

# Propagation of chaos in a network of Fitz-Hugh-Nagumo

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IMSV

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1. Model: FitzHugh-Nagumo
2. Propagation of chaos and our results
3. Strategy
  - Coupling method
  - First theorem: synchronous coupling
  - Second theorem: non-synchronous coupling
4. Conclusion

# What do we model?

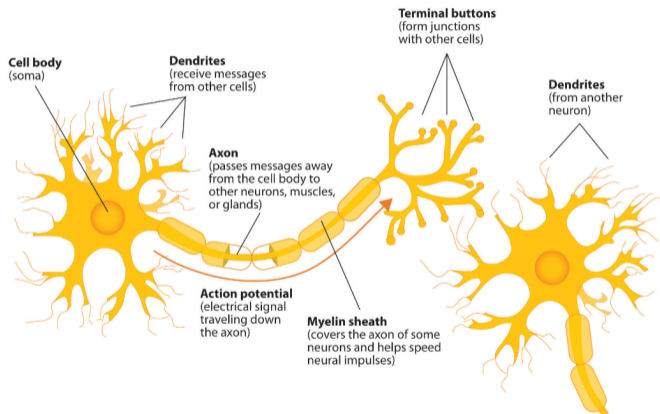
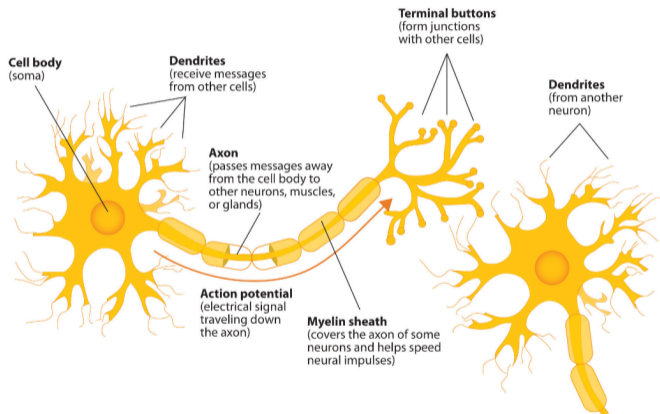


Figure: Neurons and a connexion

# What do we model?



Main elements:

- ▶ Membrane potential
- ▶ Incoming signal
- ▶ Outgoing signal

Figure: Neurons and a connexion

# What do we model?

- ▶ Model membrane potential
- ▶ Noise from the presynaptic current
- ▶ Noise from the conductance dynamic
- ▶ Transmission of potential between neurons via synapses

## What do we model?

- ▶ Model membrane potential  $X$
- ▶ Noise from the presynaptic current
- ▶ Noise from the conductance dynamic
- ▶ Transmission of potential between neurons via synapses

### Deterministic model

$$\begin{cases} dX_t = (X_t - (X_t)^3 - C_t - \alpha)dt \\ dC_t = (\gamma X_t - C_t + \beta)dt \\ Z_t = (X_t, C_t). \end{cases}$$

$C$  is a recovery variable.  $\alpha, \beta, \gamma$  fixed.

## What do we model?

- ▶ Model membrane potential  $X$
- ▶ Noise from the presynaptic current  $B^X$
- ▶ Noise from the conductance dynamic  $B^C$
- ▶ Transmission of potential between neurons via synapses

### Stochastic model

$$\begin{cases} dX_t = (X_t - (X_t)^3 - C_t - \alpha)dt + \sigma_x dB_t^X \\ dC_t = (\gamma X_t - C_t + \beta)dt + \sigma_c dB_t^C \\ Z_t = (X_t, C_t). \end{cases}$$

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## What do we model?

