Breaking the dice

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29 April 2014

The sagas

Contrary to popular belief, the Norse sagas are not legends or tales. They are a historical genre, biographies of men of valour. There are two main strands:

- the Icelandic sagas, describing the settlement of Iceland from Norway, and the discovery of Greenland and America. The most important ones are the saga of Egil Skalagrimsson and the saga of Burnt-Njaal
- Heimskringla, the lives of the kings of Norway. Their author, Snorri Sturluson (1178-1241), must rank as one of the greatest writers of all times. The excerpt I am going to show you is from the saga of Olav Haraldson (995-1030), also known as Saint Olav

It is said in Iceland that any man must read Burnt-Njaal once a year.

Rolling the dice

- Thorstein the Learned says that there was a settlement on the island of Hising which had alternatively belonged to Norway and to Gotland. So the kings agreed to draw lots: they would throw dice, and the highest score would win the island. The Swedish king threw two sixes, and said there was no point for king Olav to throw. But king Olav replied, shaking the dice in his hand: "*There are still two sixes in the dice, and it is a trifling matter for God my Lord to have them turn up*". He threw, and two sixes turned up.

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- Then the king of Sweden threw, and again two sixes turned up. Thereupon king Olav threw threw, and one die turned up a six, but the other one split in two, so it turned up a seven.

The breaking point

This is *the moment your model fails* : the unexpected occurs, the unplanned-for: you discover that things are not really what you supposed them to be. According to a former US Secretary of Defense, factors determining future events can be classified into:

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Mathematical models in finance (or any model at all) take into account *known* unknowns, mostly through probability theory. But they cannot account for *unknown* unknowns. In the following I will describe three breaking points: the saga of mathematical finance.

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Rolling the dice: 1973-2001

The standard model for the price process S_t

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW$$

This was first suggested by *Bachelier* (1900) on the basis that prices change like the square root of time: $\frac{dS}{S} \simeq \mu + \sigma\sqrt{dt}$. This formula was the basis of the famous Black and Scholes formula for option pricing. The term dW is Brownian motion: basically a mathematical construction for modelling a continuous rolling of the dice.
The standard model states that the market is just a lotterywhere someone (who?) is permanently rolling the dice, and where the traders are trying their luck. There is a non-losing martingale in that lottery: one can always hedge, i.e. *completely eliminate the market risk.*

The breaking point: 2001

- the Enron stock bursts
- the dot.com bubble bursts

The bottom fell out of the market, and there was no way anyone could hedge. You had to get off the train. So there is a new game in town: board the train just before the others do, leave the train just before the others do.
The breaking of the dice reveals a flaw in the model: you may not find a buyer when you need it most. As a consequence, you may be powerless to hedge. This is the liquidity risk:
Various efforts are made to patch the flawl:

- replace Brownian motion by other probability laws (Mandelbrot fractals, stable laws)

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- model the train game itself: mean field games

Mean field games

A new mathematical theory developed by Jean-Michel Lasry and Pierre-Louis Lions to model situations where individuals interact with a crowd. Since they are part of the crowd themselves, there is an inbuilt reflexivity into the model.
The following example (due to Yves Achdou) shows a crowd leaving a theater after a performance. There are two doors on the side.

- **case 1**: both doors are opened immediately, so everyone knows it
- **case 2**: one door only will be opened, and when they start moving, they don't know which one

The second case is the risky one.

Fueling bubbles: 2001-2008

The Greenspan era:

- the regulator provides easy money (low interest rates), so a bubble (housing) can start when the old one (internet) bursts
- investors ride the train, and when the train stops, the Fed makes sure another one is starting, so investors just board the new one
- the market provides insurance against defaults (CDS, MBS)

The breaking point: 2008

- Bear Stearns, AIG, Lehmann Brothers, Northern Rock, Royal Bank of Scotland, Dexia, Fortis, Natixis...
- so this train stops, but this time no other train is leaving the station
- on the contrary, another train stops, the one carrying sovereign debt: Greece, Ireland, Spain, Cyprus, ...

General flight to safe and liquid assets. No lending, no investment. The breaking of the dice reveals a flaw in the model:

Moral hazard

- Liquidity problems again
- But another issue appears: leverage :
  - Lehmann Brothers 31. RBS 21
  - RBS in 2008 has total assets of 1 900 billion pounds, more than any other company in the world, more than the GNP of Great Britain, and equity of 91 billion

Why did banks take on so much risk ? Limited liability: bear the positive side, and not the negative side:

- some institutions are too big to fail: governments to not want another Lehmann
- conflicting interests: the moral hazard problem
  - corporate governance: management vs. shareholders
  - the Chinese wall: investment banking vs. retail banking
  - titrization of loans and mortgages: the seller does not bear the risk

Never give a sucker an even break

The standard model revisited to accomodate limited liability

$$dX_t = \mu_t dt + \sigma dW$$

X_t is revenue and accrues to me, μ_t is decided by my agent, at a cost for himself, and σdW is exogeneous (random noise). I can't observe my agent's effort μ_t, all I can observe is the output X_t .

- If it is high, it may be due to chance, or because the agent worked hard
- If it is low, it may be due to chance, or because the agent worked hard.

No monitoring, no clawing back past compensations. How am I to devise a compensation scheme which will induce the agent to maximize my net revenue ?

Historical examples:

- sharecropping: I share the revenue with my agent (50% ?)
- farming: he pays me a fixed rent and keeps the revenue for himself

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- Value of monitoring, accountability, and prudential regulation
- Reemergence of Keynes: money as a shelter (store of value)