CREEP TIDE THEORY

Synchronization, Dissipation , Circularization

S.Ferraz-Mello (USP)
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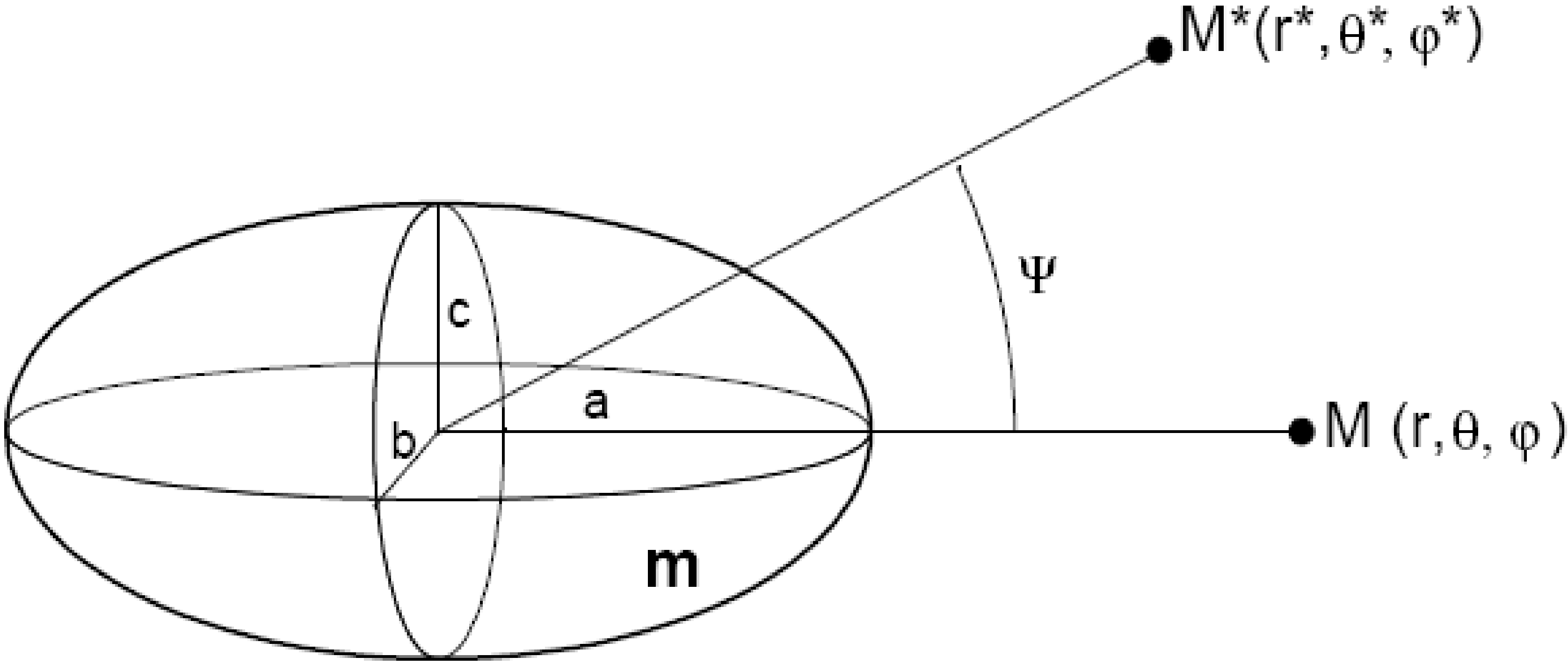
G.H.Darwin, 1880

Goldreich, 1963
Kaula, 1964
Alexander, 1973
Zahn, 1977
Mignard, 1979
Hut, 1981
Eggleton &c, 1998
Mardling & Lin, 2002
Ferraz-Mello &c, 2008 (**REVIEW**)
and many others

Static tide (Jeans prolate spheroid)

$$\epsilon = \frac{a}{b} - 1 = \frac{15}{4}\left(\frac{M}{m}\right)\left(\frac{R}{r}\right)^3.$$



Potential at M*

$$\mathbf{U} = -\frac{Gm}{r^*} - \frac{k_f G M R^5}{2r^3 r^{*3}}(3\cos^2\Psi - 1);$$

$k_f = 15A/4mR^2$ (Fluid Love number)

Dynamical tide

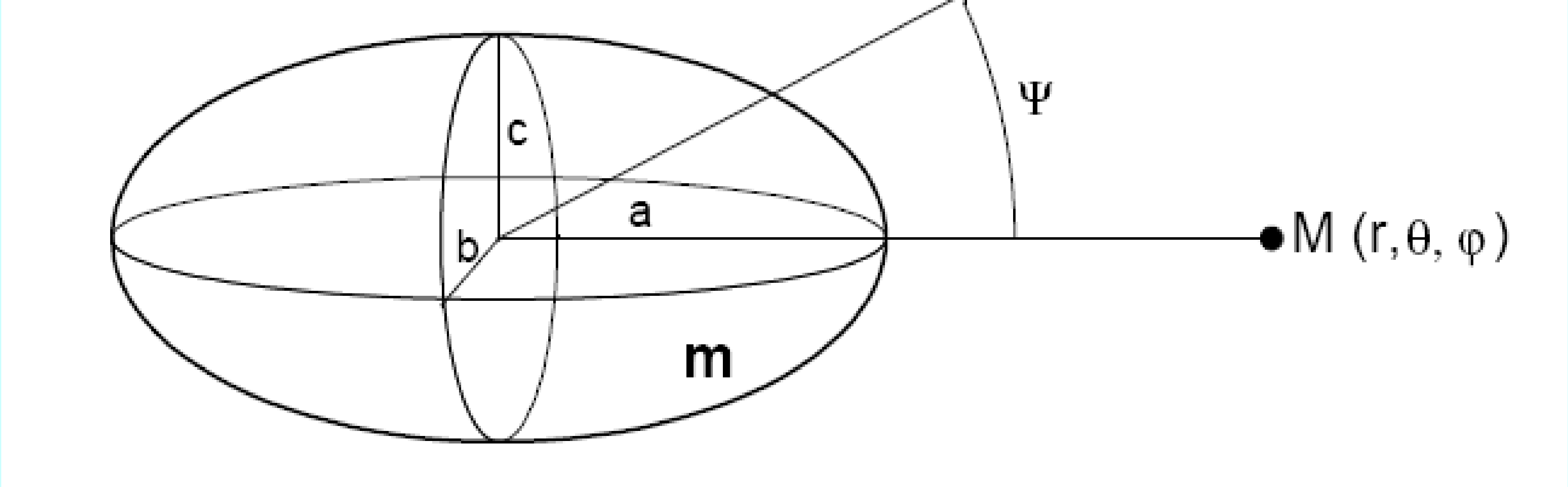
Introduces phase lags:

$$\cos\psi \rightarrow \cos(\psi-\varepsilon)$$

Decomposes :

$$\cos(\psi-\varepsilon) \rightarrow \cos\psi + \varepsilon\sin\psi$$

elastic+anelastic



Force acting on M (due to elastic tide)

$F_1 = \mathcal{F}(r)$ → torque=0

$F_2 = 0$ → <W>=dissipation=0

$F_3 = 0$ → de/dt=0

Recent theories (of the anelastic tide):

Efroimsky, Lainey, Williams (2007-2012)
Remus et al. (2012)
Ogilvie & Lin (2004)
Greenberg (2009)
S.Ferraz-Mello (2012-2013)
Correa, Boué, Laskar & Rodríguez (2014)

- All of them depart in a larger or lesser extent from Darwin's theory.

