

Correction of Assignment 1
Friday May 27th

Step 1. Finding α

Anna is willing to pay up to 5,65 \$ for a lottery ticket that gives her 100 \$ one with probability 0,1. This means that that getting 5,65 \$ now and getting 100 \$ in one month with probability 0,1 give her the same utility. Write this as an equation:

$$u(5,65) = \frac{1}{10}u(100)$$

We know that Anna's utility function is $u(m)^\alpha$. Writing this in the equation, we get

$$\begin{aligned}(5,65)^\alpha &= \frac{1}{10}(100)^\alpha \\ 5,65 &= \frac{1}{10^{1/\alpha}}100 \\ 10^{1/\alpha} &= \frac{100}{5,65}\end{aligned}$$

Take the (decimal) logarithm of both sides:

$$\begin{aligned}\frac{1}{\alpha} &= 2 - \log 5,65 = 2 - 0,75 = 1,25 \\ \alpha &= 0,8\end{aligned}$$

Step 2. Finding r

The same ticket in one month's time gives $\frac{1}{1+r}$ the utility of the same ticket today:

- utility of ticket today: $(5,65)^\alpha$
- utility of ticket in one month $\frac{1}{1+r}(5,65)^\alpha$

If Anna is willing to pay 5,09 \$ for the last ticket, this means that it gives her the same utility as 5,09 \$. Write the equation:

$$\begin{aligned}\frac{1}{1+r}u(5,65) &= u(5,09) \\ \frac{1}{1+r}(5,65)^\alpha &= (5,09)^\alpha \\ 1+r &= \left(\frac{5,65}{5,09}\right)^\alpha = 1,0871\end{aligned}$$

So the psychological discount rate for one month is 1,09
Now we answer the questions

Question 1. There are two outcomes to the lottery

- 50 \$ with a probability 5% gives a utility $0,05u(50) = 0,05(50)^{0,8}$
- 200\$ with a probability 20% gives a utility $0,2u(200) = 0,2(200)^{0,8}$

The amount of money m that gives the same utility is:

$$\begin{aligned} u(m) &= 0,05(50)^{0,8} + 0,2(200)^{0,8} \\ &= 1,143 + 13,863 = 15 \end{aligned}$$

Replacing $u(m)$ by $m^{0,8}$, this becomes $m^{0,8} = 15$, so

$$m = 15^{1/0,8} = 15^{1,25} = 29,51$$

So Anna is willing to pay 29,51 \$ for the ticket

Question 2 One year is twelve months. So for the same ticket in one year, Anna gets the utility

$$\frac{15}{(1+r)^{12}} = \frac{15}{1,0871^{12}} = \frac{15}{2,72} = 5,51$$

The corresponding amount of money m is given by $m^{0,8} = 5,51$, hence:

$$m = 5,51^{1,25} = 8,44$$

So Anna is willing to pay 8,44 \$ for the same ticket in one year

Question 3 The utility of the bet to Anna is $\frac{p}{1+r}u(200)$ with p the probability of Real winning. If she pays 100 \$ for it, it means that the utility of 100 \$ now is the same as the utility of the bet:

$$\begin{aligned} u(100) &= \frac{p}{1+r}u(200) \\ 100^{0,8} &= p \frac{200^{0,8}}{1,0871} \\ p &= 1,0871 \left(\frac{1}{2}\right)^{0,8} = 1,0871 \times 0,5743 \\ p &= 0,624 \end{aligned}$$

Her probability of Real winning is at least 62,4%