, together with estimates on regularity and entropy production. As an application we prove stability and convergence of spectral methods for the Boltzmann equation, when the discretization parameter is large enough (with explicit bound).

[62] F. Filbet, C. Negulescu, C. Yang. Numerical study of a nonlinear heat equation for plasma physics. *Int. J. Comp. Math.* 89, 8, 1060-1082, 2012.

This paper is devoted to the numerical approximation of a nonlinear temperature balance equation, which describes the heat evolution of a magnetically confined plasma in the edge region of a tokamak. The nonlinearity implies some numerical difficulties, in particular long time behavior, when solved with standard

methods. An efficient numerical scheme is presented in this paper, based on a combination of a directional splitting scheme and the IMEX scheme introduced in [Filbet and Jin]

[63] F. Filbet, A. Rambaud. Analysis of an Asymptotic Preserving Scheme for Relaxation Systems. A paraître dans *Math. Mod. Num. Anal.*

We study the convergence of a class of asymptotic preserving numerical schemes initially proposed by F. Filbet & S. Jin and G. Dimarco & L. Pareschi in the context of nonlinear and stiff kinetic equations. Here, our analysis is devoted to the approximation of a system of transport equations with a nonlinear source term, for which the asymptotic limit is given by a conservation laws. We investigate the convergence of the approximate solution $(u_h^\varepsilon, v_h^\varepsilon)$ to a nonlinear relaxation system, where $\varepsilon > 0$ is a physical parameter and h represents the discretization parameter. Uniform convergence with respect to ε and h is proven and error estimates are also obtained. Finally, several numerical tests are performed to illustrate the accuracy and efficiency of such a scheme.

[64] T. Goudon, A. Moussa, L. He, P. Zhang. The Navier-Stokes-Vlasov-Fokker-Planck system near equilibrium. *SIAM J. Math. Anal.* 42, 5, 2177-2202, 2010.

This paper is concerned with a system that couples the incompressible Navier-Stokes equations to the Vlasov-Fokker-Planck equation. Such a system arises in the modeling of sprays, where a dense phase interacts with a disperse phase. The coupling arises from the Stokes drag force exerted by a phase on the other. We study the global-in-time existence of classical solutions for data close to an equilibrium. We investigate further regularity properties of the solutions as well as their long time behavior. The proofs use energy estimates and the hypoelliptic structure of the system.

[65] T. Goudon, J. J. Nieto, O. Sanchez, J. Soler. Vanishing viscosity regimes and nonstandard shock relations for semiconductor superlattices models. *SIAM J. Appl. Math.* 71, 1, 180-199, 2011.

This paper is concerned with the analysis of asymptotic problems from discrete drift-diffusion models describing charge transport in semiconductor superlattices. The regimes we are interested in lead to balance laws. However, the nonconservative structure of the discrete system might produce defect measure terms in the limit process. These defect terms, concentrated on the shock discontinuities, can be related to non standard jump relations (in contrast with the usual Rankine-Hugoniot law) when considering discontinuous solutions and wave fronts.

[66] M. P. Gualdani, S. Mischler, C. Mouhot. Factorization for non-symmetric operators and exponential H-theorem. Prépublication.

We present a factorization method for estimating resolvents of non-symmetric operators in Banach or Hilbert spaces in terms of estimates in another (typically smaller) "reference" space. This applies to a class of operators writing as a "regularizing" part (in a broad sense) plus a dissipative part. Then in the Hilbert case we combine this factorization approach with an abstract Plancherel identity on the resolvent into a method for enlarging the functional space of decay estimates on semigroups. In the Banach case, we prove the same result however with some loss on the norm. We then apply these functional analysis approach to several PDEs: the Fokker-Planck and kinetic Fokker-Planck equations, the linear scattering Boltzmann equation in the torus, and, most importantly the linearized Boltzmann equation in the torus (at the price of extra specific work in the latter case). In addition to the abstract method in itself, the main outcome of the paper is indeed the first proof of exponential decay towards global equilibrium (e.g. in terms of the relative entropy) for the full Boltzmann equation for hard spheres, conditionnally to some smoothness and (polynomial) moment estimates. This improves on the result in [Desvillettes-Villani, Invent. Math., 2005] where the rate was "almost exponential", that is polynomial with exponent as high as wanted, and solves a long-standing conjecture about the rate of decay in the H-theorem for the nonlinear Boltzmann equation, see for instance [Cercignani, Arch. Mech, 1982] and [Rezakhanlou-Villani, Lecture Notes Springer, 2001].

[67] R. G. Iagar, P. Laurençot. Positivity, decay, and extinction for a singular diffusion equation with gradient absorption. *J. Funct. Anal.* 262, 7, 3186-3239, 2012.

We study qualitative properties of non-negative solutions to the Cauchy problem for the fast diffusion equation with gradient absorption

$$\partial_t u - \Delta_p u + |\nabla u|^q = 0 \quad \text{in} \quad (0,\infty)\times\mathbb{R}^N,$$

where $N \geq 1$, $p \in (1,2)$, and $q > 0$. Based on gradient estimates for the solutions, we classify the behavior of the solutions for large times, obtaining either positivity as $t \to \infty$ for $q > p - N/(N+1)$, optimal decay estimates as $t \to \infty$ for $p/2 \leq q \leq p - N/(N+1)$, or extinction in finite time for $0 < q < p/2$. In addition, we show how the diffusion prevents extinction in finite time in some ranges of exponents where extinction occurs for the non-diffusive Hamilton-Jacobi equation.