Anelastic Tide

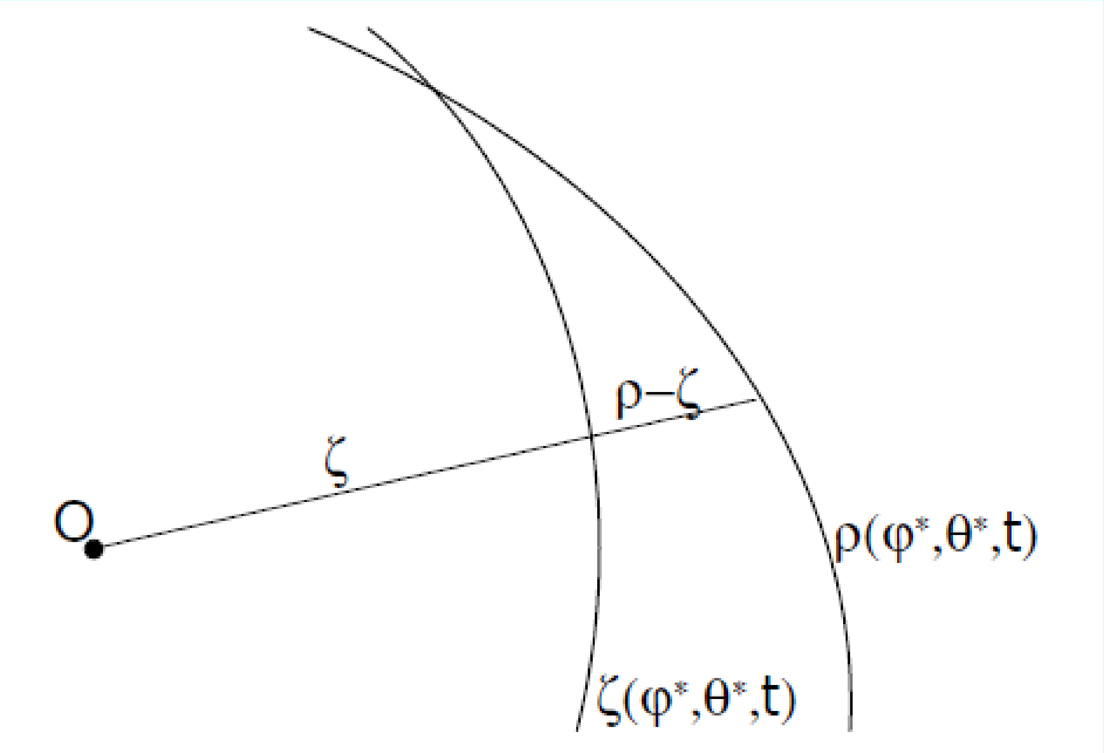
New theory

Aim:

- **Substitution of plugged**

ad hoc lags by one physical law

S.Ferraz-Mello
DDA 2012 (astro-ph 1205.3957)
Cel.Mech.Dyn.Astron.116,109(2013)



ANSATZ

$$\dot{\zeta} = \gamma(\rho - \zeta).$$

ζ….. Surface of the body
ρ….. Surface of instantaneous equilibrium (**VIRTUAL**)

$$\dot{\zeta} = \gamma(\rho - \zeta).$$

↔ Newtonian **CREEP**

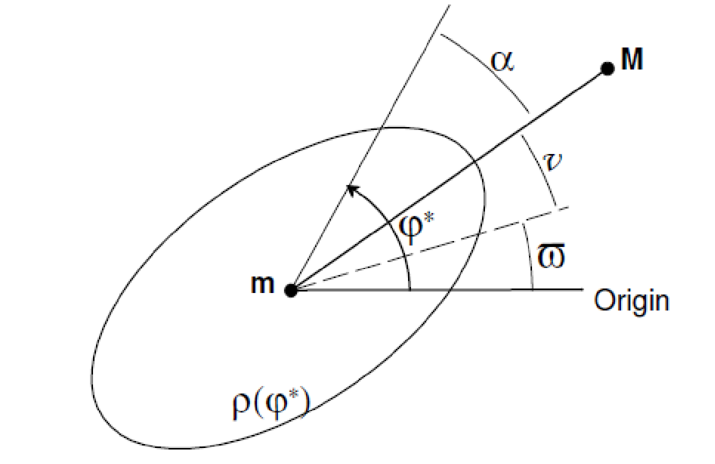
$$\gamma = \frac{wR}{2\eta} = \frac{3gm}{8\pi R^2 \eta}$$

Relaxation factor (critical frequency)

Approx. Solution of the Navier-Stokes equation

Ref: Happel and Brener, Low Reynolds number Hydrodynamics, Kluwer, 1973 + **Darwin, 1879**

$$\dot{\zeta} + \gamma\zeta = \gamma R' + \frac{15\gamma R \sin^2\theta^*}{8}\left(\frac{M}{m}\right)\left(\frac{R}{a}\right)^3\left(\frac{a}{r}\right)^3 \cos(2\varphi^* - 2\varpi - 2v).$$



O.D.E. for ζ(t)

$$\varpi = \varpi_0 - \Omega t.$$

Simplifications **(here!)**:

homogeneous bodies

equator = orbit plane; $\theta^* = \pi/2$

solution

$$\zeta = Ce^{-\gamma t} + R' + \frac{15\mu R \sin^2\theta^*}{8}\left(\frac{M}{m}\right)\left(\frac{R}{a}\right)^3 \sum_{k=-N}^{N} \frac{E_k(e)\cos(2\bar{\alpha} + k\ell - \sigma_k)}{\sqrt{\mu^2 + (\nu + kn)^2}}$$

= Superposition of tidal bulges

Prolateness:

$$\epsilon_k = \frac{15}{4} E_k(e)\cos\sigma_k \left(\frac{M}{m}\right)\left(\frac{R}{a}\right)^3$$

$$E_k(e) = \frac{1}{2\pi\sqrt{1-e^2}} \int_0^{2\pi} \frac{a}{r} \cos\left(2v + (k-2)\ell\right)\, dv.$$

