Identifying Individual Preferences from Group Behavior

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The individual consumption model

A single individual acting with certain outcomes:

$$\begin{matrix} \max U(x) \\ p\, x \leq 1 \end{matrix} \implies x(p)$$

If $U(x)$ is C^2, if $U''(x)$ is positive definite and the maximum $x(p)$ is attained at an interior point, we get the *individual demand function* $p \to x(p)$, from R^M into itself, which is C^1 and satisfies the Walras law:

$$p\, x(p) = 1$$

The Slutsky relations

Conversely, given a C^1 map $p \to x(p)$ which satisfies the Walras law, it is an individual demand function iff it the associated *Slutsky matrix*:

$$[S(x)]^{m,n} := \frac{\partial x^m}{\partial p_n} - \sum_k x^m \frac{\partial x^n}{\partial p_k} p_k$$

is symmetric and negative definite.
The map $p \to x(p)$ can be inverted to a map $x \to p(x)$, and the vectors $p(x)$ are orthogonal to a family of hypersurfaces which foliate R^M : these are the *indifference curves* $U(x) = \mathrm{cst}$. So the preference relation can be recovered

A perfect theory...

We have:

- a mathematical model
- testable implications: a map $p \longrightarrow x(p)$ is *rationalizable* (i.e. arises as an individual demand function from utility maximization) iff it is satisfy the Slutsky relations
- non-parametric identifiability: the preference relation $\preceq$ can be recovered from the demand function $p \rightarrow x(p)$

Two caveats apply

- we assume that the whole function $x(p)$ can be observed. In practice, data sets are finite. One will have to choose a demand function from a parametrized family, and adjust the parameters
- all these results are local, i.e. they hold in some unspecified neighbourhood of $(\bar{p}, x(\bar{p}))$, not in R_+^M

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- Kapan (2009) also rejects the unitary model on Turkish data collected during a period of high inflationobs

A model for group behavior

A group is a collection of individuals $h = 1, ..., H$. It buys $\xi \in R^M$ on the market and transforms it into private and public goods within the group:

$$\xi \text{ market consumption} \Longrightarrow \begin{array}{c} (x_1, ..., x_H) \text{ private good} \\ X \text{ public good} \end{array}$$

via a group production function:

$$f(-\xi, x_1, ..., x_H, X) \leq 0$$

The group is bound by a budget constraint:

$$\pi\xi \leq 1$$

Group efficiency

Individual h has a utility function $U^h(x_1, ..., x_H, X)$. The actual production/redistribution is the result of negociations within the group. Failing further information, the group functions as a black box to the econometrician. The basic assumption of Browning and Chiappori is that the group is *efficient*: there are Pareto multipliers $\mu_h \geq 0$, depending on π, such that the outcome is given by:

$$\left.\begin{array}{c} \max_{\xi, x_1, ..., x_H, X} \sum_h \mu_h(\pi)\, U^h(x_1, ..., x_H, X) \\ f(-\xi, x_1, ..., x_H, X) \leq 0 \\ \pi\xi \leq 1 \end{array}\right\} \Longrightarrow \xi(\pi)$$

Examples

Public goods only, no group production:

$$\left.\begin{array}{c} \max_X \sum_h \mu_h(P)\, U^h(X) \\ PX \leq 1 \end{array}\right\} \Longrightarrow X(P)$$

Private goods only, no group production:

$$\left.\begin{array}{c} \max_{x_1,\ldots,x_H} \sum_h \mu_h(p)\, U^h(x_h) \\ p\sum x_h \leq 1 \end{array}\right\} \Longrightarrow (x_1(p), \ldots, x_H(p))$$

No group production:

$$\left.\begin{array}{c} \max_{x_1,\ldots,x_H,X} \sum_h \mu_h(p,P)\, U^h(x_h, X) \\ p\sum x_h + PX \leq 1 \end{array}\right\} \Longrightarrow (x_1(p,P), \ldots, x_H(p,P), X(p,$$

Set $(x_1(p,P), \ldots, x_H(p,P), X(p,P)) = \xi(\pi)$

Testable consequences

- (Browning-Chiappori) If $\xi(\pi)$ is the demand function of a group with H members, then its Slutsky matrix is the sum of a symmetric, negative definite matrix and a matrix of rank at most $H-1$

$$S(\pi) = \Sigma(\pi) + \sum_{k=1}^{H-1} u_k v_k'$$

- The BC condition is always satisfied when $2H+1 \geq M$. For a group with H members, we require at least $2H+1$ goods.
- On FAMEX data, $M=8$: food in the home, food outside the home, services, masculine clothing, feminine clothing, transportation, leisure and vice. So we can test the BC condition up to $H=3$. Indeed, it is verified for couples with a probability of 44.3%

