Institut d'Astrophysique de Paris
Unité mixte de recherche 7095 *CNRS - Université Pierre et Marie Curie*

Workshop on Astronomy and Dynamics
at the occasion of Jacques Laskar's 60th birthday

Testing Modified Gravity with Planetary Ephemerides

Luc Blanchet

Gravitation et Cosmologie ($\mathcal{G}\mathbb{R}\varepsilon\mathbb{C}\mathcal{O}$)
Institut d'Astrophysique de Paris

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Evidence for dark matter in astrophysics

1. Oort [1932] noted that the sum of observed mass in the vicinity of the Sun falls short of explaining the vertical motion of stars in the Milky Way
2. Zwicky [1933] reported that the velocity dispersion of galaxies in galaxy clusters is far too high for these objects to remain bound for a substantial fraction of cosmic time
3. Ostriker & Peebles [1973] showed that to prevent the growth of instabilities in cold self-gravitating disks like spiral galaxies, it is necessary to embed the disk in the quasi-spherical potential of a huge halo of dark matter
4. Bosma [1981] and Rubin [1982] established that the rotation curves of galaxies are approximately flat, contrarily to the Newtonian prediction based on ordinary baryonic matter

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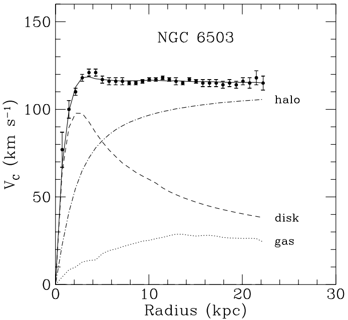
Rotation curves of galaxies are approximately flat



- For a circular orbit we expect

$$v(r) = \sqrt{\frac{G\,M(r)}{r}}$$

- The fact that $v(r)$ is constant implies that beyond the optical disk

$$\boxed{M_{\text{halo}}(r) \simeq r \qquad \rho_{\text{halo}}(r) \simeq \frac{1}{r^2}}$$

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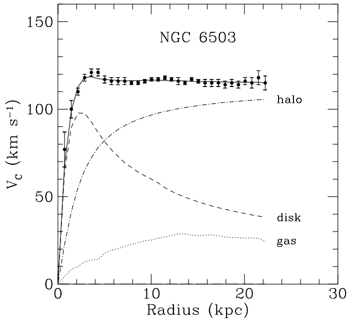

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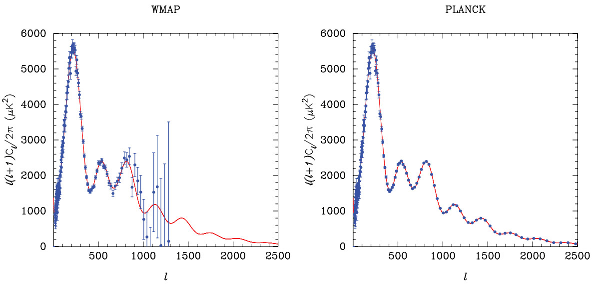
The cosmological concordance model Λ-CDM



This model brilliantly accounts for:

- The mass discrepancy between the dynamical and luminous masses of clusters of galaxies
- The precise measurements of the anisotropies of the cosmic microwave background (CMB)
- The formation and growth of large scale structures as seen in deep redshift and weak lensing surveys
- The fainting of the light curves of distant supernovae

Challenges with CDM at galactic scales

The CDM paradigm faces severe challenges when compared to observations at galactic scales [McGaugh & Sanders 2004, Famaey & McGaugh 2012]

1. Unobserved predictions
   - Numerous but unseen satellites of large galaxies
   - Phase-space correlation of galaxy satellites
   - Generic formation of dark matter cusps in galaxies
   - Tidal dwarf galaxies dominated by dark matter
2. Unpredicted observations
   - Correlation between mass discrepancy and acceleration
   - Surface brightness of galaxies and the Freeman limit
   - Flat rotation curves of galaxies
   - Baryonic Tully-Fisher relation for spirals
   - Faber-Jackson relation for ellipticals

