INPOP planetary ephemerides: from 2003 to 2015

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with the contribution of:

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Planetary ephemerides

Theory of planetary (and usually Moon) motions

What for?

- celestial mechanics and reference frames
- tests of fundamental physics
- planetology: physics of asteroids, Moon
- solar physics

- preparation of space missions
- paleoclimatology and geological time scales

- other topics: preparation of stellar occultations, public outreach
### 3 generations of planetary ephemerides

<table>
<thead>
<tr>
<th></th>
<th>Gaillot 1913</th>
<th>DE200/VSOP 1983</th>
<th>INPOP10a 2011</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>angle Earth-</td>
<td>distance km</td>
<td>angle Earth-</td>
</tr>
<tr>
<td>Mercury</td>
<td>1</td>
<td>450</td>
<td>0.050</td>
</tr>
<tr>
<td>Venus</td>
<td>0.5</td>
<td>100</td>
<td>0.050</td>
</tr>
<tr>
<td>Mars</td>
<td>0.5</td>
<td>150</td>
<td>0.050</td>
</tr>
<tr>
<td>Jupiter</td>
<td>0.5</td>
<td>1400</td>
<td>0.1</td>
</tr>
<tr>
<td>Saturn</td>
<td>0.5</td>
<td>3000</td>
<td>0.1</td>
</tr>
<tr>
<td>Uranus</td>
<td>1</td>
<td>12700</td>
<td>0.2</td>
</tr>
<tr>
<td>Neptune</td>
<td>1</td>
<td>22000</td>
<td>0.2</td>
</tr>
<tr>
<td>Pluto</td>
<td>1</td>
<td>24000</td>
<td>0.2</td>
</tr>
</tbody>
</table>
How did Intégrateur Numérique Planétaire de l’Observatoire de Paris start in 2003?

JPL numerical DE

- End of 70's: First positions of planets deduced from sc ⇒ DE200 numerical integration
- Mars orbiter accuracy ⇒ Up to 300 asteroids to be included (Standish and Fienga, 2002)
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**BDL analytical VSOP**
- Analytical planetary ephemerides: VSOP (Bretagnon 1982, Bretagnon 1987)
- 1998: First attempt of VSOP direct fit to observations (Fienga 1999)
- 2002: ⇒ Extension of VSOP series / MGS, Viking Pathfinder observations (Fienga and Simon 2002)
- VSOP2013 (Simon et al. 2013): \(\approx 2.4 \times 10^6\) terms @ first order of mass
How did INPOP start?

Long term ephemerides

Phil. Trans. R. Soc. Lond. A (1999) 357, 1735–1759

The limits of Earth orbital calculations: geological time-scale use

by Jacques Laskar
Astronomie et Systèmes Dynamiques, CNRS–Bureau des Terres, 77 Av. Denfert-Rochereau, 75014 Paris, France

The orbital motion of the planets in the Solar System is chaotic. As a result, initially close orbits diverge exponentially with a characteristic Lyapunov time of 5 Ma. This sensitivity to initial conditions will limit the possibility of obtaining an accurate solution for the orbital and precessional motion of the Earth over more than 35–50 Ma. The principal sources of uncertainty in the model are reviewed here. It appears that at present the largest source of error could reside in the lack of knowledge of the value of the precession due to the oblateness (\( J_2 \)) of the Sun. Nevertheless, for the calibration of geological time-scale, this limitation can be overcome to some extent if one considers in the geological data the signature of the outer orbital motion which is predictable on a much longer time-scale. Moreover, it will be possible to observe in the geological records the trace of the transverse \((s_4 - s_3) - 2(s_4 - g_3)\) secular resonance to the \((s_4 - s_3) - (g_4 - g_3)\) detection and dating of these passages should induce extremely high the dynamical models for the orbital evolution of the Solar System.