- ▶ Model membrane potential  $X$
- ▶ Noise from the presynaptic current  $B^X$
- ▶ Noise from the conductance dynamic  $B^C$
- ▶ Transmission of potential between neurons via synapses: interaction  $K_X$  and  $K_C$

### Stochastic model with interactions

We consider  $N$  neurons and for each  $i \leq N$ ,  $X^{i,N}$  and  $C^{i,N}$  their quantities.

$$\begin{cases} dX_t^{i,N} = (X_t^{i,N} - (X_t^{i,N})^3 - C_t^{i,N} - \alpha)dt + \frac{1}{N} \sum_{j=1}^N K_X(Z_t^i - Z_t^j)dt + \sigma_x dB_t^{i,X} \\ dC_t^{i,N} = (\gamma X_t^{i,N} - C_t^{i,N} + \beta)dt + \frac{1}{N} \sum_{j=1}^N K_C(Z_t^i - Z_t^j)dt + \sigma_c dB_t^{i,C} \\ Z_t^{i,N} = (X_t^{i,N}, C_t^{i,N}). \end{cases} \quad (1)$$



## State of the art : FitzHugh-Nagumo

- ▶ León, Samson (2018). *"Hypoelliptic stochastic FitzHugh–Nagumo neuronal model: mixing, up-crossing and estimation of the spike rate"*
  - ▶ Noise only on  $C$  (conductance dynamic)
  - ▶ Study of equations for one neuron.

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  - ▶ Study on  $N$  neurons (existence, uniqueness, stationary solution, ...)

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  - ▶ Study on  $N$  neurons (existence, uniqueness, stationary solution, ...)
- ▶ Baladron, Fasoli, Faugeras, Touboul (2012). *Mean-field description and propagation of chaos in networks of Hodgkin-Huxley and FitzHugh-Nagumo neurons*, completed by Bossy, Faugeras and Talay (2015) *Clarification and Complement to (...)*
  - ▶ Noise only on  $X$
  - ▶ Propagation of chaos and convergence

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# Idea of propagation of chaos

- ▶ Exchangeable particles (symmetric law)

## Chaos (Kac<sup>1</sup>, Sznitman<sup>2</sup>)

At fixed  $k$ ,  $k$  particles in a set of  $N$  particles are asymptotically i.i.d. when  $N$  tends to infinity.

## Propagation of chaos

At fixed  $t$ , if the system of  $N$  particles is initially chaotic, then it stays chaotic at time  $t$ .

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<sup>1</sup>Mark Kac. “Foundations of Kinetic Theory”. In: *Third Berkeley Symposium on Mathematical Statistics and Probability*. 1956.

<sup>2</sup>Alain-Sol Sznitman. “Topics in Propagation of Chaos”. In: *Ecole d’Eté de Probabilités de Saint-Flour XIX — 1989*.

# Framework

## Assumptions

$\sigma_C > 0$  or/and  $\sigma_X > 0$ .

Lipschitz interaction.

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Lipschitz interaction.

## Notations

Initial distribution :  $(\mu_0)^{\otimes N}$ .

$\mu_t^{k,N}$  : marginal distribution at time  $t$  of the first  $k$  neurons in a network of  $N$  neurons with this initial distribution.

We will measure the distance between  $\mu_t^{k,N}$  and  $\bar{\mu}_t^{\otimes k}$  where  $\bar{\mu}_t$  is a specific measure.

## Definition of $\bar{\mu}_t$ ?

We denote  $K * \nu(x) = \int K(x - y)\nu(dy)$ .

$\frac{1}{N} \sum_{j=1}^N K_X(Z_t^i - Z_t^j)$  can be seen as  $K_X * \left( \frac{1}{N} \sum_{j=1}^N \delta_{Z_t^j} \right)$  applied on  $Z_t^i$ .



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$$\begin{cases} dX_t^{i,N} = (X_t^{i,N} - (X_t^{i,N})^3 - C_t^{i,N} - \alpha)dt + \frac{1}{N} \sum_{j=1}^N K_X(Z_t^i - Z_t^j)dt + \sigma_x dB_t^{i,X} \\ dC_t^{i,N} = (\gamma X_t^{i,N} - C_t^{i,N} + \beta)dt + \frac{1}{N} \sum_{j=1}^N K_C(Z_t^i - Z_t^j)dt + \sigma_c dB_t^{i,C} \\ Z_t^{i,N} = (X_t^{i,N}, C_t^{i,N}). \end{cases}$$

↓

$$\begin{cases} d\bar{X}_t = (\bar{X}_t - (\bar{X}_t)^3 - \bar{C}_t - \alpha)dt + K_X * \bar{\mu}_t(\bar{Z}_t)dt + \sigma_x d\bar{B}_t^X \\ d\bar{C}_t = (\gamma \bar{X}_t - \bar{C}_t + \beta)dt + K_C * \bar{\mu}_t(\bar{Z}_t)dt + \sigma_c d\bar{B}_t^C \\ \bar{Z}_t = (\bar{X}_t, \bar{C}_t) \\ \bar{\mu}_t = \text{Law}((\bar{X}_t, \bar{C}_t)), \end{cases}$$