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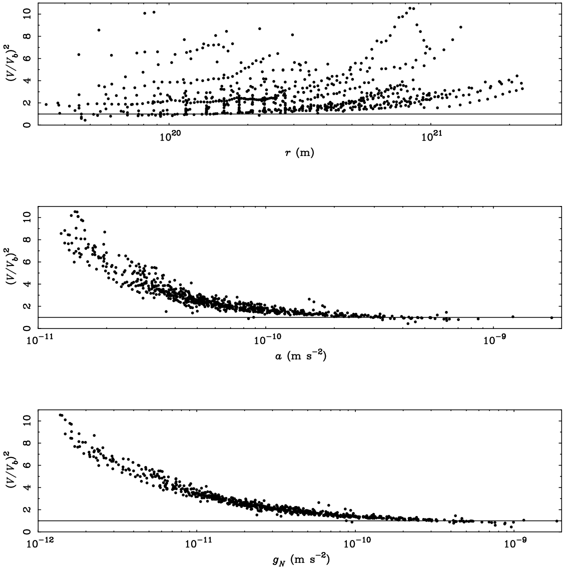
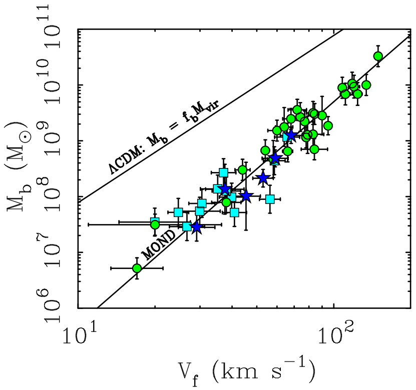
All these challenges are mysteriously solved (sometimes with incredible success) by the MOND empirical formula [Milgrom 1983]

Mass discrepancy versus acceleration



Baryonic Tully-Fisher relation [Tully & Fisher 1977, McGaugh 2011]



We have approximately $V_\mathrm{f} \simeq \left(G\,M_\mathrm{b}\,a_0\right)^{1/4}$ where $a_0 \simeq 1.2\times 10^{-10}\mathrm{m/s^2}$ is very close (mysteriously enough) to typical cosmological values

$$a_0 \simeq 1.3\,a_\Lambda \quad \text{with} \quad a_\Lambda = \frac{c^2}{2\pi}\sqrt{\frac{\Lambda}{3}}$$

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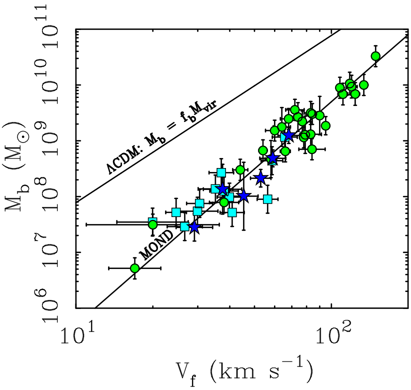

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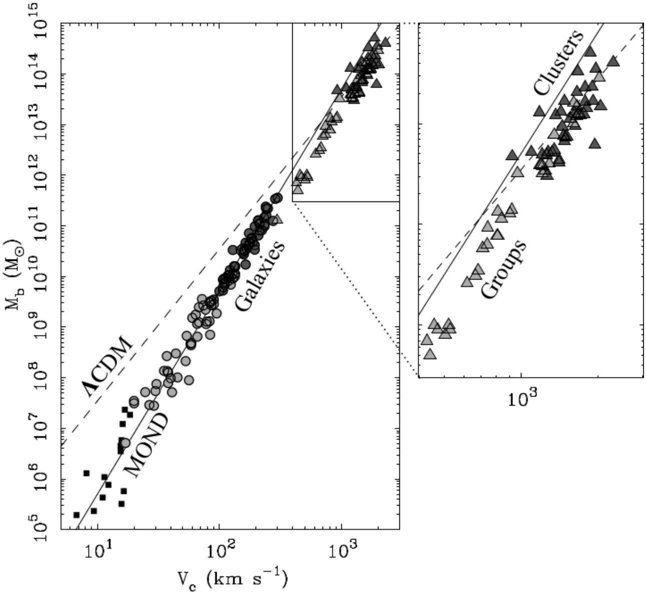
MOND versus Λ-CDM



Modified Poisson equation [Milgrom 1983, Bekenstein & Milgrom 1984]