Keywords: precession; obliquity; orbital evolution; chaos; Solar System; Milankovitch cycles; palaeoclimates
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Long term ephemerides

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Keywords: precession; obliquity; orbital evolution; chaos; Solar System; Milankovitch cycles; palaeoclimates

In 2003, start of the Intégrateur Numérique Planétaire de l'Observatoire de Paris, INPOP
### INPOP Evolution since 2003

<table>
<thead>
<tr>
<th>Version</th>
<th>Description</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>INPOP08</td>
<td>4D planetary ephemerides: TT-TDB</td>
<td>TT-TDB 1st release</td>
</tr>
<tr>
<td></td>
<td>(Fienga et al. 2009)</td>
<td><a href="http://www.imcce.fr/inpop">www.imcce.fr/inpop</a></td>
</tr>
<tr>
<td></td>
<td>a priori sigma fit</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fitted to planetary data and LLR</td>
<td></td>
</tr>
<tr>
<td>INPOP10a</td>
<td>289 asteroids, no mean density, ring</td>
<td>Long-term La2010</td>
</tr>
<tr>
<td></td>
<td>(Fienga et al. 2011)</td>
<td>145 (GM_{ast}, GM_{ring})</td>
</tr>
<tr>
<td></td>
<td>Direct fit with constraints</td>
<td>GM_{\odot}, J_2^{\odot}, EMRAT, Tests of GR</td>
</tr>
<tr>
<td></td>
<td>Improvement of outer planet orbits</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fixed AU, (\beta, \gamma, \dot{\varpi}, \dot{\Omega})</td>
<td></td>
</tr>
<tr>
<td>INPOP10e</td>
<td>Direct fit with constraints + a priori sigma</td>
<td>GAIA last release</td>
</tr>
<tr>
<td></td>
<td>(Fienga et al. 2013)</td>
<td>152 (GM_{ast}, GM_{ring})</td>
</tr>
<tr>
<td></td>
<td>Solar corona studies and corrections</td>
<td>GM_{\odot}, J_2^{\odot}, EMRAT</td>
</tr>
<tr>
<td></td>
<td>(Verma et al. 2013)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Test of ICRF link with pulsar surveys</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Use of raw MGS tracking data (GINS)</td>
<td></td>
</tr>
<tr>
<td>INPOP13a</td>
<td>MESSENGER independant</td>
<td>Tests of GR</td>
</tr>
<tr>
<td></td>
<td>(Verma et al. 2014)</td>
<td>150 (GM_{ast}, GM_{ring})</td>
</tr>
<tr>
<td></td>
<td>orbit determination</td>
<td>GM_{\odot}, J_2^{\odot}, EMRAT</td>
</tr>
<tr>
<td></td>
<td>(Fienga et al 2015)</td>
<td></td>
</tr>
</tbody>
</table>
INPOP today, INPOP13c

- **Numerical integration of the** (Einstein-Imfeld-Hoffmann, $c^{-4}$ PPN approximation) equations of motion.

\[ \ddot{x}_{\text{Planet}} = \sum_{A \neq B} \mu_B \frac{r_{AB}}{\|r_{AB}\|^3} + \ddot{x}_{\text{GR}}(\beta, \gamma, c^{-4}) + \ddot{x}_{\text{AST},300} + \ddot{x}_{J_2^\odot} \]

- Adams-Cowell in extended precision
- 8 planets + Pluto + Moon + asteroids (point-mass, ring), GR, $J_2^\odot$, Earth rotation (Euler angles, specific INPOP)
- Moon: orbit and librations
- Simultaneous numerical integration TT-TDB, TCG-TCB
- Fit to observations in ICRF over 1 cy (1914-2014)
- GAIA ESA planetary ephemerides
- Asteroid physics, Tests of gravity, solar physics
More than 1 century of data

- **Mercury**: independent analysis of MESSENGER data (Verma et al. 2014)
- **Saturn**: Cassini VLBI and radio tracking
- **Jupiter**: Galileo VLBI and 5 s/c flybys
- **Venus, Mars**: VEX, MEX, MGS, MRO, MO, ...

