Mathematics in the social sciences

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Plan of the Lectures

1. What is a mathematical model ?
2. Towards one-dimensional (wo)man
3. Individual freedom and collective action
    1. The role of institutions
    2. Playing games
    3. Markets and their inadequacies

I will not broach the important topic of statistics

What is a mathematical model ?

What are the social sciences ?

Galileo Galilei *Il Saggiatore* (1623)

"La Filosofia è scritta in questo grandissimo libro, che continuamente ci sta aperto innanzi à gli occhi (io dico l'universo) ma non si può intendere se prima non s'impara a intender la lingua, e conoscer i caratteri, ne' quali è scritto. Egli è scritto in lingua matematica, e caratteri son triangoli, cerchi, & altre figure geometriche, senza i quali mezi è impossibile a intenderne vmanamente parola; senza questi è vn'aggirarsi vanamente per vn'oscuro laberinto "

The basic example: planetary motion

- the correct language to express the facts: Kepler's three laws
- mathematical proof and physical causality: Newtons's law of gravitation

Can something similar be done for the social sciences ?

Modernity

Modernity is marked by a series of separations:

- nature (including animals) vs humans (hence the idea of social sciences)
- society vs. individuals (Protestantism, Hegel), hence the separation between sociology and psychology
- imperfect reality vs. perfect ideas (Plato), mortal body vs. immortal soul (Christianity, Descartes)

The first unitary philosopher is Spinoza (1632-1677) but he is ahead of our times, which are still embedded in the duality thinking

Social laws vs. physical laws

Graeber, *The utopia of rules*, (2015) *The hidden reality of human life is the fact that the world doesn't just happen. It isn't a natural fact, even though we tend to treat it as if it is: it exists because we all collectively produce it*

Society is a network of games. Social sciences study the rules of the games.

Graeber *The ultimate, hidden truth of the world is that it is something that we make and could just as easily make differently*

An example: kinship

Kinship is the basic relation in society: who is my mother, who is my father. It goes with the basic rule: thou shalt not marry thy mother or thy father.

Note first that kinship also is a social production: in ancient Rome, I can disavow a baby born from my wife (*suscepit liberum*) and I can recognize as my child someone born from a woman I have not slept with (*adoptio*)

Kinship among the Warlpirli

Levy-Strauss, *Les structures élémentaires de la parenté* (1949).

There are eight clans, A, B, C, D and a, b, c, d . If you are from clan A you can only marry someone from clan a, and if you are from clan a you can only marry someone from clan A.
The clan of the children is determined by the mother: B if the mother is A, and d if the mother is a

The cardboard model

The mathematical model

We define two maps S (spouse) and M (mother) from$\{A, B, C, D\} \cup \{a, b, c, d\}$ to itself:

$$S(A) = a,\ S(B) = b,\ S(C) = c,\ S(D) = d$$
$$S(a) = A,\ S(b) = B,\ S(c) = C,\ S(d) = D$$
$$M(A) = D,\ M(B) = A,\ M(C) = B,\ M(D) = C$$
$$M(a) = b,\ M(b) = c,\ M(c) = d,\ M(d) = a$$

We note that:

$$MS \neq SM$$
$$S^2 = Id,\ M^4 = Id$$
$$(SM)^2 = Id$$
$$(MS)^2 = Id$$

This is a model for the kinship relation among the Warlpirli, with the understanding that the equality symbol $=$ means '*belong to the same clan*'

Mathematical models in the social sciences

Can we answer specific questions ?

Have we learned something universal about human nature ?

Towards one-dimensional (wo)man

The birth of finance

Although all societies use symbols and arrange them in complex stuctures, no mathematics were directly involved. In agrarian societies, they are used as a tool to keep track of flows and stocks of goods.

During the late Middle Ages in Europe, the development of banking in Germany and Italy leads to a major invention: finance, which will transform the world and the way we understand it

What is finance ?

Economics is the study of production and distribution of material goods.

Finance is the study of the production and flow of a single good, money, which is fongible and storable.

1. It is a very restricted domain, but it central to the functioning of modern societies.
2. It is very open to mathematics
3. If a price can be set to everything, then everything can be reduced to finance.
4. This is the birth certificate of one-dimensional (wo)man

1. Luca Pacioli (1457-1517) introduces double-entry bookkeping (*partita doppia*): every activity can now be expressed in the balance sheet (for example work and capital)
2. the Catholic Church finally abandons its prohibition of charging interest: money now stretches into the future

When time becomes money

An interest rate of r per annum means that the banker will give you $\frac{m}{(1+r)^N}$\$ today in exchange for m\$ in$N$ years. So a disaster that will cost you m\$ in N years is worth only $\frac{m}{(1+r)^N}$\$ today.