[68] R. G. Iagar, P. Laurençot. Eternal solutions to a singular diffusion equation with critical gradient absorption. Prépublication.

The existence of nonnegative radially symmetric eternal solutions of exponential self-similar type $u(t,x) = e^{-p\beta t/(2-p)} f_\beta(|x|e^{-\beta t};\beta)$ is investigated for the singular diffusion equation with critical gradient absorption $\partial_t u - \Delta_p u + |\nabla u|^{p/2} = 0$ in $(0,\infty)\times\mathbb{R}^N$ where $2N/(N+1) < p < 2$. Such solutions are shown to exist only if the parameter β ranges in a bounded interval $(0,\beta_*]$ which is in sharp contrast with well-known singular diffusion equations such as $\partial_t\phi - \Delta_p\phi = 0$ when $p = 2N/(N+1)$ or the porous medium equation $\partial_t\phi - \Delta\phi^m = 0$ when $m = (N-2)/N$. Moreover, the profile $f(r;\beta)$ decays to zero as $r \to \infty$ in a faster way for $\beta = \beta_*$ than for $\beta \in (0,\beta_*)$ but the algebraic leading order is the same in both cases. In fact, for large r, $f(r;\beta_*)$ decays as $r^{-p/(2-p)}$ while $f(r;\beta)$ behaves as $(\log r)^{2/(2-p)} r^{-p/(2-p)}$ when $\beta \in (0,\beta_*)$.

[69] X. Lu, C. Mouhot. On Measure Solutions of the Boltzmann Equation part I: Moment Production and Stability Estimates. *J. Diff. Eq.* 252, 3305-3363, 2012.

The spatially homogeneous Boltzmann equation with hard potentials is considered for measure valued initial data having finite mass and energy. We prove the existence of *weak measure solutions*, with and without angular cutoff on the collision kernel; the proof in particular makes use of an approximation argument based on the Mehler transform. Moment production estimates in the usual form and in the exponential form are obtained for these solutions. Finally for the Grad angular cutoff, we also establish uniqueness and strong stability estimate on these solutions.

[70] A. Mellet, S. Mischler, C. Mouhot. Fractional diffusion limit for collisional kinetic equations. *Arch. Rat. Mech. Anal.* 199, 493-525, 2011.

This paper is devoted to diffusion limits of linear Boltzmann equations. When the equilibrium distribution function is Maxwellian distribution, it is well known that for an appropriate time scale, the small mean free path limit gives rise to a diffusion equation. In this paper, we consider situations in which the equilibrium distribution function is a heavy-tailed distribution with infinite variance. We then show that for an appropriate time scale, the small mean free path limit gives rise to a fractional diffusion equation.

[71] S. Mischler, C. Mouhot. About Kac's Program in Kinetic Theory. *C. R. Math. Acad. Sci. Paris* 349, 1245-1250, 2011.

This paper is devoted to the study of propagation of chaos and mean-field limits for systems of indistinguable particles, undergoing collision processes. The prime examples we will consider are the many-particle jump processes of Kac and McKean giving rise to the Boltzmann equation. We solve the conjecture raised by Kac, motivating his program, on the rigorous connection between the long-time behavior of a collisional many-particle system and the one of its mean-field limit, for bounded as well as unbounded collision rates. Motivated by the inspirative paper by Grünbaum , we develop an abstract method that reduces the question of propagation of chaos to that of proving a purely functional estimate on generator operators (*consistency estimates*), along with differentiability estimates on the flow of the nonlinear limit equation (*stability estimates*). This allows us to exploit dissipativity at the level of the mean-field limit equation rather than the level of the particle system (as proposed by Kac). Using this method we show: (1) Quantitative estimates,

that are uniform in time, on the chaoticity of a family of states. (2) Propagation of *entropic chaoticity*. (3) Estimates on the time of relaxation to equilibrium, that are *independent of the number of particles in the system*. Our results cover the two main Boltzmann physical collision processes with unbounded collision rates: hard spheres and *true* Maxwell molecules interactions. The proof of the *stability estimates* for these models requires significant analytic efforts and new estimates.