<table>
<thead>
<tr>
<th>Method</th>
<th>Years</th>
<th>V, Ma, J, S</th>
<th>Me, J, S, U, N</th>
<th>Me, V, Ma</th>
<th>Me, V</th>
<th>J, S, U, N, P</th>
<th>Moon</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>S/C VLBI</td>
<td>1990-2010</td>
<td>1/10 mas</td>
<td>0.1/1 mas</td>
<td>0.1/1 mas</td>
<td>0.1/1 mas</td>
<td>300 mas</td>
<td>300 mas</td>
<td>1 cm</td>
</tr>
<tr>
<td>S/C Flybys</td>
<td>1976-2014</td>
<td>1/10 mas</td>
<td>1/10 mas</td>
<td>1/30 m</td>
<td>2/30 m</td>
<td>1 km</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S/C Range</td>
<td>1976-2014</td>
<td>1/10 mas</td>
<td>1/10 mas</td>
<td>2/30 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct range</td>
<td>1965-1997</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optical</td>
<td>1914-2014</td>
<td>1/10 mas</td>
<td>1/10 mas</td>
<td>1/30 m</td>
<td>2/30 m</td>
<td>1 km</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LLR</td>
<td>1980-2014</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
INPOP and space missions

Providing ephemerides to ESA: GAIA

- Specific GR development for GAIA (TT-TDB, TCG-TCB)
- Navigation and data analysis ephemerides
- INPOP10e (Fienga et al. 2013)
- Expected new INPOP release for Gaia in 2020
Providing ephemerides to ESA: GAIA

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- Navigation and data analysis ephemerides
- INPOP10e (Fienga et al. 2013)
- Expected new INPOP release for Gaia in 2020

Using S/C navigation for INPOP construction

- since INPOP10e, direct analysis of S/C tracking data for constraining planet orbits
- MGS: INPOP10e + new constraints on solar plasma (Verma et al. 2013)
- MESSENGER: accurate orbit determination \( \Rightarrow \) INPOP13a (Verma et al. 2014)
- MESSENGER: New constraints over \( \beta, \gamma, J_2^\odot, \frac{\dot{G}}{G} \)
- Cassini, Juno, Bepi-Colombo, JUICE
INPOP and the solar physics (Verma et al. 2013)
# INPOP13 accuracy versus DE, EPM (I)

<table>
<thead>
<tr>
<th>Name</th>
<th># Perturbers</th>
<th># fitted masses</th>
<th>Ring</th>
<th>(GM_{\odot})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TNO</td>
<td>Main belt</td>
<td>TNO</td>
<td>Main belt</td>
</tr>
<tr>
<td>INPOP13c</td>
<td>0</td>
<td>150</td>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td>DE430</td>
<td>0</td>
<td>343</td>
<td>0</td>
<td>343</td>
</tr>
<tr>
<td>EPM2011</td>
<td>21</td>
<td>301</td>
<td>0</td>
<td>21 + 3 density classes</td>
</tr>
</tbody>
</table>

- **Differences in dynamical modeling and adjustments**
- **INPOP13c data sets \(\approx\) DE430 data sets BUT**
  - 2 specific data analysis for MESSENGER
  - new JPL analysis for CASSINI \(\rightarrow\) Independent analysis in progress
- **EPM2011 older data sets without MESSENGER data and recently published Cassini VLBI**
INPOP13 accuracy versus DE, EPM (II)

- \((\text{INPOP13} - \text{DE430}) \leq 50\% \text{ of (O-C)}\)

- Limit for considering an ephemeris significantly different from INPOP13c or DE430
INPOP and the asteroids

From the INPOP point-of-view:

- Mars is crucial for INPOP
- 35% INPOP data from Mars orbiters
- big amount of perturbers with unknown masses!
- limiting factor for PE extrapolation ...
- ... but also for other applications!