Cayley functions

$$\sigma_k = \arctan\left(\frac{kn + \nu}{\gamma}\right).$$

Phase of the forced terms

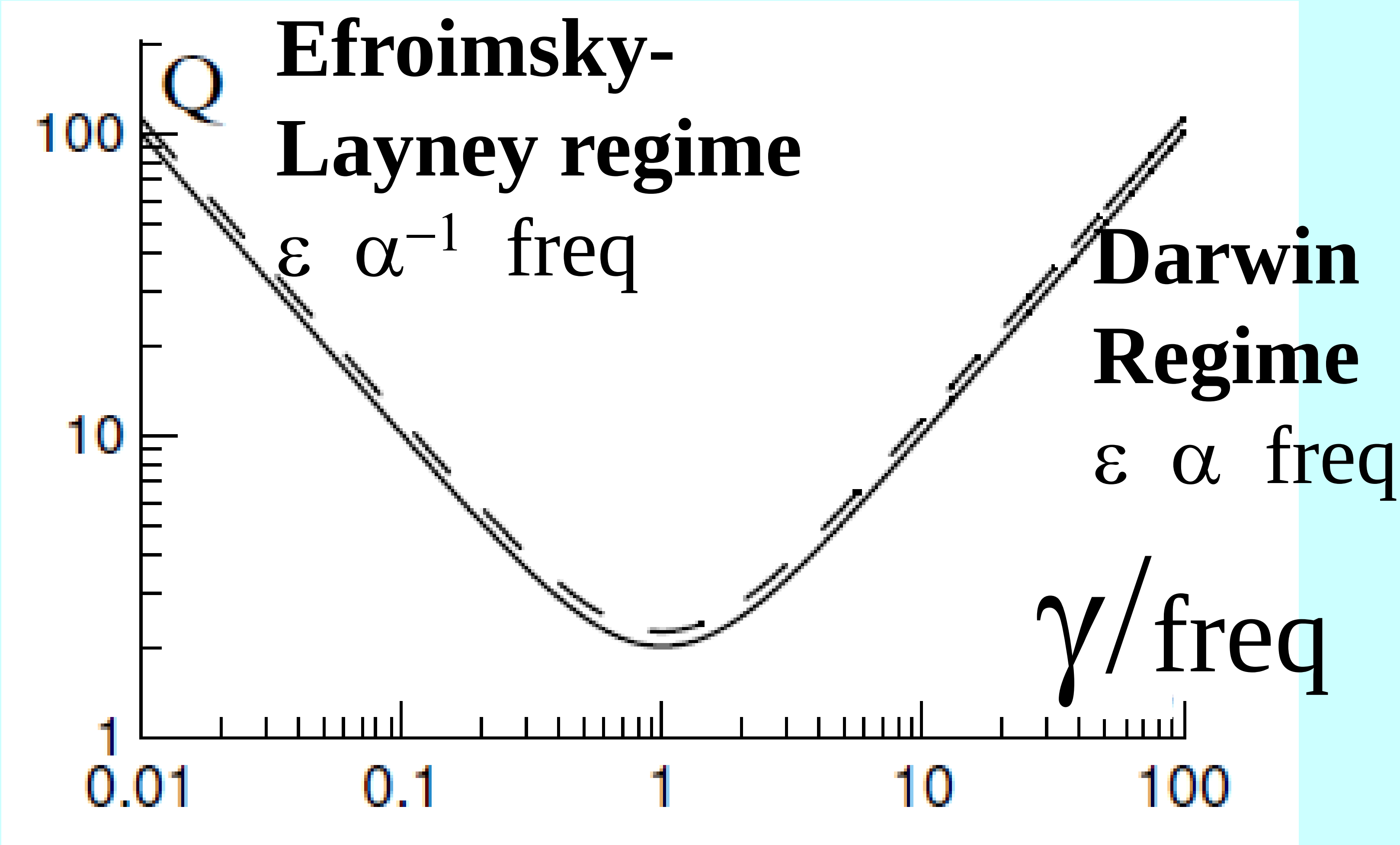
F = grad U → dW/dt = **F.v**

Energy dissipation

$$< \dot{W} >_{\mathrm{orb}} = -\frac{3k_f G M^2 R^5 n}{4a^6} \sum_{-N}^{N} (2-k) E_k^2(e) \sin 2\sigma_k$$

Comparing to Darwinian linear theories:

$$\mathbf{Q} = \frac{k_2}{k_f}\left(\chi + \frac{1}{\chi}\right) \qquad \chi = \mathbf{Freq}/\boldsymbol{\gamma}$$



Free rotation freq $= \nu$ (semi-diurnal)

Pseudo-synchronous freq $=$ n (monthly)

Torque

$$M_2 = \sum_{k=-N}^{N} -\frac{2k_f GMmR^2\epsilon_k}{5r^3}\sin(2v + (k-2)\ell - \sigma_k)$$

$$\dot{\nu} = -\frac{3GM\overline{\epsilon}_\rho}{2a^3}\sum_{k=2-N}^{2+N} E_{2,2-k}\sum_{j=-N-2+k}^{N-2+k} E_{2,2-k+j}\left(\sin 2\overline{\sigma}_k \cos j\ell + 2\cos^2\overline{\sigma}_k \sin j\ell\right)$$

Where ν is the semi-diurnal frequency $= 2\Omega - 2\dot{\lambda}$

and $$\sin 2\overline{\sigma}_k = \frac{2\gamma\left(\nu + (2-k)n\right)}{\gamma^2 + \left(\nu + (2-k)n\right)^2}$$

FIRST-order non-linear o.d.e.

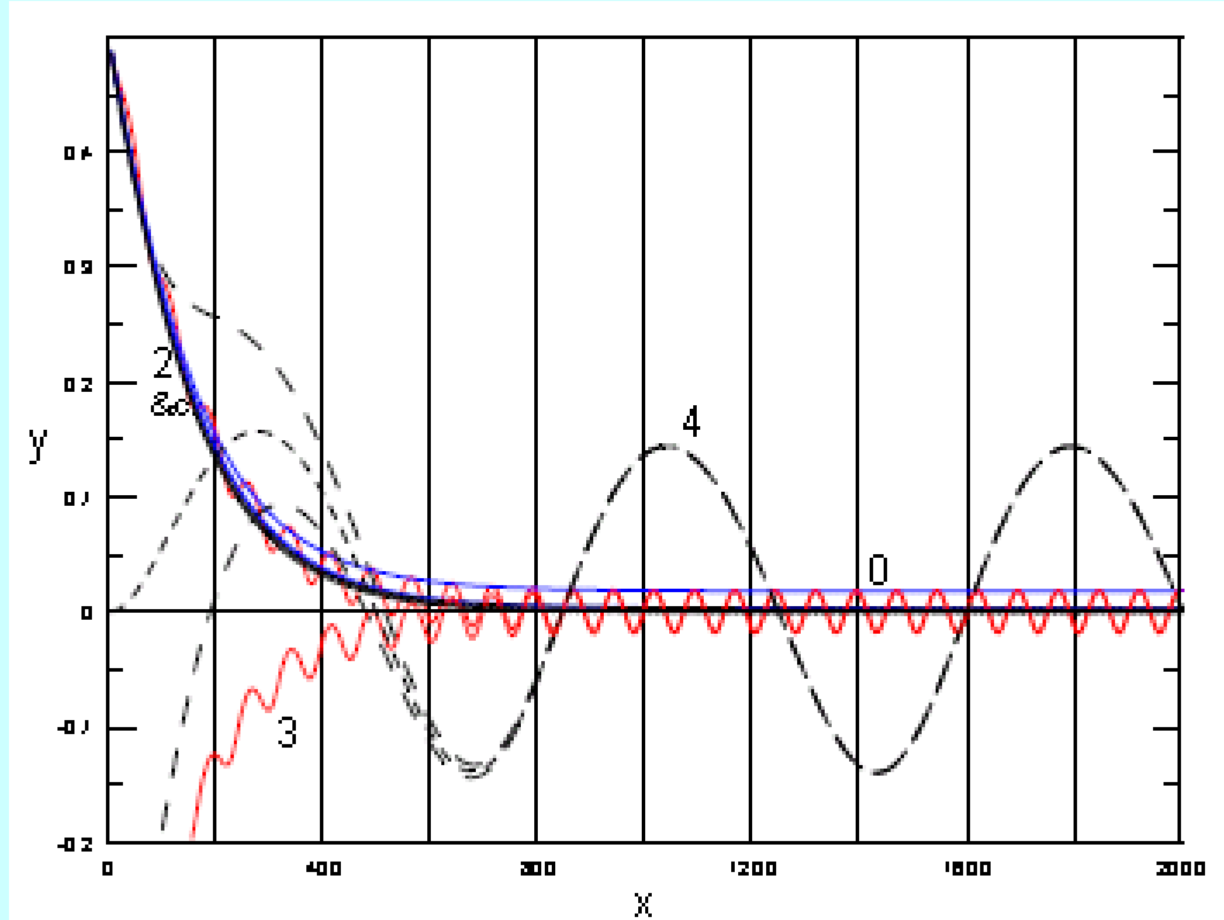
SYNCHRONIZATION

Simulations
near ν=0

(normalized variables)