Let us compute the value today of 1 Million Euros in 50, 100 and 200 years for various values of r

years	50	100	200
10%	8519	73	$< 0,01$
4,6%	105540	11140	124
1,4%	500000	250000	62000

The Stern Review on Climate Change (2006) takes $r = 1,3\%$ for normative reasons whereas Nordhaus (Nobel Prize 2018) takes for r the market rate, which is close (at the time) to 4,6%. This is one of the many reasons Nordhaus is overoptimistic. See his Nobel lecture slides. https://www.nobelprize.org/uploads/2018/10/nordhaus-slides.pdf

Mastering the future

Interest rates turn time into money. By using them, one can make decisions which deferred consequences. The higher the interest rate, the less the future is worth. In the case of climate change, for instance, with high interest rates

- Strict interpretation: you can insure against disasters very cheaply
- Broad interpretation: what happens in one century is no concern today

The birth of probability theory

Probability theory arises from the idea of fairness. Everyone understands what a 50/50 chance is. Blaise Pascal (1623-1662) drew the consequences of that idea and invented the calculus of probability.

Le problème des partis du chevalier de Méré: *Deux joueurs jouent à un jeu de hasard en 3 parties gagnantes, chacun ayant misé la même somme d'argent* m *; or il se trouve que le jeu est interrompu avant que l'un des deux joueurs ait obtenu 3 victoires et ainsi remporté la victoire et de ce fait la totalité des enjeux soit* $2m$*. Comment, dans ces circonstances, doit-on partager les enjeux ?*

Pascal's answer

Lettre de Pascal à Fermat (16 July 1654): *Voici à peu près comment je fais pour savoir la valeur de chacune des parties, quand deux joueurs jouent, par exemple, trois parties, et chacun a mis 32 pistoles au jeu. Posons que le premier en ait deux et l'autre un ; ils jouent maintenant une partie, dont le sort est tel que, si le premier la gagne, il gagne tout l'argent qui est au jeu, savoir, 64 pistoles ; si l'autre la gagne, ils sont deux parties à deux parties, et par conséquent, s'ils veulent se séparer, il faut qu'ils retirent chacun leur mise, savoir, chacun 32 pistoles. Considérez donc, Monsieur, que si le premier gagne, il lui appartient 64 ; s'il perd, il lui appartient 32. Donc s'ils ne veulent point hasarder cette partie, et se hasarder sans la jouer, le premier doit dire : ' Je suis sûr d'avoir 32 pistoles, car la perte même me les donne ; mais pour les 32 autres, peut-être je les aurais, peut-être vous les aurez ; le hasard est égal ; partageons donc ces 32 pistoles par la moitié, et me donnez, outre cela, mes 32 qui me sont sûres.' Il aura 48 pistoles et l'autre 16*

The calculus of probabilities

Pascal's method can be extended by creating a binomial tree. Every node of the tree has a probability of winning, expressed in percentage

If my probability of winning is p, then my fair share of the pot is p. If a lottery ticket gives you a probability p of winning m\$ then its fair price is mp\$

Consider a lottery which proceeds as follows. A fair coin is thrown.

1. If it is heads the first time, you win 2$ and the game stops.If it is tails, the coin is thrown again
2. If it is heads the second time, you win 4$ and the game stops. If it is tails, the coin is thrown again
3. If it is heads the third time, you win 8$ and the game stops. If it is tails, the coin is thrown again

And so on. If it heads the nth time, you win $2^n$$ and the game stops. If it is tails, the coin is thrown again What is the fair price of a ticket ?

Dealing with uncertainty

People do not want to pay the fair price for large gains with small probabilities.

To take this into account, Nicolas Bernoulli in 1713 proposed to replace the gain m by m^{α}, with $0 < \alpha < 1$. This reduces the psychological impact of large gains, because m^{α} then is much smaller than m.

Doubling your gain does not double the satisfaction you derive from it: changing m to $2m$ changes m^{α} to $2^{\alpha} m^{\alpha}$,with $2^{\alpha} < 2$

Saint-Peterburg revisited

Instead of paying the fair price:

$$\sum_{n=1}^{\infty} \frac{2^n}{2^n}$$

the individual will pay

$$\sum_{n=1}^{\infty} \frac{2^{n\alpha}}{2^n} = \frac{1}{2^{1-\alpha} - 1}$$

Each individual is characterized by his/hers own α. From observing how much the individual is willing to pay for the lottery ticket, one can deduce his/her α

The utility function

The function $m \to m^{\alpha}$ is called the *utility function* of the individual. It can be any function $u(m)$, provided it concave, increasing and smooth:

- Increasing, expressing the fact that the individual is insatiable
- Concave, expressing the fact that the individual prefers 50 to an $1/2$ chance of gaining 100.

The last fact is called risk aversion, and α is often called the risk-aversion coefficient

How correct is the model ?

Kahnemann and Tversky

- Individuals are risk-averse wrt to gains, but risk-prone wrt losses
- Individuals attribute excessive weight to events with low probability (hence the price of lottery tickets)

Utilitarism

Finance has taught us how to take into account deferred consequences (interest rates). Money bets have taught us how to take into account uncertainty (expected utility). The idea of utilitarism is to extend these concepts to economics, where there are many goods instead of just one, and even to the whole of human behaviour.