Non-linear stochastic differential equation of McKean-Vlasov type

## Definition of $\bar{\mu}_t$ ?

For  $1 \leq i \leq N$ ,

$$\begin{cases} d\bar{X}_t^i = (\bar{X}_t^i - (\bar{X}_t^i)^3 - \bar{C}_t^i - \alpha)dt + K_X * \bar{\mu}_t(\bar{Z}_t^i)dt + \sigma_X d\bar{B}_t^{i,X} \\ d\bar{C}_t^i = (\gamma\bar{X}_t^i - \bar{C}_t^i + \beta)dt + K_C * \bar{\mu}_t(\bar{Z}_t^i)dt + \sigma_C d\bar{B}_t^{i,C} \\ \bar{Z}_t^i = (\bar{X}_t^i, \bar{C}_t^i) \\ \bar{\mu}_t = \text{Law}((\bar{X}_t^1, \bar{C}_t^1)), \end{cases}$$

Then  $\bar{\mu}_t^{\otimes k}$  is the law of  $(\bar{Z}_t^1, \dots, \bar{Z}_t^k)$ .

- ▶ Same type of drift.
- ▶ Same idea for the interaction.
- ▶ Brownian noise.

## Result: Propagation of chaos

Let  $K_X$  be  $L_X$ -Lipschitz,  $K_C$  be  $L_C$ -Lipschitz. Let's notice  $\sigma_X$  or  $\sigma_C$  can be null.

Theorem (Non uniform in time  
propagation of chaos (C., Le Bris) )

*There exist explicit  $C_1, C_2 > 0$ , such that  
for all good probability measures  $\mu_0$  on  $\mathbb{R}^2$*

$$\mathcal{W}_1 \left( \mu_t^{k,N}, \bar{\mu}_t^{\otimes k} \right) \leq C_1 e^{C_2 t} \frac{k}{\sqrt{N}}.$$

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Theorem (Uniform in time propagation of chaos (C., Le Bris))

*Under some condition on  $L_X$  and  $L_C$ , there exist explicit  $B_1 > 0$ , such that for all good probability measures  $\mu_0$  on  $\mathbb{R}^2$*

$$\mathcal{W}_1 \left( \mu_t^{k,N}, \bar{\mu}_t^{\otimes k} \right) \leq B_1 \frac{k}{\sqrt{N}}.$$

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# Strategy: Coupling method

## Controlling the Wasserstein distance

$$\mathcal{W}_1(\nu, \nu') = \inf_{\pi \in \Pi(\nu, \nu')} \int d(z, z') \pi(dz, dz')$$

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$$\mathcal{W}_1(\nu, \nu') = \inf_{\pi \in \Pi(\nu, \nu')} \int d(z, z') \pi(dz, dz')$$

Two choices:

- ▶ choice of coupling  $\pi$
- ▶ choice of distance (or major bound of)  $d$



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## Strategy: Coupling method

### Naive idea: Synchronous coupling

$$\mathcal{W}_1 \left( \mu_t^{k,N}, \bar{\mu}_t^{\otimes k} \right) \leq \mathbb{E} \left[ \frac{1}{N} \sum_{i=1}^N |X_t^{i,N} - \bar{X}_t^i| + |C_t^{i,N} - \bar{C}_t^i| \right]$$

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Study the dynamics  $d(X_t^{i,N} - \bar{X}_t^i)$  and  $d(C_t^{i,N} - \bar{C}_t^i)$ .

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Study the dynamics  $d(X_t^{i,N} - \bar{X}_t^i)$  and  $d(C_t^{i,N} - \bar{C}_t^i)$ .