$$y = \nu/\gamma$$

$$x = (n/\gamma)(t - t_0)$$



Parameter

$$\log_{10} n/\gamma$$

$$< \dot{\Omega} > = -\frac{45GM^2R^3}{16ma^6}\sum_{k=-N}^{N} E_k^2(e)\sin 2\sigma_k$$

$$\sin 2\sigma_k = \frac{2\gamma(\nu + kn)}{\gamma^2 + (\nu + kn)^2}.$$

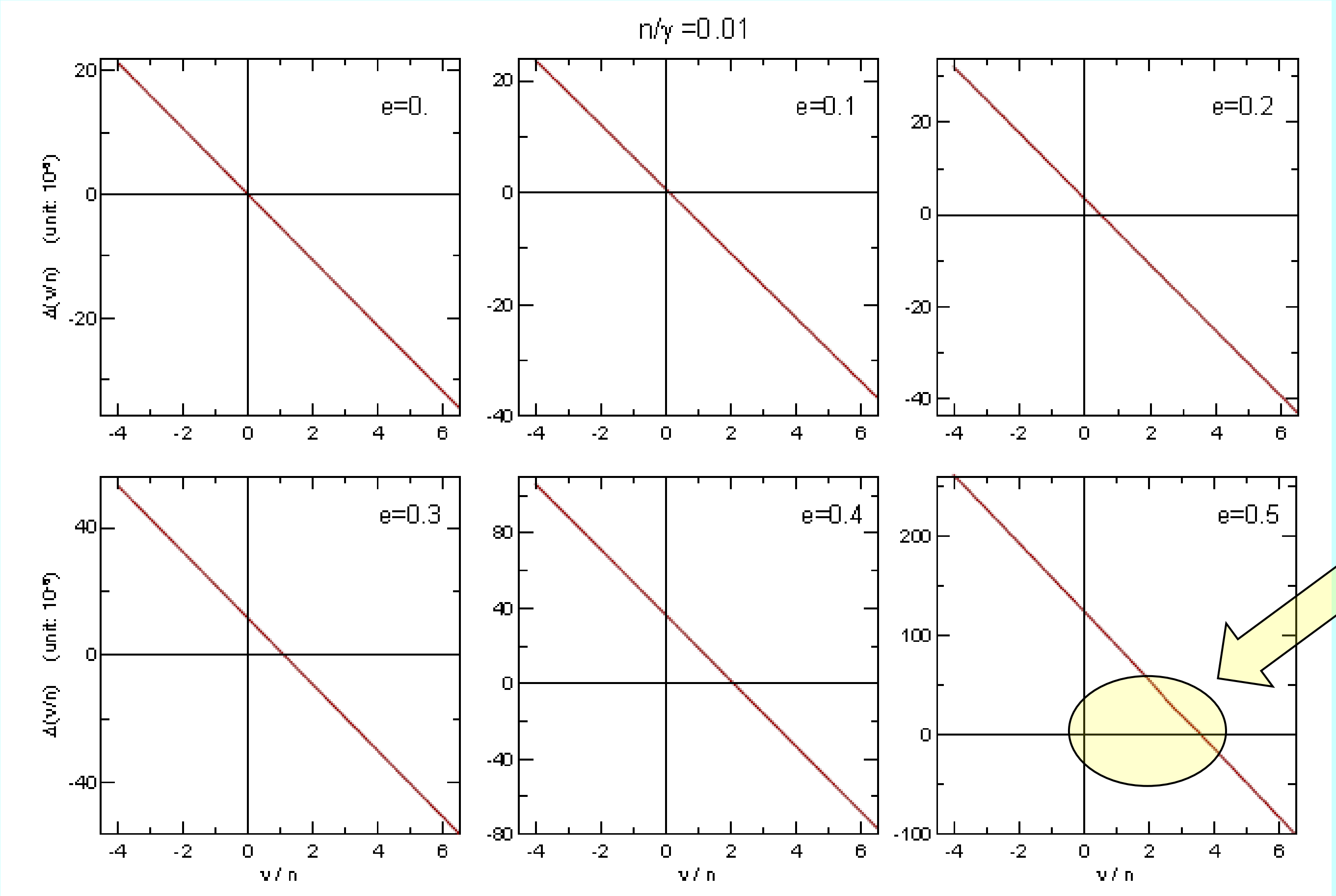
- If $< \dot{\Omega} >= 0,$ Stationary or pseudo-synchronous solution

$$\Omega = n + \frac{6n\gamma^2}{n^2+\gamma^2}\, e^2 + 3n\gamma^2\frac{226n^6 + 1453n^4\gamma^2 + 28n^2\gamma^4 + \gamma^6}{8(n^2+\gamma^2)^3(4n^2+\gamma^2)}\, e^4 + \mathcal{O}(e^6)$$