The economic approach to human behaviour

Gary Becker (1976) *The economic approach to human behaviour*

Individuals continuously decide between alternatives. Each choice is a set of numbers $x = (x_1, ..., x_N) \in \mathbb{R}^N$. Typically, these are possible consumptions, where x_n denotes the quantity chosen of good N.
There are several possible cases:

- the decision has immediate and certain consequences (immediate delivery)
- the decision has deferred consequences (future delivery)
- the decision has uncertain consequences (contingent claim)

Homunculus

This is the standard mathematical model used in economics. Each individual is characterized by

- his/her *utility function* $u(x) = u(x_1, ..., x_n)$. Note that $2u$, u^2 of more generally $\varphi \circ u$ for any increasing function φ will give the same ordering.
- his/her *psychological interest rate* r
- the *probability* $p(\omega)$ he/she assigns to an uncertain event ω

The latter is quite interesting: human beings can assign probabilities to events which have never happened before and will never happen again (bets). This is the basis of another approach to probability theory (de Finetti)

The standard model

Between two certain and immediate decisions $x = (x_1, ..., x_N)$ and $y = (y_1, ..., y_N)$ the individual chooses x if

$$u(x) > u(y)$$

- $x(\omega_1)$ which will materialize in t_1 years if a certain event ω_1 happens before
- $y(\omega_1)$ which will materialize in t_2 years if a certain event ω_2 happens before,

the individual chooses x if:

$$\frac{1}{(1+r)^{t_1}} p(\omega_1)\, u(x(\omega_1)) > \frac{1}{(1+r)^{t_2}} p(\omega_2)\, u(y(\omega_2))$$

And it all adds up

Does it work ? The psychogical interest rate

Experimental psychology shows that people are impatient: they prefer pleasent events to happen early rather than late, and unpleasant ones late rather than early. This is consistent with a psychological discount rate $\alpha_t < 1$ defined by:

Definition

The individual is indifferent between getting a quantity u_t of utility at time t and getting a quantity u_0 now if: $u_0 = \alpha_t u_t$

Impatience means that $\alpha_t < 1$ and $\alpha_{t+1} < \alpha_t$

- For homunculus, there is some $r > 0$ such that $\alpha_t = \frac{1}{(1+r)^t}$ for all t.

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- More realistic is $\alpha_t = \frac{1}{1+kt}$ for some $k > 0$

Homunculus is consistent

Should I start jogging every day ? If I do I get a pain in the ass today, say $-p$, and a long-term benefit tomorrow, say b.Let us compute:

- If I start running today, the present benefit is $-p+\alpha_1 b<0$ provided $\alpha_1<p/b$. I will not run today

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- But when tomorrow happens, I discover that it is today, and promise myself I will start tomorrow
- With homunculus, $\alpha_1 = \alpha_2/\alpha_1$, so this cannot happen.

Does it work ? The a priori probability

It is a fact that people take bets, which means that they are able to put probabilities on events which have never happened before (a world cup final in soccer) and that one hopes will never happen (blow-up of Challenger 7, or Fukushima) This has been used as a foundation for probability theory (Dutch books, Ramsey and de Finetti)
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- The Rumsfeld statement
- The Ellsberg example

Risk

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- Risk is a social construction:
  1. The risk of terrorism is vastly overblown
  2. The risk of climate change is either not understood or vastly underblown

Does it work ? The utility function

Yes! Fundamental work of Browning, Chiappori, Ekeland.

According to Popper (1934), a theory is scientific if and only if it can be disproved, that is, and experiment can be set up, the result of which is predicted by the theory. It the experimental result differs from the predicted one, the theory is disproved. It not, the theory is not disproved (but not proved) and can be used until a new experiment is set up
Does there exist such an experiment in economic theory ?

Consumer behaviour

Suppose an individual has an amount w to spend on N goods. The unit price of good n is p_n. What basket does he/she choose? According to the theory, he/she will solve the following problem:

$$\max u(x_1, ..., x_n)$$
$$\sum_{n=1}^{N} p_n x_n \leq w$$
$$x_n \geq 0$$

Since u is concave, there is a unique solution $x(p)$. In fact, these are N functions of N variables $x_n(p_1, ..., p_n)$ $1 \leq n \leq N$.

The Antonelli-Slutsky conditions

Introduce the following matrix:

$$S_{ij} = \frac{\partial x^i}{\partial p_j} - \sum p_n \frac{\partial x^i}{\partial p_n} x^j$$

It was first established by Antonelli, then forgotten and rediscovered by Slutsky, that:

Theorem

There is a function $u(x)$ such that $x(p)$ solves problem $(\mathcal{P})$ if and only if the matrix S is symmetric:

$$S_{ij} = S_{ji}$$

The Browning-Chiappori-Ekeland test

- It was proved by Chiappori and Ekeland (1999) that the Slutsky conditions are in fact sufficient: if they are satisfied, then the consumer does have a utility function. With some additional requirements on the Slutsky matrix, the utility function will be concave

Lesson: households are NOT individual.