Define  $\bar{B}_t^{i,X} = B_t^{i,X}$  and  $\bar{B}_t^{i,C} = B_t^{i,C}$ .

$$\begin{aligned} d(X_t^{i,N} - \bar{X}_t^i) &= (\text{Drift on } X_t^{i,N})dt + \frac{1}{N} \sum_{j=1}^N K_X(Z_t^i - Z_t^j)dt \\ &\quad - (\text{Drift on } \bar{X}_t^i)dt - K_X * \bar{\mu}_t(\bar{Z}_t^i)dt \end{aligned}$$

# Strategy: Coupling method

Naive idea: Synchronous coupling

Steps:

## Strategy: Coupling method

### Naive idea: Synchronous coupling

Steps:

1) Denote  $r_t^i = |X_t^{i,N} - \bar{X}_t^i| + |C_t^{i,N} - \bar{C}_t^i|$ . We want to bound  $\mathbb{E}(\frac{1}{N} \sum_{i=1}^N r_t^i)$ . Then

$$dr_t^i \leq \left[ Cr_t^i + \left| \frac{1}{N} \sum_{j=1}^N K_X(Z_t^i - Z_t^j) - K_X * \bar{\mu}_t(\bar{Z}_t^i) \right| + \left| \frac{1}{N} \sum_{j=1}^N K_C(Z_t^i - Z_t^j) - K_C * \bar{\mu}_t(\bar{Z}_t^i) \right| \right] dt.$$

## Strategy: Coupling method

### Naive idea: Synchronous coupling

Steps:

2) Decompose interaction part

$$\left| \frac{1}{N} \sum_{j=1}^N K_X(Z_t^i - Z_t^j) - K_X * \bar{\mu}_t(\bar{Z}_t^i) \right| \leq \left| \frac{1}{N} \sum_{j=1}^N K_X(Z_t^i - Z_t^j) - \frac{1}{N} \sum_{j=1}^N K_X(\bar{Z}_t^i - \bar{Z}_t^j) \right| + \left| \frac{1}{N} \sum_{j=1}^N K_X(\bar{Z}_t^i - \bar{Z}_t^j) - K_X * \bar{\mu}_t(\bar{Z}_t^i) \right|.$$

Controlled thanks to Lipschitz property

## Strategy: Coupling method

### Naive idea: Synchronous coupling

Steps:

3) Finally

$$d\mathbb{E}(r_t^i) \leq \left( C\mathbb{E}(r_t^i) + C \frac{1}{\sqrt{N}} \left( \mathbb{E}(\|\bar{Z}_t\|_1^2) \right)^{1/2} \right) dt.$$

Good bounds on  $\mathbb{E}(\|\bar{Z}_t\|_1^2)$  + Gronwall's lemma  $\Rightarrow$  first theorem proved (non uniform in time)



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## Strategy: Coupling method

### Improved idea: Mixed coupling

(Here  $\sigma_X > 0$ ). Consider, for an adequate  $\delta$ :

$$\mathcal{W}_1 \left( \mu_t^{k,N}, \bar{\mu}_t^{\otimes k} \right) \leq \mathbb{E} \left[ \frac{1}{N} \sum_{i=1}^N f(|X_t^{i,N} - \bar{X}_t^i| + \delta |C_t^{i,N} - \bar{C}_t^i|) \right]$$

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Study the dynamic  $d(X_t^{i,N} - \bar{X}_t^i)$  and  $d(C_t^{i,N} - \bar{C}_t^i)$ .

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Study the dynamic  $d(X_t^{i,N} - \bar{X}_t^i)$  and  $d(C_t^{i,N} - \bar{C}_t^i)$ .

In the subspace  $\{(X_t^{i,N} - \bar{X}_t^i) = 0\}$ : deterministic contraction.  $\Rightarrow$  Synchronous coupling

In the orthogonal subspace  $\Rightarrow$  Reflection coupling (maximization of variance)

**Non-synchronous coupling.** Mixed reflection coupling and synchronous one.

## Strategy: Coupling method

### Improved idea: Mixed coupling

Consider two white noises for each  $i$ :  $B^{i,sc,X}$ ,  $B^{i,rc,X}$ .