Limits: **Efroimsky-Lainey** $\gamma << n$ → $\Omega = n$

Darwin $\gamma >> n$ → $\Omega = n(1+6e^2+..)$

Map $\quad y(x) \rightarrow y(x+2\pi/\gamma)-y(x)$



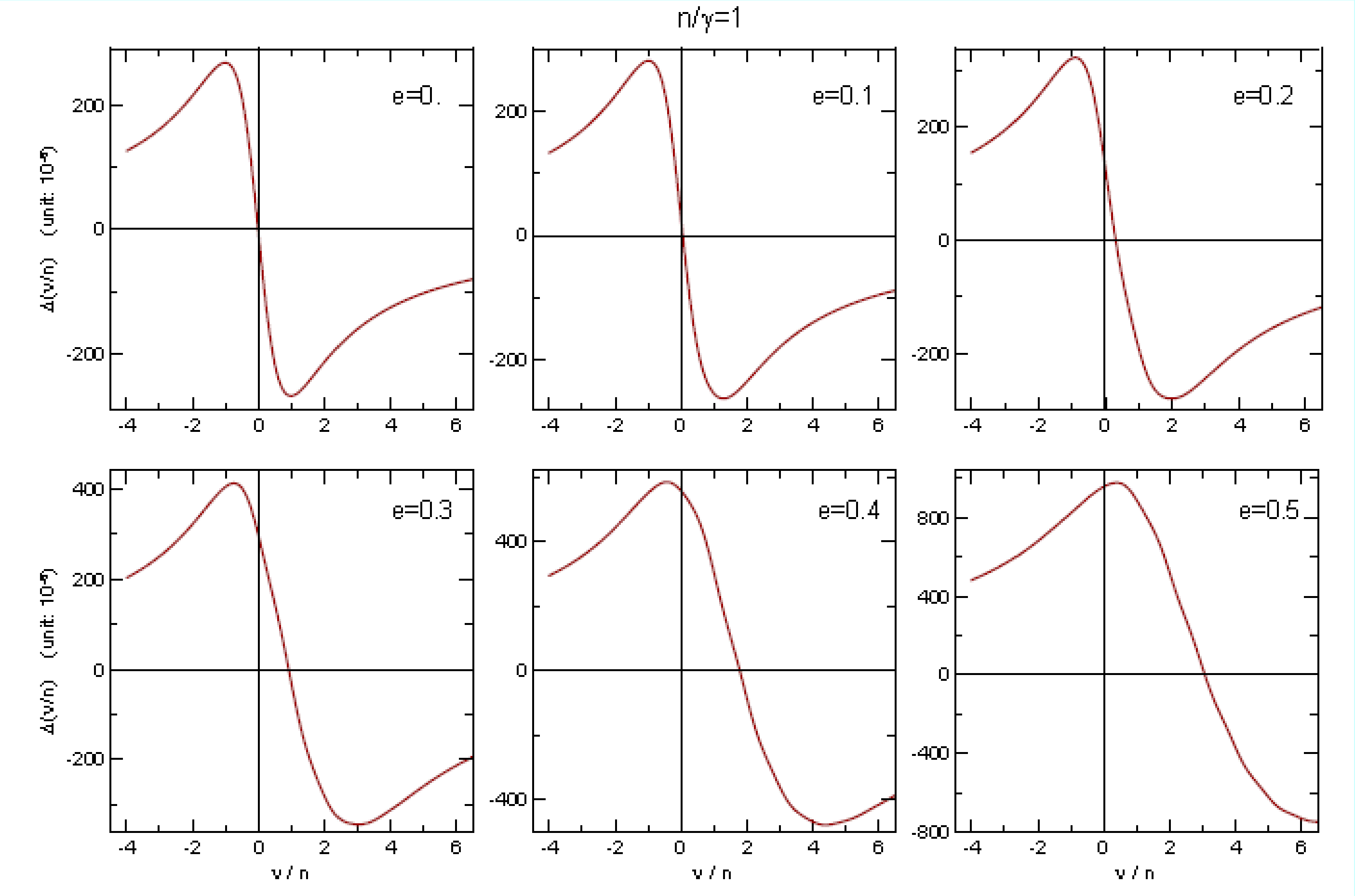
The intersections are attractors

N.B.

These attractors are **supersynchronous**

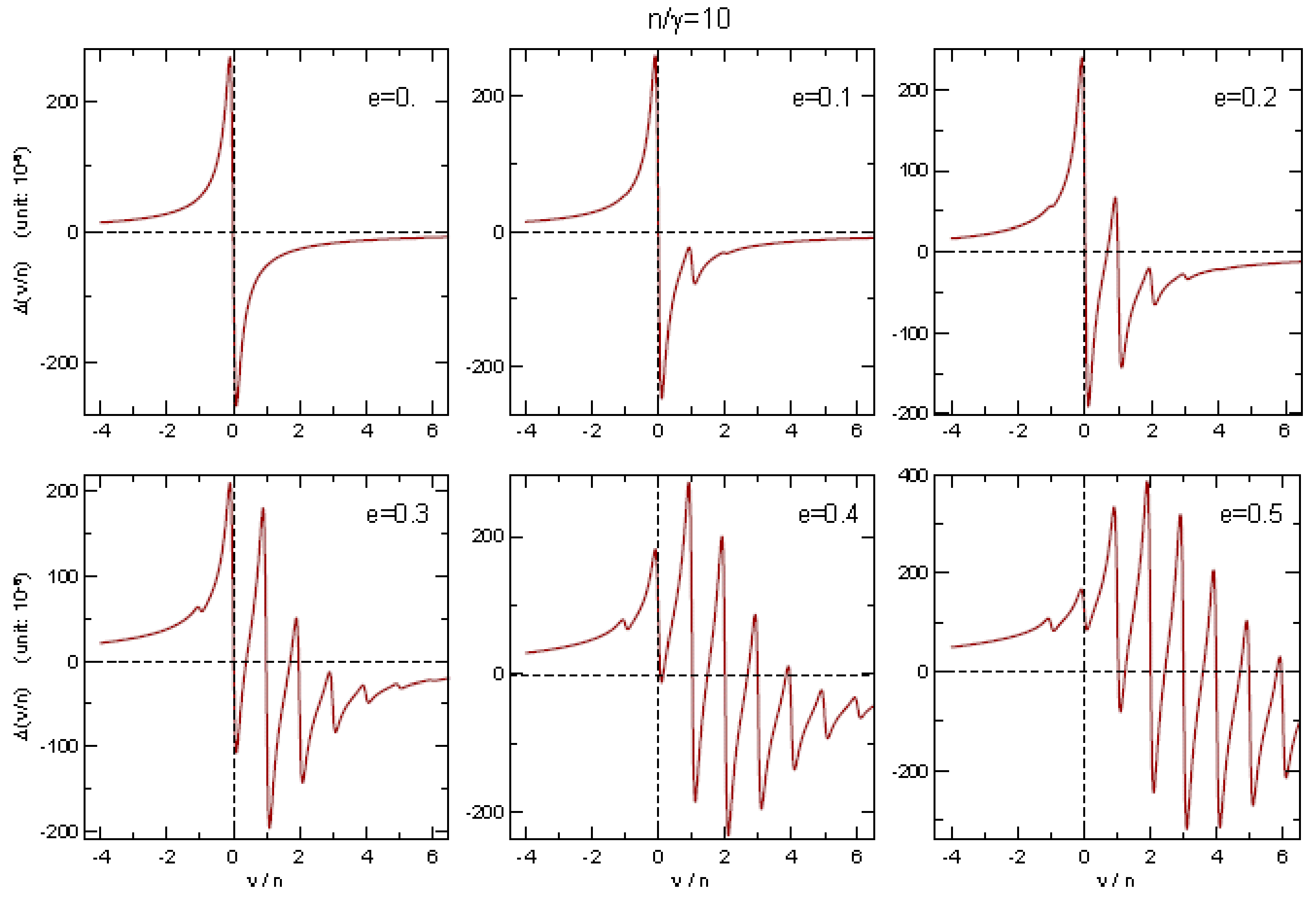
$\Omega = n + 6ne^2$

$\gamma >> n$ (ex: stars, hot Jupiters)