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- Chiappori and Ekeland extended the theory to couples, and they find that couples satisfy the extended relations with a probability of 88%

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Nature, culture or training ?

There is the prevalent and unspoken idea of the individual as a rational soul piloting a body with irrational impulses (stoicism, christianism), preferably a 40-year old white male working in finance. However:

- Descartes (Discours de la Méthode) *"nous avons tous été enfants avant d'être hommes"*
- Graeber (The utopia of rules) *"It normally takes a great deal of work to turn a newborn baby into a person - someone with a name and social relationships (mother, father, ...) and a home, towards others have responsibilities, who can someday be expected to have responsibilities to them as well"*

The production of individuals

- The production of persons is a major activity of society: rites (initiation into adulthood: first communion, graduation), laws (a *sans-papiers* is a human being but not a person, but a corporation is a person but not a human being but not a person), education, information media, advertisements

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- The BCE result shows to what extent this program is successful in Canada concerning household consumption: people are educated into buying

Economic Growth

All human beings are identical, they live for ever, and their aim in life is to maximize the utility he/she derives from consuming. All types of consumption are aggregated into a single good c, the supply of which depends on time t, so the representative individual seeks to maximize the criterion:

$$J(c) := \int_0^\infty e^{-rt} u(c_t)\, dt. \tag{1}$$

However, the good c has to be produced, so that at every time t the representative individual has to decide between saving and consuming. Saving goes into augmenting the capital k :

$$\frac{dk}{dt} = f(k) - c, \quad k(0) = k_0, \tag{2}$$

Note that

- There is no concern about diversity in societies

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- There is no concern about diversity in societies
- There is no concern about diversity in generations
- Production is not subject to physical constraints
- Human life is not subject to biological constraints

Unlimited Growth

Population is allowed to grow at an exponential rate n and technological progress at an exponential rate g.

Standard growth theory (Ramsey, 1928) states that the economy (total production + total consumption) then grows at an exponential rate $n + g$ while consumption per capita grows at the rate g

https://www.ceremade.dauphine.fr/ ekeland/Articles/09-09-CahiersCDD2.pdf

Climate change: the Nordhaus model

The criterion $J(c)$ is unchanged:

$$J(c) = \int_0^\infty e^{-rt} u(c)\, dt \tag{3}$$

The dynamics of the economy are now:

$$\frac{dk}{dt} = [D(s) - h(A)]\, f(k) - c \tag{4}$$

$$\frac{ds}{dt} = Af(k) - \beta(s - \bar{s}) \tag{5}$$

$$k(0) = k_0, \quad s(0) = s_0 \tag{6}$$

Here s_t is the stock of carbon in the atmosphere at time t. In the absence of production, it evolves according to $\frac{ds}{dt} = -\beta(s - \bar{s})$, so that $s_t \to \bar{s} > 0$, where $\bar{s}$ is the natural (pre-industrial) equilibrium stock of carbon. Of course, we have $\beta > 0$. The production process emits carbon, and $A_t > 0$ is the emission rate at time t. It is a policy tool, meaning that it is chosen by the planner, but it is costly: resources have to be diverted from investment and consumption to abate emissions. The cost $h(A)$ is on

Interpretation

$D(s)$ is the damage function. In the Nordhaus model, it depends on temperature, and there is a climate module to determine the latter, but we skip this step and take directly the stock of carbon as a proxy for the temperature. The functions $f(k)$, $h(A)$ and $D(s)$ satisfy the following:

- $f : [0, \infty] \to [0, \infty]$ is increasing, strictly concave, TBD: $f'(k) \to \infty$ when $k \to 0$ and $f'(k) \to 0$ when $k \to \infty$
- $h : [0, 1] \to [0, 1]$ is decreasing, $h(A) \to 1$ when $A \to 0$ and $h(1) = 0$
- $D : [\bar{s}, \infty] \to [0, 1]$ is decreasing, with $D(\bar{s}) = 1$ and $D \to 0$ when $s \to \infty$

We now perform the same analysis we did in the preceding section. Note that there are now two control variables, c, and A, and two state variables, k, and s.

Theorem

Suppose (k_∞, s_∞) *is an optimal stationary state. Then* $(s_\infty, k_\infty, A_\infty, c_\infty)$ *must satisfy*

$$\begin{aligned} r &= \left[D\left(s_\infty\right) - h\left(A_\infty\right) + h'\left(A_\infty\right)A_\infty\right] f'\left(k_\infty\right), \\ D'\left(s_\infty\right) f\left(k_\infty\right) &= \left(r+\beta\right) h'\left(A_\infty\right), \\ A_\infty &= \frac{\beta\left(s_\infty - \bar{s}\right)}{f\left(k_\infty\right)}, \\ c_\infty &= \left[D\left(s_\infty\right) - h\left(A_\infty\right)\right] f\left(k_\infty\right). \end{aligned}$$

Individual freedom and collective action 1:

The role of institutions

Birth of the individual

https://www.overleaf.com/project/622f3139fdbd2308bfaa8bed
Can we construct society by aggregating individuals according to certain rules, as we construct molecules and solids from atoms ? The problem is that since the 17th century there is no force binding the individuals/atoms together

- The Romans distinguished *auctoritas* from *potestas*.
- During the Middle Ages, the pope held auctoritas and the king potestas.
- After the Reformation, auctoritas disappears, and there is nothing left to bind potestas.
- Peace of Westphalia (1648)

The social contract

The problem: given a collection of independent individuals, rational souls piloting a body with irrational impulses (stoicism, christianism), how can one enact rules which will enable them to live together as well as possible ?