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Consider two functions  $\varphi_{sc}$  and  $\varphi_{rc} : \mathbb{R}^+ \rightarrow [0, 1]$  such that  $\varphi_{rc}^2 + \varphi_{sc}^2 = 1$ :

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Consider two functions  $\varphi_{sc}$  and  $\varphi_{rc} : \mathbb{R}^+ \rightarrow [0, 1]$  such that  $\varphi_{rc}^2 + \varphi_{sc}^2 = 1$ :

$$\begin{aligned} dB_t^{i,X} &= \varphi_{sc} (|X_t^{i,N} - \bar{X}_t^i|) dB_t^{i,sc,X} + \varphi_{rc} (|X_t^{i,N} - \bar{X}_t^i|) dB_t^{i,rc,X} & d\bar{B}_t^{i,C} &= dB_t^{i,C} \\ d\bar{B}_t^{i,X} &= \varphi_{sc} (|X_t^{i,N} - \bar{X}_t^i|) dB_t^{i,sc,X} - \varphi_{rc} (|X_t^{i,N} - \bar{X}_t^i|) dB_t^{i,rc,X} \end{aligned}$$

Then:

$$dB_t^{i,X} - d\bar{B}_t^{i,X} = 2\varphi_{rc} (|X_t^{i,N} - \bar{X}_t^i|) dB_t^{i,rc,X}$$

## Strategy: Coupling method

### Improved idea: Mixed coupling

Two Lyapunov functions:

1) Sufficient to have good bounds on moments of  $Z_t^i$  and  $\bar{Z}_t^i$ .

$$H(z) = \frac{1}{2}\gamma x^2 + \beta x + \frac{1}{2}c^2 + \alpha c + H_0$$



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$$H(z) = \frac{1}{2}\gamma x^2 + \beta x + \frac{1}{2}c^2 + \alpha c + H_0$$

2) Need better control of the noise:

$$\tilde{H}(z) = \int_0^{H(z)} \exp(a\sqrt{u}) du = \frac{2}{a^2} \exp\left(a\sqrt{H(z)}\right) \left(a\sqrt{H(z)} - 1\right) + \frac{2}{a^2}.$$

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We define new "distance":

$$\rho\left((z_j, z'_j)_{1 \leq j \leq N}\right) = \frac{1}{N} \sum_{i=1}^N f\left(r(z_i, z'_i)\right) \left(1 + \epsilon \tilde{H}(z_i) + \epsilon \tilde{H}(z'_i) + \dots\right)$$

with concave  $f$ .

## Strategy: Coupling method

### Improved idea: Mixed coupling

With this coupling and this distance, we work on the dynamic of  $e^{ct} \rho \left( (Z_t^{j,N}, \bar{Z}_t^j)_{1 \leq j \leq N} \right)$ .  
Good choice of parameters + contraction in various spaces + Gronwall's lemma  $\Rightarrow$  Second theorem

## State of the art: Mixed coupling

- ▶ Lindvall, Rogers (1986). *"Coupling of Multidimensional Diffusions by Reflection"*
- ▶ Eberle, Zimmer (2016). *"Sticky couplings of multidimensional diffusions with different drifts"*
  - ▶ Discuss the approach of sticky couplings
  - ▶ Provide total variation bounds
  - ▶ Prove the contraction of process on a linear subspace
- ▶ Eberle, Guillin, Zimmer (2019). *"Couplings and quantitative contraction rates for Langevin dynamics"*
  - ▶ applied to Langevin equation without interaction
- ▶ Pre-print: Arnaud Guillin, Pierre Le Bris, and Pierre Monmarché (2021). *"Convergence Rates for the Vlasov-Fokker-Planck Equation and Uniform in Time Propagation of Chaos in Non Convex Cases"*

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- ▶ Other types of interaction
- ▶ Environmental noise:  $dZ_t^i = f(Z_t^i)dt + \sigma dB_t$



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## Perspectives

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- ▶ Environmental noise:  $dZ_t^i = f(Z_t^i)dt + \sigma dB_t$
- ▶ Individual parameters  $\alpha_i, \beta_i, \gamma_i, \dots$

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- ▶ Other types of interaction
- ▶ Environmental noise:  $dZ_t^i = f(Z_t^i)dt + \sigma dB_t$
- ▶ Individual parameters  $\alpha_i, \beta_i, \gamma_i, \dots$
- ▶ Synchronization of neurons

Thank you

- ▶ Laetitia Colombani and Pierre Le Bris. *Chaos Propagation in Mean Field Networks of FitzHugh-Nagumo Neurons*. Submitted. June 2022. arXiv: 2206.13291