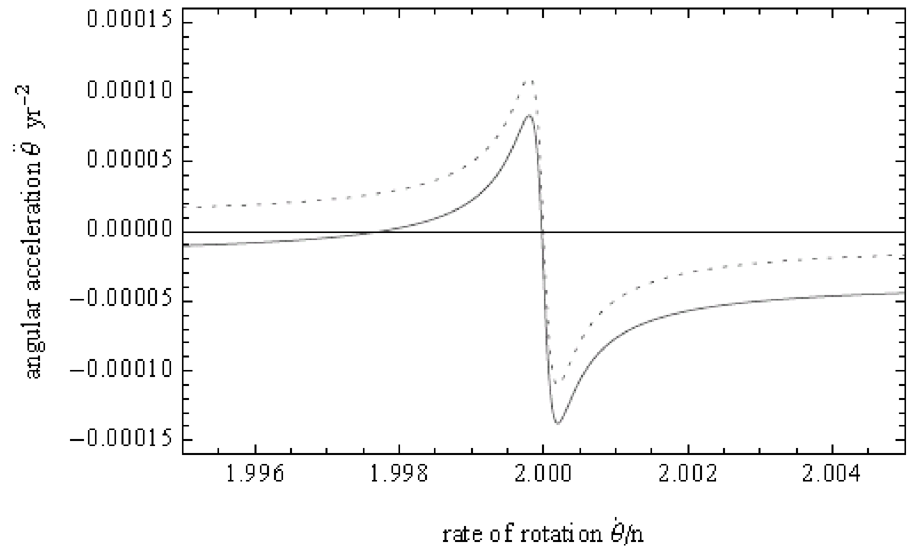
$\gamma \sim n$

Ref: SFM, DDA 2014 and CeMDA (to be pub.)
Correia et al. A&A 2013

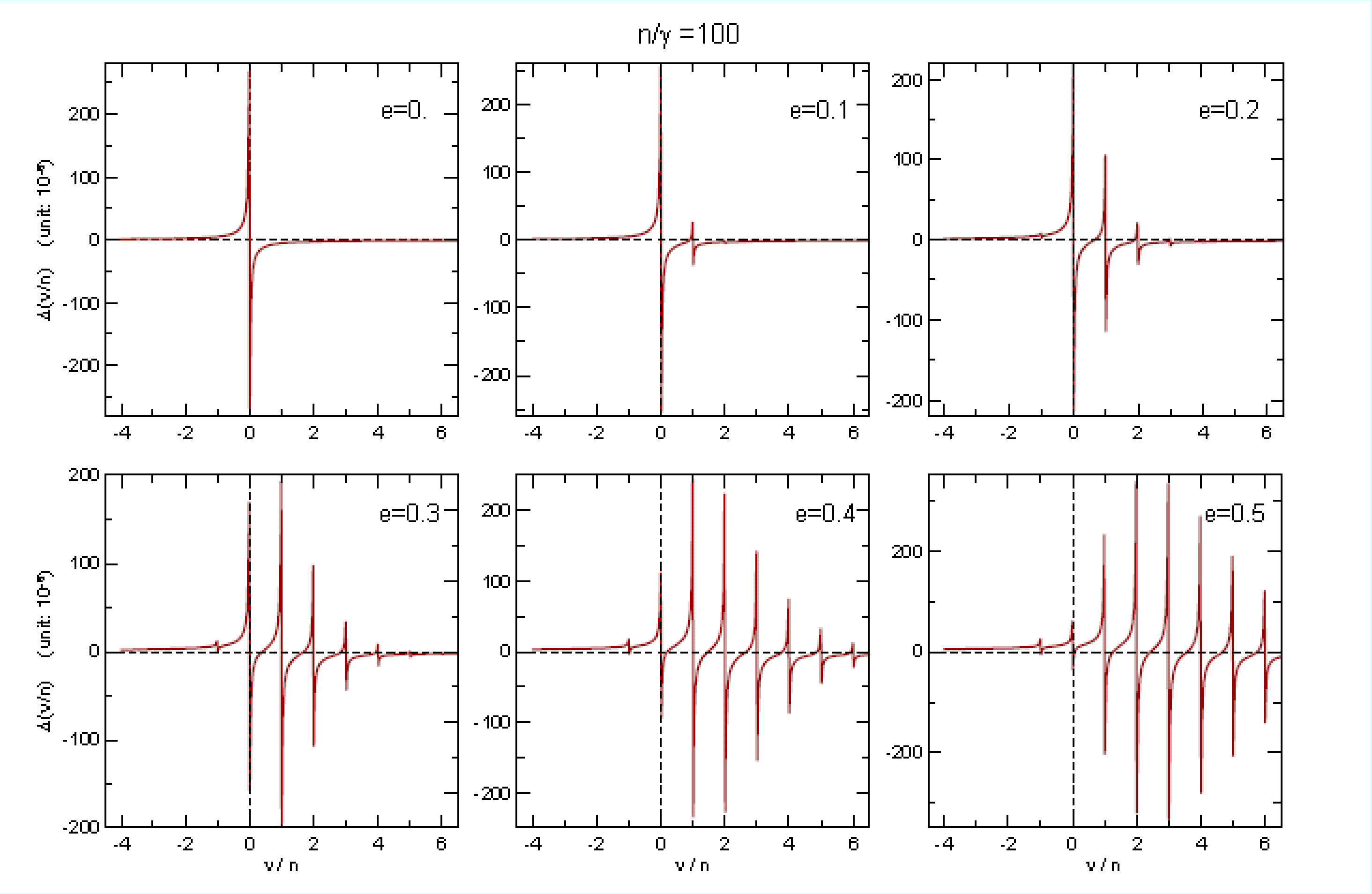


$\gamma = n/10$ (ex: Mercury, distant satellites)

New attractors at $\nu = n, 2n, 3n, \ldots$



Makarov & Efroimsky, ApJ. 2013

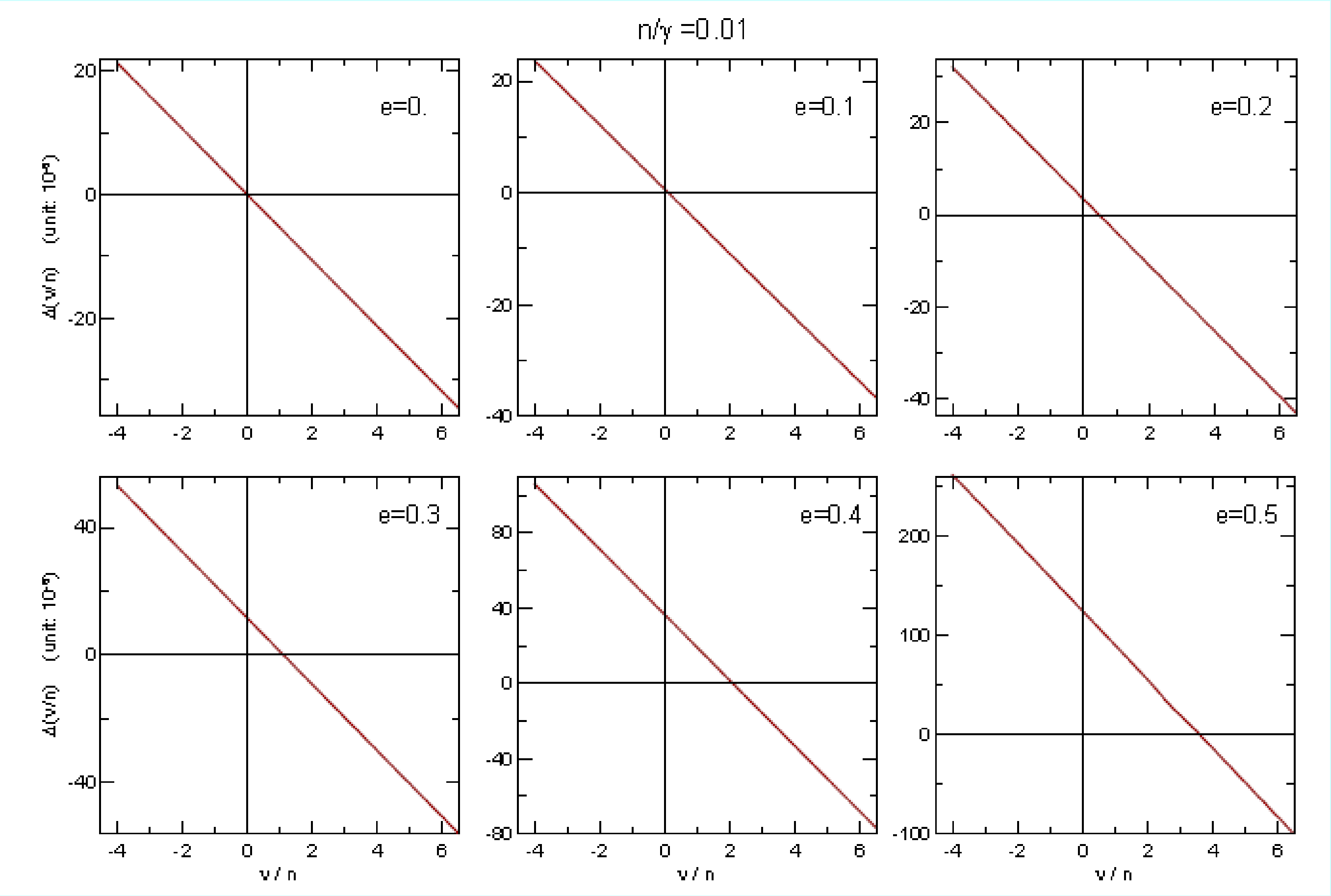


$\gamma << n$ (ex: Moon, Titan)

attractors at ν = -n, 0, n, 2n, 3n,....