The Hobbes solution: (*The Leviathan*) (1651): in order to avoid *the war of all against all*, individuals give up part of their freedom to the ruler, thereby legitimizing potestas. The only bound of potestas is insurrection: when the yoke becomes too harsh the people may install a new potestas. See also Pascal and Spinoza. This inaugurates the social contract tradition, from

Rousseau, *Du Contrat Social* (1762) to Rawls *A theory of Justice* (1971)

An unnatural problem ?

Montesquieu *Lettres Persanes*(1721) *"Je n'ai jamais ouï parler du droit public, qu'on n'ait commencé par rechercher soigneusement quelle est l'origine des sociétés; ce qui me parait ridicule. Si les hommes n'en formaient point, s'ils se quittaient et se fuyaient les uns les autres, il faudrait en demander la raison, et chercher pourquoi ils se tiennent séparés: mais ils naissent tous liés les uns aux autres; un fils est né auprès de son père, et il s'y tient: voila la société et la cause de la société"*

What we will not discuss

The central concepts which are debated are freedom, justice, and the common good.

- Freedom, as we understand it (political freedom) cannot even be phrased in the utilitarian framework. At best, it is misunderstood as the freedom to consume: how often have you seen an advertisment stating *I am free to choose ...* . ?
- Justice consists in *treating equals equally and unequals unequally, in proportion to relevant similarities and differences* (Aristoteles, Nicomachean Ethics). Modern works are John Rawls, *A theory of justice,* (1971) and Amartya Sen, *The idea of justice,* (2009)
- There have recently been many studies about inequalities, following Thomas Piketty, *Le capital au XXIème siècle*

Rawls and Sen

Rawls' theory:

- The principles of justice

Sen's position: *The strong perception of manifest injustice applies to adult human beings as well (as children). What moves us, reasonably enough, is not the realization that the world falls short of being completely just – which few of us expect – but that there are clearly remediable injustices around us which we want to eliminate*.

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     - *(a) to the greatest benefit of the least advantaged, consistent with the just savings principle, and*
     - *(b) attached to offices and positions open to all under conditions of fair equality of opportunity.*
- These principles are agreed upon in a social contract negociated under the veil of ignorance

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Collective Decision-Making

Consider an assembly of 60 deciding by majority rule. They are facing three choices A, B, C. Individual preferences are as follows:

$A > B > C$	23
$B > C > A$	17
$B > A > C$	2
$C > A > B$	10
$C > B > A$	8

Is this a quirk of majority rule ? No, it is a standard issue of all voting procedures

Arrow's impossibility theorem

Suppose we have a voting rule which satisfies the three reasonable conditions:

- it applies to all situations (any number of choices, any set of preferences)
- unanimity rule: if everyone prefers A to B then the group prefers A to B
- irrelevant alternatives: if the group prefers A to B, and the individual preferences are changed in a way that preserves the individual rankings of A and B, the group will still prefer A to B

then the rule is dictatorial: there is an individual who chooses for everyone:

Collectives cannot agree on the common good

The common good cannot possibly be what the collective decides on

Practical conclusion

- the procedure is as important as the individual preferences
- individuals will resort to strategic voting

The utilitarian definition

Suppose we have I individuals, $i = 1, ..., I$ facing a set of decision $x \in R^N$. Individual i has a utility function u_i and is given x_i. We define the *welfare function* of the group, U, by:

$$W(x_1, ..., x_I) = \sum_{i=1}^{I} u_i(x_i)$$

It plays the role of a utility function for the group. The best decision according to that welfare function will be the one that maximises W, namely the allocation $\bar{x}_i$ defined by:

$$W(\bar{x}_1, ..., \bar{x}_I) \geq W(x_1, ..., x_I) \quad \forall x \in D$$

The common good is not uniquely defined!

However, note that $2u_1$ also is a utility function for $i = 1$, leading to another welfare function W':

$$W(x_1, ..., x_I) = 2u_1(x_1) + \sum_{i=2}^{I} u_i(x_i)$$

and to another best decision $\bar{x}' \neq \bar{x}$. What is the common feature among all these possible best points?

Pareto optimality

Stems from Vilfredo Pareto (1848-1923)

Definition

A distribution $(x_1, ..., x_I)$ is Pareto optimal if any other distribution gives less to one member of the group

$$\forall (y_1, ..., y_I), \quad \exists j : u_j(x_j) > u_j(y_j)$$

For instance, one cannot give more to one member without giving less to another

Theorem

For any choice of individual utilities, the points x which maximise welfare are Pareto optima

Pareto optimality as efficiency

A Pareto optimum is efficient: there is no waste. Economic theory does not tell you about the common good, it tells you about not throwing anything away.