Identifiability: first obstacle

- Conversely, if a map $\pi \to \xi(\pi)$ satisfies the Browning-Chiappori condition and the Walras law $\pi\xi(\pi) = 1$, and if its Jacobian matrix $D\xi$ is of full rank, then $\xi(\pi)$ is the demand function of a group with public goods only, and of a group with private goods only
- The *private* or *public* nature of intragroup consumption cannot be tested from aggregate data on group demand

Identifiability: second obstacle

It will be convenient to take (p, X) as independent variables instead of (p, P)

Conditional sharing rules:

$$\left.\begin{array}{c}\max_{x_1,\dots,x_H,X} \sum_h \mu_h(p,P)\, U^h(x_h, X) \\ p\sum x_h + PX \leq 1\end{array}\right\} \Longrightarrow \rho^h(p,X) = px_h$$

Collective indirect utility:

$$W^h(p,X) = \max\left\{ U^h(x_h, X) \mid px^h = \rho^h(p,X) \right\}$$

- Knowing the W^h allows to assess the impact on each group member of changes in p and X. For welfare purposes, it is enough to recover the W^h instead of the U^h. However, even in this restricted sense, identifiability does not obtain:
- The collective indirect utilities are *not* uniquely determined by the knowledge of group demand

Identifiability under exclusion

- If, for each h, agent h does not consume good h, then, generically, the H collective indirect utilities are ordinally identifiable
- If $\left(W^1, ..., W^H\right)$ and $\left(\tilde{W}^1, ..., \tilde{W}^H\right)$ are two sets of collective indirect utilities which rationalize the collective demand $\xi(\pi)$, and if

$$\frac{\partial W^1}{\partial \pi_1} = \frac{\partial \tilde{W}^1}{\partial \pi_1} = 0$$

then there is a function φ such that:

$$\tilde{W}^1(\pi, X) = \varphi\left(W^1(\pi, X)\right)$$

- Examples of exclusive goods: leisure, clothing

Further Issues

- *distribution factors* are variables which affect some individual's power within the group; when they are observable, they facilitate identifiability
- the same methods can be applied to markets with large number of consumers (thus joining the Sonnenschein Mantel Debreu literature), or with small numbers of consumers (Ekeland-Djitte), or to incomplete markets (Chiappori-Ekeland) or to markets with non-budgetary constraints (Aloqeili)
- we do *not* know how to extend these results to the case of decision under uncertainty. The challenge is to separate individual preferences from subjective probabilities

The mathematical structure

The first-stage decision process is

$$\max_{X,\rho} \sum_h \mu^h V^h \left(p, X, \rho^h\right)$$
$$\sum \rho^h + P\ X \leq 1$$

and the second-stage decision process is:

$$V^h \left(p, X, \rho^h\right) = \max \left\{ U^h \left(x_h, X\right) \ \middle|\ p x_h \leq \rho^h \right\}$$

leading to:

$$\begin{array}{c} \sum_h \gamma_h D_p V^h = -\sum_h x_h = -x \\ \sum_h \gamma_h D_X V^h = P \end{array} \iff \begin{array}{c} \sum_h \gamma_h D_p W^h = -x - D_p \left(XP\left(p, X\right)\right) \\ \sum_h \gamma_h D_X W^h = P - D_p \left(XP\left(p, X\right)\right) \end{array}$$

In words, a given vector field has to be decomposed into a linear combination of gradients

The mathematical tools

Let $\pi \longrightarrow \xi(\pi)$ be a map from R^M into itself satisfying the Walras law $\pi\, \xi(\pi) = 1$. We associate with it the 1-form ω defined by:

$$\omega = \sum_{m=1}^{M} \xi^m d\pi_m$$

The following are equivalent:

- ξ satisfies the symmetry part of the Browning-Chiappori condition for a group with H members
- the Slutsky matrix is the sum of a symmetric matrix and a matrix of rank $H - 1$
- there are functions $\lambda_1(\pi), ..., \lambda_H(\pi)$ and $W^1(\pi), ..., W^H(\pi)$ such that $\omega = \sum_{h=1}^{H} \lambda_h dW^h$
- ω satisfies the Darboux condition:

$$\omega \wedge (d\omega)^H = 0$$

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