Ω = n/2, n, 3n/2, 2n, 5n/2, etc.

STARS (high γ)



Then, for solar-type stars

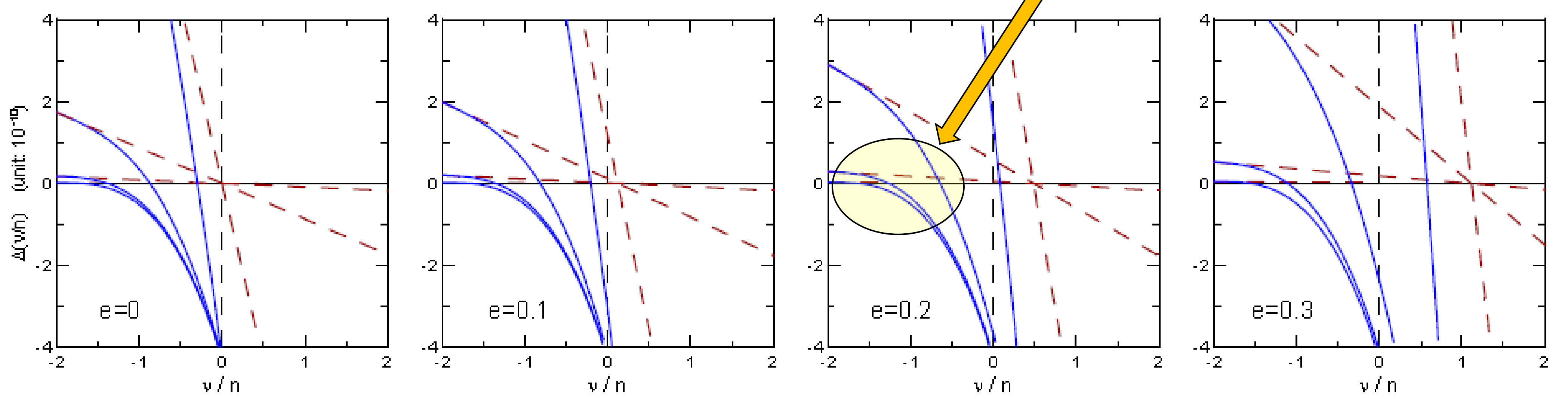
$$\dot{\Omega} = -f_P B_W \Omega^3$$

where

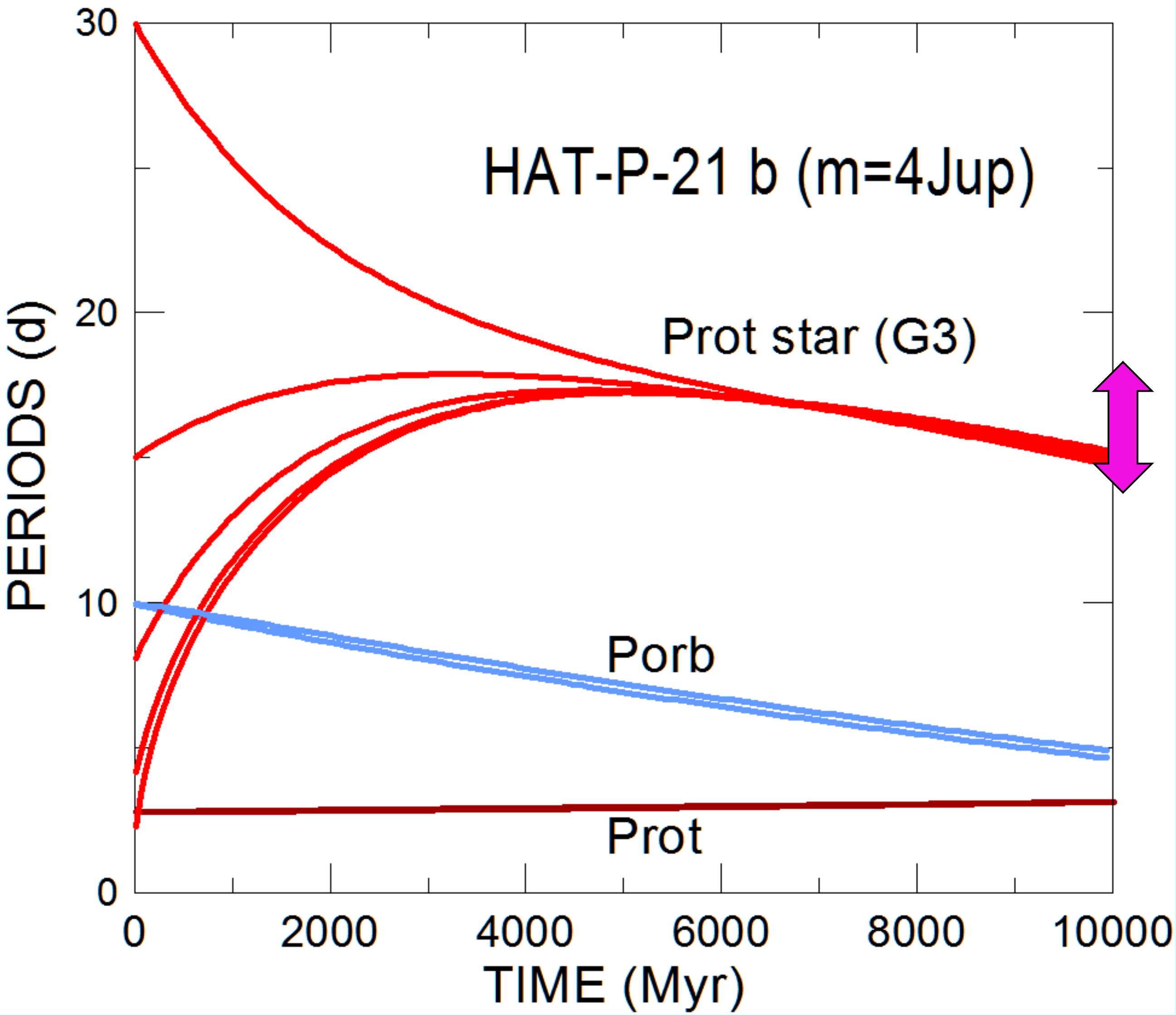
$$B_W = 2.7 \times 10^{47} \frac{1}{C} \sqrt{\left(\frac{R}{R_\odot}\frac{M_\odot}{m}\right)} \qquad \text{(cgs units)}$$