Note that a Pareto optimum can be vastly unequal. However, theoreticians such as Friedrich von Hayek hold that efficiency so difficult to achieve that there is no point about worrying further. Increase the size of the cake before thinking of how to share it.

Achieving efficiency is not easy

Achieving efficiency is not easy. Mancur Olson, *The logic of collective action* (1965), gives the following example

There is a group with 1000 members. A collective action is proposed, typically a strike, the success of which depends on the number of participants.

- if n members participate, the (sure) gain is n\$ for each member of the group
- the cost of participating is 100\$ (for the participants only)

Free riding

Let us be clever:

- you get n\$ whether you participated or not,

The end result is no one participates and that by being clever, everyone misses an opportunity to ge 900\$ each. This is what happens with climate change

Free riding

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- but non-participants do not have to pay the 100\$cost.

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Free riding

Let us be clever:

- you get n\$ whether you participated or not,
- but non-participants do not have to pay the 100\$cost.
- he best strategy is not to participate and enjoy the work of others (free riding)

The end result is no one participates and that by being clever, everyone misses an opportunity to ge 900\$ each. This is what happens with climate change

The importance of institutions

Two reasons institutions are important:

- The procedures for decision-making are crucial and as important for the decision as individual preferences
- It is to the benefit of everyone to create institutions that will prevent free-riding (passager clandestin)

Individual freedom and collective action 2: Playing games

Moving around in a city

- N inhabitants
- they all own a car
- n use it: the larger n is, the slower is the traffic
- public transportation: buses

Moving around in a city

- N inhabitants
- they all own a car
- n use it: the larger n is, the slower is the traffic
- public transportation: buses
- public transportation: trams or metro

Formalization

N individuals

Individual n is characterized by:

- his/her available decisions D_n
- his/her utilility function $u_n(d_1, ..., d_n)$

Definition

A set of decisions $(\bar{d}_1, ..., \bar{d}_N)$ is a Nash equilibrium if, for every n :

$$u_n(\bar{d}_1, ..., \bar{d}_{n-1}, d_n, \bar{d}_{n+1}, ..., \bar{d}_N) \geq u_n(\bar{d}_1, ..., \bar{d}_{n-1}, \bar{d}_n, \bar{d}_{n+1}, ..., \bar{d}_N)$$

He/she has no regrets: considering what the others have done, he/she has done as well as possible, con

Examples of equilibria

The free rider problem

The transportation problem

How to get out of a bad equilibrium: the need for coercion.

Game theory

Max Weber: two steps of rationality:

- Rationality of purpose: what do I want ?
- Rationality of means: how shall I get it ?

In life situations there are usually a multiplicity of purposes. Games are situations where the first step has been solved : the purpose is well-defined and quantified, so that mathematics can be used

Games can be created by institutions which implement the rules. Need at least two players (chess), often more (athletics, soccer). Can be cooperative (soccer) or not (chess)

Interesting particular case: whatever one wins, the other loses. They allow for a matrix representation, where player 1 chooses the line and player 2 choose the column. In the resulting place, the number is player 1 gain, which is also player 2 loss

1

0	1	2
7	8	3
6	5	4

Interesting particular case: whatever one wins, the other loses. They allow for a matrix representation, where player 1 chooses the line and player 2 choose the column. In the resulting place, the number is player 1 gain, which is also player 2 loss

1.

$$\begin{matrix} 0 & 1 & 2 \\ 7 & 8 & 3 \\ 6 & 5 & 4 \end{matrix}$$

2. Morra cinese (sasso/forbici/carta)

Mixed strategies

Penalty kicks in soccer
Elementary form: Left, Right ; equuilibrium is even probabilities
Professional form: Left, Center, Right, equilibrium probabilities depend on the players

Chiappori, P.-A., S. Levitt, and T. Groseclose (2002). *"Testing Mixed-Strategy Equilibria When Players Are Heterogeneous: The Case of Penalty Kicks in Soccer ."* American Economic Review, 92 (4): 1138-1151.

Theorem

Every two-person zero-sum game with finite decision sets has an equilibrium in mixed strategies

Player n chooses decision d_n with the equilibrium probability p_n over D_n

The prisoner's dilemma

1. Cooperate with the police or not ?

$$\begin{matrix} -1|-1 & -3|1 \\ 1|-3 & -5|-5 \end{matrix}$$

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2. The role of suspicion

1. Cooperate with the police or not ?

$$\begin{array}{cc} -1|-1 & -3|1 \\ 1|-3 & -5|-5 \end{array}$$

2. The role of suspicion
3. It is not important what I plan to do. What is important is what the other player thinks I will do

The nuclear world

1. First-strike capability

$$\begin{array}{cc} -1|-1 & 0|-1 \\ -1|0 & 0|0 \end{array}$$

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4. https://d3n8a8pro7vhmx.cloudfront.net/ican/pages/1549/attachments/Enough-is-Enough-Global-Nuclear-Weapons-Spending-2020-published-13052020.pdf?1589365383

The case of warfare

Warfare is a life situation where there is no ambiguity of purpose (survival), and for this reason it is considered part of game theory. The first pionneers in game theory were military writers and historians

- Sun-Zi (400 BC ?) *The art of warfare*
- Thucydides son of Oloros (460-400 BC ?) *The Peloponnesian War*
- Francesco Guicciardini *Storia d'Italia*

Strategic thinking: put yourself in the opponent's shoes and anticipate.