[72] S. Mischler, C. Mouhot. Kac's Program in Kinetic Theory. A paraître dans *Invent. Math.*

This paper is devoted to the study of mean-field limit for systems of indistinguables particles undergoing collision processes. As formulated by [Kac, 1956] this limit is based on the chaos propagation, and we (1) prove and quantify this property for Boltzmann collision processes with unbounded collision rates (hard spheres or long-range interactions), (2) prove and quantify this property *uniformly in time*. This yields the first chaos propagation result for the spatially homogeneous Boltzmann equation for true (without cut-off) Maxwell molecules whose "Master equation" shares similarities with the one of a Lévy process and the first quantitative chaos propagation result for the spatially homogeneous Boltzmann equation for hard spheres (improvement of the convergence result of [Sznitman, 1984]). Moreover our chaos propagation results are the first uniform in time ones for Boltzmann collision processes (to our knowledge), which partly answers the important question raised by Kac of relating the long-time behavior of a particle system with the one of its mean-field limit, and we provide as a surprising application a new proof of the well-known result of gaussian limit of rescaled marginals of uniform measure on the N-dimensional sphere as N goes to infinity (more applications will be provided in a forthcoming work). Our results are based on a new method which reduces the question of chaos propagation to the one of proving a purely functional estimate on some generator operators (consistency estimate) together with fine stability estimates on the flow of the limiting non-linear equation (stability estimates).

[73] S. Mischler, C. Mouhot, B. Wennberg. A new approach to quantitative propagation of chaos for drift, diffusion and jump processes. Prépublication.

This paper is devoted the the study of the mean field limit for many-particle systems undergoing jump, drift or diffusion processes, as well as combinations of them. The main results are quantitative estimates on the decay of fluctuations around the deterministic limit and of correlations between particles, as the number of particles goes to infinity. To this end we introduce a general functional framework which reduces this question to the one of proving a purely functional estimate on some abstract generator operators (consistency estimate) together with fine stability estimates on the flow of the limiting nonlinear equation (stability estimates). Then we apply this method to a Boltzmann collision jump process (for Maxwell molecules), to a McKean-Vlasov drift-diffusion process and to an inelastic Boltzmann collision jump process with (stochastic) thermal bath. To our knowledge, our approach yields the first such quantitative results for a combination of jump and diffusion processes.

[74] S. Motsch, L. Navoret. Numerical simulations of a non-conservative hyperbolic system with geometric constraints describing swarming behavior. *Multiscale Mod. Simul.* 9, 3, 1253-1275, 2011.

The Vicsek model is a very popular individual based model which describes collective behavior among animal societies. A macroscopic version of the Vicsek model has been derived from a large scale limit of this individual based model. In this work, we want to numerically validate this Macroscopic Vicsek model (MV). To this aim, we compare the simulations of the macroscopic and microscopic models one with each other. The MV model is a non-conservative hyperbolic equation with a geometric constraint. Due to the lack of theory for this kind of equations, we derive several equivalents for this system leading to specific numerical schemes. The numerical simulations reveal that the microscopic and macroscopic models are in good agreement provided that we choose one of the proposed formulations based on a relaxation of the geometric constraint. This confirms the relevance of the macroscopic equation but it also calls for a better theoretical understanding of this type of equations.

[75] C. Mouhot, E. Russ, Y. Sire. Fractional Poincaré inequalities for general measures. *J. Math. Pures Appl. 95*, 72-84, 2011.

We prove a fractional version of Poincaré inequalities in the context of $\mathbb{R}^n$ endowed with a fairly general measure. Namely we prove a control of an L^2 norm by a non local quantity, which plays the role of the gradient in the standard Poincaré inequality. The assumption on the measure is the fact that it satisfies the classical Poincaré inequality, so that our result is an improvement of the latter inequality. Moreover we also quantify the tightness at infinity provided by the control on the fractional derivative in terms of a weight growing at infinity. The proof goes through the introduction of the generator of the Ornstein-Uhlenbeck semigroup and some careful estimates of its powers. To our knowledge this is the first proof of fractional Poincaré inequality for measures more general than Lévy measures.

[76] C. Mouhot, C. Villani. On Landau damping. *Acta Math.* 207, 29-201, 2011.

Going beyond the linearized study has been a longstanding problem in the theory of Landau damping. In this paper we establish exponential Landau damping in analytic regularity. The damping phenomenon is reinterpreted in terms of transfer of regularity between kinetic and spatial variables, rather than exchanges of energy; phase mixing is the driving mechanism. The analysis involves new families of analytic norms, measuring regularity by comparison with solutions of the free transport equation; new functional inequalities; a control of nonlinear echoes; sharp scattering estimates; and a Newton approximation scheme. Our results hold for any potential no more singular than Coulomb or Newton interaction; the limit cases are included with specific technical effort. As a side result, the stability of homogeneous equilibria of the nonlinear Vlasov equation is established under sharp assumptions. We point out the strong analogy with the KAM theory, and discuss physical implications.

[77] T. Rey. Blow up Analysis for Anomalous Granular Gases. *SIAM J. Math. Anal.* 44, 3, 1544-1561, 2012.

We investigate in this article the long-time behaviour of the solutions to the energy-dependent, spatially-homogeneous, inelastic Boltzmann equation for hard spheres. This model describes a diluted gas composed of hard spheres under statistical description, that dissipates energy during collisions. We assume that the gas is "anomalous", in the sense that the energy dissipation increases when the temperature decreases. This allows the gas to cool down in finite time. We study the existence, uniqueness and attractiveness of blow up profiles for this model and the cooling law associated, generalizing the classical Haff's Law for granular gases. To this end, we give some new estimates about the third order moment of the inelastic Boltzmann equation with drift term and we introduce new strongly "non-linear" self-similar variables.