MOND takes the form of a modified Poisson equation

$$\boldsymbol{\nabla} \cdot \left[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{fonction MOND}} \boldsymbol{g} \right] = -4\pi\, G\, \rho_{\text{baryon}} \quad \text{avec} \quad \boldsymbol{g} = \boldsymbol{\nabla} U$$

- The Newtonian regime is recoved when $g \gg a_0$
- In the MOND regime $g \ll a_0$ we have $\boxed{\mu \simeq g/a_0}$

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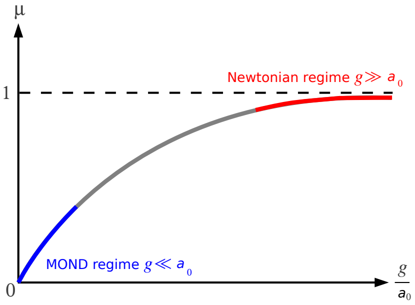

Modified gravity theories

1. Generalized Tensor-Scalar theory (RAQUAL) [Bekenstein & Sanders 1994]
2. Tensor-Vector-Scalar theory (TeVeS) [Bekenstein 2004, Sanders 2005]
3. Generalized Einstein-Æther theories [Zlosnik *et al.* 2007, Halle *et al.* 2008]
4. Khronometric theory [Blanchet & Marsat 2011, Sanders 2011, Barausse *et al.* 2015]
5. Bimetric theory (BIMOND) [Milgrom 2012]

- These theories contain non-standard kinetic terms parametrized by an arbitrary function which is linked *in fine* to the MOND function
- In some cases they have stability problems associated with the fact that the Hamiltonian is not bounded from below [Clayton 2001, Bruneton & Esposito-Farèse 2007]
- Generically they have problems to recover the cosmological model Λ-CDM at large scales and the spectrum of CMB anisotropies [Skordis, Mota *et al.* 2006]

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Dielectric analogy of MOND [Blanchet 2006]

- In electrostratics the Gauss equation is modified by the polarization of the dielectric (dipolar) material

$$\nabla \cdot \Big[\underbrace{(1+\chi_e)\boldsymbol{E}}_{\boldsymbol{D}\text{ field}}\Big] = \frac{\rho_e}{\varepsilon_0} \qquad \Longleftrightarrow \qquad \nabla \cdot \boldsymbol{E} = \frac{\rho_e + \rho_e^{\text{polar}}}{\varepsilon_0}$$

- Similarly MOND can be viewed as a modification of the Poisson equation by the polarization of some dipolar medium

$$\nabla \cdot \left[\mu\left(\frac{g}{a_0}\right)\boldsymbol{g}\right] = -4\pi\, G\, \rho_{\text{b}} \qquad \Longleftrightarrow \qquad \nabla \cdot \boldsymbol{g} = -4\pi\, G \left(\rho_{\text{b}} + \underbrace{\rho^{\text{polar}}}_{\text{dark matter}}\right)$$

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Modified dark matter theories

1. Dipolar dark matter in standard GR [Blanchet & Le Tiec 2008, 2009]
2. Bimetric theories with two species of DM particles [Bernard & Blanchet 2014]
3. Massive bigravities with dark matter sector [Blanchet & Heisenberg 2015]

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What about the Solar System scale?

1. In spherical symmetry the MOND equation becomes

$$\mu\left(\frac{g}{a_0}\right) g = g_{\mathrm{N}} \equiv \frac{GM_\odot}{r^2}$$

2. Suppose MOND approaches the Newtonian regime like

$$\mu\left(\frac{g}{a_0}\right) = 1 - k\left(\frac{a_0}{g}\right)^q \quad \text{when} \quad g \to \infty$$