Individual freedom and collective action 3:
Markets and their inadequacies

A moment in history

- Economic growth
- Global trade

An illusion:

- Adam Smith (1776): "This division of labour, from which so many advantages are derived, is not originally the effect of any human wisdom, which foresees and intends that general opulence to which it gives occasion. It is the necessary, though very slow and gradual consequence of a *certain propensity in human nature* which has in view no such extensive utility; the propensity to truck, barter, and exchange one thing for another".
- Anthropologists have never found any propensity to truck and barter. The basic relation is the gift (Marcel Mauss, *Essai sur le Don*) which has to be repaid at some later point in order to keep social relations alive
- Money, salaried work, concepts so familiar as to seem natural, are not. Most societies functioned on credit, and artisans sold their work, not their time

How did it happen?

The first steps of industrial society: sugar and the plantation economy
Climate course, p.247
https://www.ceremade.dauphine.fr/~ekeland/Articles/06-2021/Climat.pdf

Production driven by consumption
Consumption driven by production
The world system: energy, production, consumption

Economic growth as the aim of policy

- In line with economic theory, human beings are seen exclusively as consumers
- The more you consume, the better off you are, hence the desire for growth
- GDP is a monetary measure of the market value of all the final goods and services produced in a specific time period by countries
- Y is the sum of consumption C, investment I, government expenditures G and net exports E:

$$Y = C + I + G + E$$

GDP as a measure of the common good

- The origins of GDP in WWII
- The blind side of GNP: women, and nature
- The unclear distinction between production and consumption
- The production of people

Goods

- What is an economic good (or an economic bad)? Whatever affects our utilities as consumers
- Classification of goods

	Exclusive	Non-exclusive
Rival	Private	Commons
Non-rival	Club	Public

- Private good: food, clothes
- Public good: lighthouse, knowledge
- Common good: fisheries, climate
- Club good: tolls

The production problem

- Factors of production: renewable or not
- Labor and capital as factors of production $Y = F(K, L)$
- The problem of value
- The distribution of surplus value

The consumption problem

How do you distribute goods among the population ?
For private goods: the market
It is ideologically the preferred one
The EU treaties aim to foster free and undistorted competition

Markets: simple case

Markets confronts buyers (consumers)and sellers (producers). The price is announced before the transactions begin, it is the same for everyone and everyone plans accordingly. Producers produce q at a cost $c(q)$
How do they react to the price p ?

- As p increases, consumers will buy less: this is the demand function $D(p)$ which slopes downwards
- As p increases, producers will produce more: this is the supply function $S(p)$ which slopes upwards

The equilibrium price $\bar{p}$ is at the intersection point

Are monopolies bad ?

Each producer has a cost function $c(q)$, where q is quantity produced.

- Under perfect competition , the producer will take p as given, and will try to maximise profit $pq - c(q)$, hence:

$$c'(q_1) = p$$

- If the producer has a monopoly , he can influence the price p by restricting the quantity q he/she produces. Now p becomes a decreasing function of q. He/she has now to maximise $p(q)\,q - c(q)$. This yields:

$$c'(q_2) = p(q_2) + q_2 p'(q_2) < p(q_2)$$

Are monopolies bad ?

- Let $\bar{p}$ be the (observed) market price.
  - If the producer functions under perfect competition, he produces q_1 given by $c'(q_1) = \bar{p}$
  - If the producer is a monopolist, he produces q_2 given by $c'(q_2) = \bar{p} + \alpha$ with $\alpha < 0$
- So $c'(q_2) < c'(q_1)$:
  - if $c''(q) < 0$ (c is concave, decreasing returns to scale) then $q_2 > q_1$
  - if $c''(q) > 0$ (c is concave, increasing returns to scale) then $q_2 < q_1$

Exchange economies

An exchange economy consists of N goods and I consumers: there is no production

- $x = (x_1, ..., x_N)$ is a basket of goods, with good n in quantity x_n
- the total quantity available is $X = (X_1, ..., X_N)$
- consumer i has utility $u_i(x)$ and has wealth w_i

An equilibrium is a set of prices $p = (p_1, ..., p_n)$ such that:

- each consumer i optimizes independently of the others

$$u_i(x(i)) \geq u_i(x) \text{ for all } x \text{ such that } \sum p_n x_n \leq w_i$$