and $0 < f_P < 1$. **THEN**

subsynchronous atractors



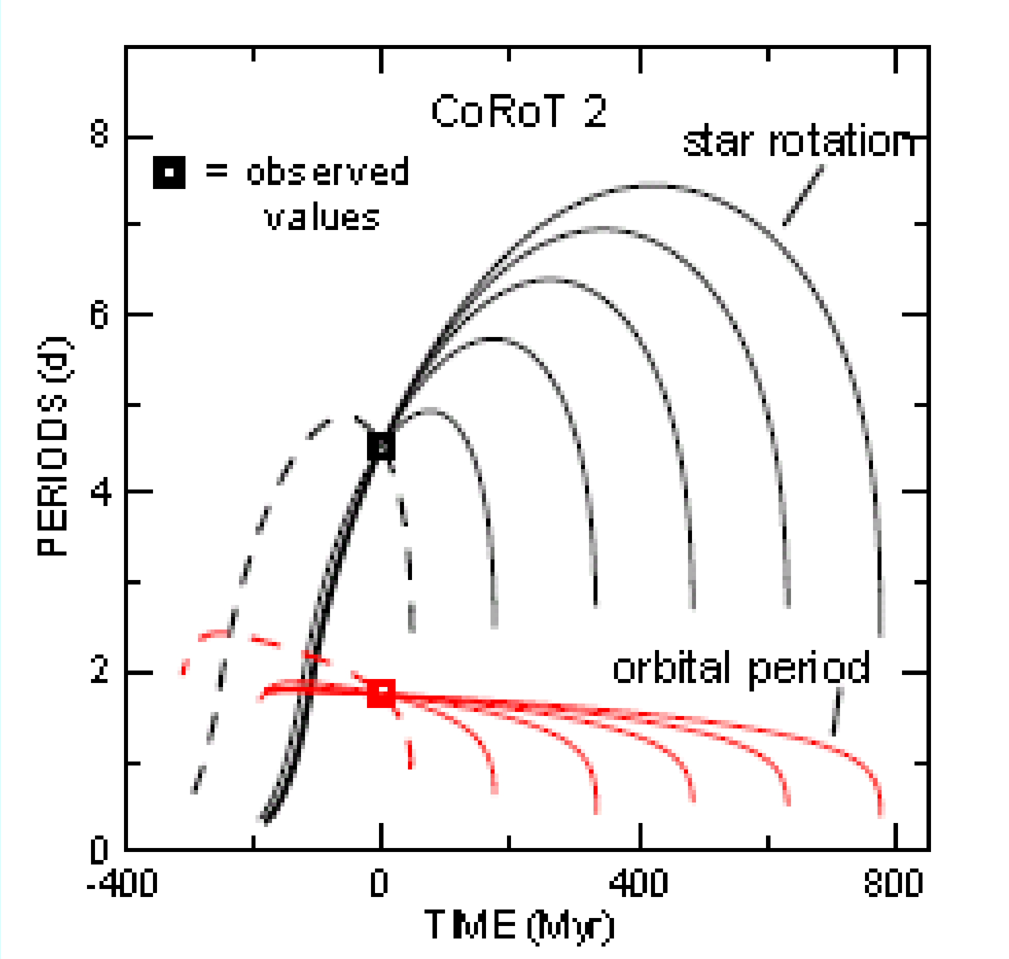
curves: $n/\gamma = 10^{-6}$ to 10^{-3} [brown: f=0; blue: f=1]



$\gamma = 25.5 s^{-1}$
e=0.228

Observed value

CoRoT 2: A young star



m_{pl}=3.3 jup

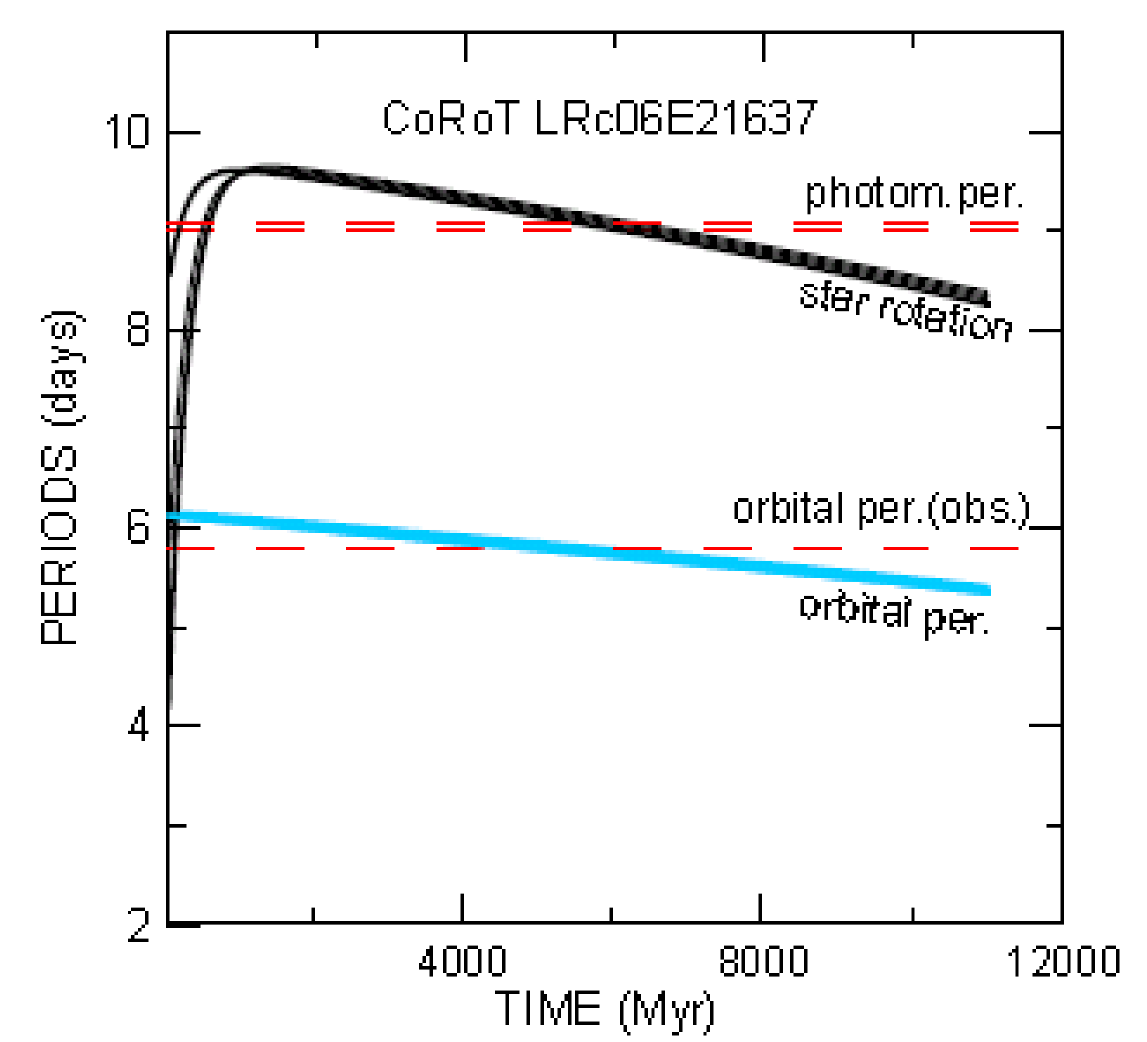
γ=20–100 s^{-1}

Result:

Age < 200 Myr

SFM et al. Astrophys. J. (in press)

CoRoT 33: A paradigm



m_{comp}=62 jup

$\gamma = 55\ s^{-1}$

SFM et al. Astrophys. J. (in press)

Two interesting F star cases:

CoRoT 15b BD (m=63.3 Jup) around a F7V star

Orbital period: 3.06 d

Star rotation: 2.9 – 3.1 d

KELT 1b BD (=27.4 Jup) around a F5 star

Orbital period: 1.217 d

Star rotation: 1.348 ± 0.4 sin I

THE END

THANKS

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