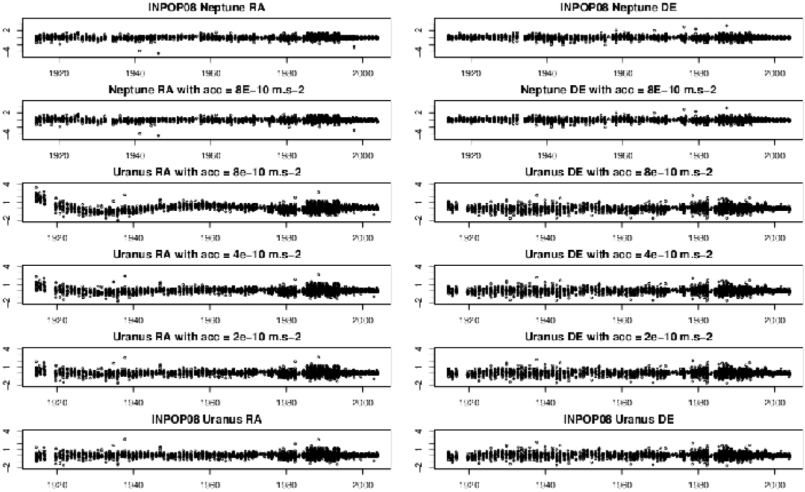
3. With $r_0 = \sqrt{GM_\odot/a_0}$ the MOND transition radius for the Sun

$$g = g_{\mathrm{N}} + k\, a_0 \left(\frac{r}{r_0}\right)^{2q-2}$$

When $q = 1$ this gives a Pioneer-like anomaly

$$a_{\mathrm{P}} = k\, a_0$$

Solar System data and Pioneer anomaly [Fienga *et al.* 2009]



The data exclude a Pioneer-like anomaly at the level $5 \times 10^{-13} \mathrm{m/s}^2$

The external field effect in MOND [Milgrom 1983]

1. Open star clusters in our Galaxy do not show evidence for dark matter despite their typical low internal gravity $g_\mathrm{i} \ll a_0$
2. In the presence of the external Galactic field g_e the MOND equation which is non-linear can be approximated by

$$\mu\left(\frac{|\boldsymbol{g}_\mathrm{i}+\boldsymbol{g}_\mathrm{e}|}{a_0}\right)\boldsymbol{g}_\mathrm{i} \approx \boldsymbol{g}_\mathrm{i}^\mathrm{Newtonian}$$

- When $a_0 \lesssim g_\mathrm{e}$ the sub-system exhibits Newtonian behaviour
- When $g_\mathrm{i} \lesssim g_\mathrm{e} \lesssim a_0$ the system is still Newtonian but with an effective Newton's constant G/μ_e

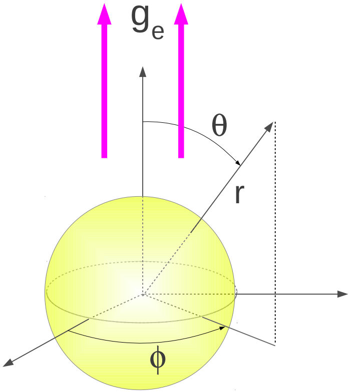
The EFE results from a violation of the strong version of the equivalence principle

The gravitational dynamics of a system is influenced by the external gravitational field in which the system is embedded

Deformation of the Sun's field by the Galactic field

The external field effect is a prediction of the non-linear Poisson equation

$$\nabla \cdot \left[\mu\left(\frac{g}{a_0}\right) \nabla U \right] = -4\pi\, G\, \rho_{\mathrm{b}}$$



The MOND field of the Sun, in the presence of the external field of the Galaxy, is deformed along the direction of the Galactic center

$$U = \boldsymbol{g}_{\mathrm{e}} \cdot \mathbf{x} + \frac{G M_\odot / \mu_{\mathrm{e}}}{r\sqrt{1 + \lambda_{\mathrm{e}} \sin^2\theta}} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

This effect influences the motion of inner planets of the Solar System

Multipole expansion of the MOND field of the Sun

1. The Newtonian physicist measures from the motion of planets the internal gravitational potential $u = U - \boldsymbol{g}_\mathsf{e} \cdot \boldsymbol{x}$ and detects the anomaly

$$\delta u = u - u_\mathsf{N} = G \int \frac{\mathrm{d}^3\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|} \, \rho_\mathsf{pdm}(\mathbf{x}', t)$$

2. Since the phantom dark matter vanishes in the strong-field regime near the Sun δu is an harmonic function and admits the multipole expansion

$$\delta u = \sum_{l=0}^{+\infty} \frac{(-)^l}{l!} \, x^L Q_L$$

   where Q_L are trace-free multipolar coefficients

3. This expansion is valid in the region inside the MOND transition radius

$$r_0 = \sqrt{\frac{GM_\odot}{a_0}} \approx 7100\,\mathrm{AU}$$

Effect in the Solar System [Milgrom 2009, Blanchet & Novak 2011]