- the total quantity X is distributed:

$$\sum \bar{x}(i) = X$$

The argument for markets

- The distribution resulting from an equilibrium is Pareto optimal . Note that it depends on the initial disctibution of wealth, which remains unchanged
- It is achieved simply by indicating a set of prices, that is N numbers $(p_1, ..., p_N)$. In contrast, allocating directly baskets to each consumer would require computing $\bar{x}(i)$ for each consumer i, that is NI numbers.
- Someone would have to do this immense computation (presumably the government) while the market does it by itself (the invisible hand). No government interference, no need for coercion (it is a self-implementing contract)
- Is it just ? ?Hayek claims yes "*If the rules of the game are fair, no one can complain about the outcome*". This forgets the initial distribution of wealth. It would be correct in a world with 100% inheritance tax and every young adult would be given the same initial endowment

Production economies

One can get closer to real economies by introducing producers . Each of them uses goods (labor, oil, capital) to produce other goods (knowledge, guns, bread). Producers maximise their profit and transmit it to their owners, who are themselves consumers. The economy then closes: if an equilibrium price is announced:

- producers choose how much to produce and give the profit to their owners
- consumers cash in their revenue and buy what they prefer among what they can afford
- and all the available goods are consumed, either for production or for consumption

The beauty of the thing is that no one worries about anything except the prices : there is no thought about coordination. Private vices (maximising profit or consumption) turn out to be public virtues

Do equilibria exist ?

Under certain circumstances, yes, the most important being:

- full information
- perfect competition
- no externalities

Note that:

- the proof is difficult and has a long history (Arrow-Debreu ,1954)
- there may be several equilibria
- there is no formal procedure to get there

Creating markets

Since the end of the Middle Ages and the colonization of the Americas, creating markets has been the preferred way Western societies have operated.

- From1500 on, successive creations of the global markets for slaves, sugar, and cotton, with immense disruption to local populations
- Between 1750 an 1850, the *Enclosure Laws* and the *Poor Laws* create in Great Britain a market for land and a market for labour
- Since 1952, the EU treaties aim to establish free and unhindered competition within the common market, and this is why the EU dismantles the monopolies of member states, such as railways, post office, and electricity

The information problem

- N persons own a car, N other persons want a car

The information problem

- N persons own a car, N other persons want a car
- The car can be either a good car or a bad one

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- Buyers are willling to pay 5 000 Euros or less
- Only the owner knows whether the car is bad or not
- The result is that (with linear utilities) if the proportion of bad cars in the market is higher than $1/5$, the market will not function

Markets are not spontaneous

Modern markets need tremendous amounts of information to function, that is, to find an equilibrium

This information must be provided by a trusted and powerful regulatory authority

It is particularly important in finance, where the information problem is acute

The externalities problem

All the equilibrium analysis relies on the fact that I am concerned only by my own actions: if I consume x and my neighbour y, then my utility is $u(x)$, not $u(x, y)$. This is not always the case.

- Consider a market for one good with N identical producers acting under perfect competition (price-takers).
- Producing q units costs $\frac{\alpha}{2}q^2$ and emits βq units of carbon
- Each producer maximises profit so the total supply is $S(p) = \frac{N}{\alpha}p$ and carbon emissions $N\frac{\beta}{\alpha}p$, which may exceed the safety threshold Q

This is called an externality. Note that the individual contributions are small $\frac{\beta}{\alpha}p$

Ways to limit emissions

- The government can set individual quotas for carbon . The total quota is Q, so every producer is limited to $\frac{Q}{N}$, so she can produce only $q = \frac{1}{\beta}\frac{Q}{N}$. Supply is diminished and the equilibrium price increases.
- The government can also set a carbon tax at a rate τ. The producer must pay τ for each ton of carbon emitted, and her cost becomes $\frac{\alpha}{2}q^2 + \gamma\beta q$. The new profit maximiser is given by $p = \alpha q + \gamma\beta$, the carbon emissions are $\frac{N\beta}{\alpha}\left(p - \gamma\beta\right)$.
- To limit emissions to Q, one sets the tax γ so that $p - \gamma\beta = \frac{\alpha Q}{\beta N}$.
- The two approaches are equivalent

The value of nature

Another problem with markets is that they recognize no intrinsic value to nature. the cost of oil is the cost of finding it, drilling it up and bringing it to market. The cost of a tree is the cost of cutting it down and bringing it to market

- Consider a forest with N trees and a yearly renewal rate of r
- The forest is owned by a logging company which is a price-takers.
- The cost of producing q logs in a year is $\frac{\alpha}{2}q^2$
- if the market price is p the logger will produce quantity $\frac{1}{\alpha}p$ each year
- if $\frac{1}{\alpha}p > rN$ then the forest will be wiped out

Fisheries

Conclusion: restoring multidimensionality

- to do mathematics, you need numbers: leads to indicators
- to do optimisation, you need a single indicator: for governments the GNP, for firms the profit
- this leads to problems because a complex system (the economy, society, nature) cannot be understood in a single number
- restore multidimensionality: at least consider several indicators, knowing that they cannot all be optimized simultaneousy, and do not neglect qualitative analysis