1. The effect is dominantly quadrupolar and grows with the distance squared

$$u = \frac{GM_\odot}{r} + \frac{1}{2} x^i x^j Q_{ij}$$

2. The quadrupole moment is aligned in the direction of the Galactic center

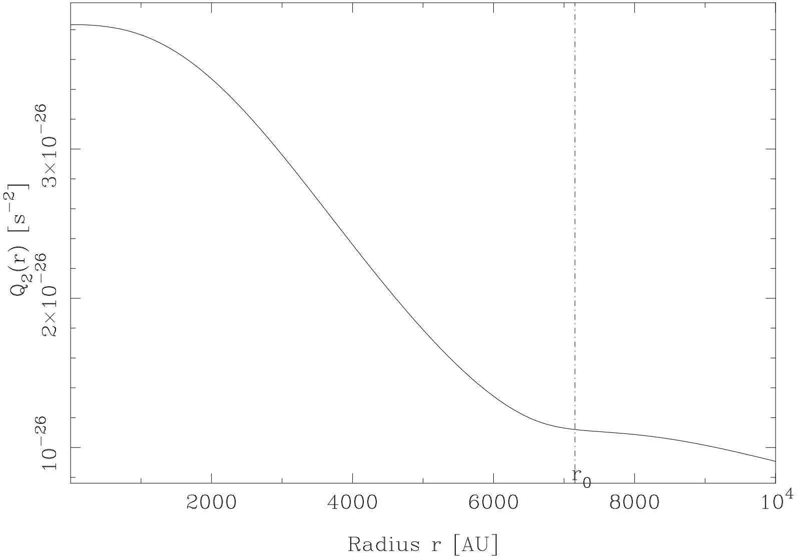
$$Q_{ij} = Q_2 \left(e_i e_j - \frac{1}{3}\delta_{ij} \right)$$

3. The quadrupole moment is computed by solving numerically the MOND equation in the presence of the external galactic field. We find

$$2.1 \times 10^{-27}\ \mathrm{s}^{-2} \lesssim Q_2 \lesssim 4.1 \times 10^{-26}\ \mathrm{s}^{-2}$$

depending on the MOND function in use

Quadrupole moment as a function of distance



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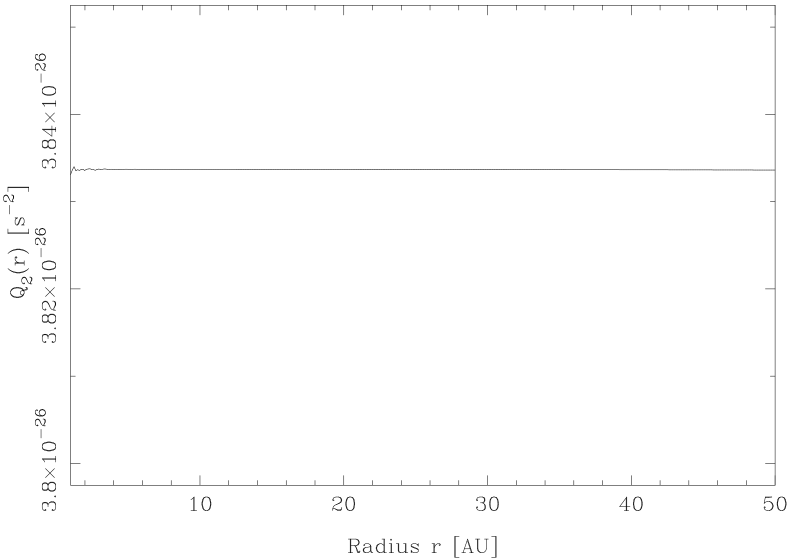

Effect on the dynamics of Solar System planets

The quadrupole effect yields a suplementary precession of the semi-major axis of planets of the Solar System [Blanchet & Novak 2011]

$$\left\langle \frac{\mathrm{d}e}{\mathrm{d}t} \right\rangle = \frac{5Q_2 e\sqrt{1-e^2}}{4n}\sin(2\tilde{\omega})$$

$$\left\langle \frac{\mathrm{d}\ell}{\mathrm{d}t} \right\rangle = n - \frac{Q_2}{12n}\Big[7 + 3e^2 + 15(1+e^2)\cos(2\tilde{\omega})\Big]$$

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Jacques agrees with these results $\Longrightarrow$

Jacques gives an exam to undergraduate students

J. Laskar *February 15, 2011* *examen-M2obs-2011.tex*1

1 MOND et les mouvements planétaires

Dans un article très récent, Luc Blanchet et Jérome Novak (MNRAS, 2010, sous presse, http ://arxiv.org/abs/1010.1349), étudient les conséquence de la théorie MOND (Modified Newtonian Dynamics) à l'échelle du Système solaire. La théorie prédit une rupture du principe d'équivalence dont une conséquence est d'induire dans la dynamique des corps du Système solaire une composante quadrupolaire dans la direction du centre galactique. Le but du problème est de calculer l'effet séculaire de cette correction sur les orbites des planètes du Système solaire.

On considère donc ici un mouvement Keplerien d'un corps de masse m (en P) autour d'un corps central fixe de masse M (en O), de Hamiltonien

$$H_0 = \frac{1}{2} m \dot{\mathbf{r}}^2 - \frac{m\mu}{r} \tag{1}$$

avec $\mu = GM$, et $\mathbf{r} = \mathbf{OP}$.

La contribution de la théorie MOND rajoute à H_0 l'energie potentielle

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Comparison with Solar System ephemerides

Predicted values for the orbital precession

Quadrupolar precession rate in mas/cy						
	Mercury	Venus	Earth	Mars	Jupiter	Saturn
μ_1	0.04	0.02	0.16	-0.16	-1.12	5.39
μ_2	0.02	0.01	0.09	-0.09	-0.65	3.12
μ_5	7×10^{-3}	3×10^{-3}	0.03	-0.03	-0.22	1.05
μ_{20}	2×10^{-3}	10^{-3}	9×10^{-3}	-9×10^{-3}	-0.06	0.3

Best published residuals for orbital precession

Postfit residuals for the precession rates in mas/cy						
	Mercury	Venus	Earth	Mars	Jupiter	Saturn
[Pitjeva 2005]	-3.6 ± 5	-0.4 ± 0.5	-0.2 ± 0.4	0.1 ± 0.5	-	-6 ± 2
[Fienga *et al.* 2009]	-10 ± 30	-4 ± 6	0 ± 0.016	0 ± 0.2	142 ± 156	-10 ± 8
[Fienga *et al.* 2010]	0.4 ± 0.6	0.2 ± 1.5	-0.2 ± 0.9	0 ± 0.1	-41 ± 42	0.2 ± 0.7

The MOND function is constrained by the precession of Saturn to be

$$\mu = \frac{g}{a_0} + o\left(\frac{g}{a_0}\right)^n \quad \text{when} \quad g \to 0$$

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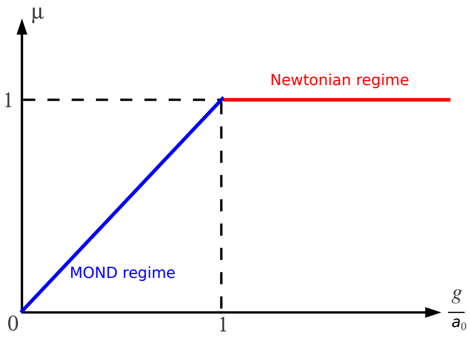
Constraining the MOND function



Solar system dynamics constraints the MOND interpolating function to be essentially exactly one when $g \geqslant a_0$ exactly linear when $g \leqslant a_0$

Conclusions

1. Λ-CDM is an extremely successful cosmological model but
   - poses the problem of the fundamental constituents of the Universe
   - faces severe challenges when compared to observations at galactic scale
2. MOND is a successful alternative to dark matter at galactic scales but
   - is based on an empirical formula not explained in terms of fundamental physics
   - does not work at galaxy cluster scale and in cosmolgy
3. Reconciling Λ-CDM at cosmological scale and MOND at galactic scale into a single relativistic theory is a great challenge
4. A non-standard form of dark matter could exist, which might explain the antinomic aspects of DM that we see at cosmological and galactic scales
5. MOND can be tested in the Solar System dynamics thanks to the external field effect by looking at precession anomalies in the motion of inner planets

Bon anniversaire Jacques!