From the planetology point-of-view:

- very few asteroids with known masses and densities
- constraints on asteroid formation
How to disentangle these accelerations?

How to identify the perturbers?

PhD thesis (P. Kuchynka)

- LS with constraints and a-priori sigmas
- More fitted masses
Distribution of asteroid masses

- About 150 asteroid masses are regularly estimated with INPOP constrained by previous estimations.
- About few tens asteroid masses are known with good accuracy from direct methods (s/c flybys, binary).
- Consistent with previous studies with a densification of the number of estimated masses: mass vs #.

INPOP13c \( \rho_S = 3.0 \pm 1.2 \) \( \rho_C = 2.4 \pm 1.1 \) g.cm\(^{-3}\)

INPOP13c: 35% of C-class with 23% TM & 41% of S-class with 17% TM
Asteroid densities

![Asteroid density plot]

- Carry (2012)
- INPOP13c \( l > 10 \text{m} \)
- INPOP13c \( 5 \text{m} < l < 10 \text{m} \)
- INPOP13c \( l < 5 \text{m} \)
The Solar system and the tests of gravity

With such accuracy, the solar system is still the ideal lab for testing gravity

Jacques Laskar workshop

INPOP planetary ephemerides
INPOP and gravity tests

In Planetary and Lunar ephemerides (like INPOP), GR plays a role in

\[
\Delta t_{SHAP} = (1 + \gamma)GM_\odot(t)\ln \frac{l_0 + l_1 + t}{l_0 + l_1 - t}
\]

\[
\Delta \dot{\omega}_{PLA} = \frac{(2\gamma - \beta + 2)GM_\odot(t)}{a(1 - e^2)c^2} + \Delta \dot{\omega}_{J_2^\odot}(J_2^\odot, a^2) + \Delta \dot{\omega}_{AST}
\]

\[
\Delta \dot{\omega}_{Moon} = \frac{(2\gamma - \beta + 2)GM_\odot(t)}{a(1 - e^2)c^2} + \Delta \dot{\omega}_{GEO} + \Delta \dot{\omega}_{SEL} + \Delta \dot{\omega}_{S,PLA}
\]

GR tests are then limited by

- Contributions by \(J_2^\odot\), Asteroids, \(2\gamma - \beta + 2\)
- Lunar and Earth physics

BUT

- Decorrelation with all the planets
- Benefit of PE global fit versus single space mission

Jacques Laskar workshop

INPOP planetary ephemerides
Specific INPOP developments for testing gravity

1- Variations/Estimations of PPN $\beta$, $\gamma$, Sun $J_2^\odot$ INPOP13a (Verma et al. 2014)

$$(\beta - 1) \times 10^5 = 0.2 \pm 2.5 \times 10^5 \quad (\gamma - 1) \times 10^5 = -0.3 \pm 2.5 \quad J_2^\odot \times 10^7 = 2.40 \pm 0.20$$
Specific INPOP developments for testing gravity

1- Variations/Estimations of PPN $\beta$, $\gamma$, Sun $J^\odot_2$

2- Simulation of a Pioneer anomaly type of acceleration $\rightarrow \ddot{x}_{\text{constant}}$ (Fienga et al. 2011)
Specific INPOP developments for testing gravity

1- Variations/Estimations of PPN $\beta$, $\gamma$, Sun $J_2^\odot$ (Fienga et al. 2011)

2- Simulation of a Pioneer anomaly type of acceleration $\rightarrow \ddot{x}_{constant}$ (Fienga et al. 2011)

3- Supplementary advance of perihelia $\dot{\varpi}$ and nodes $\dot{\Omega} \rightarrow R(\varpi(t_i), \Omega(t_i))$

\[
\begin{array}{|c|c|c|}
\hline
\Omega_{sup} & \text{INPOP08} & \text{INPOP10a} \\
\text{mas.cy}^{-1} & & \\
\hline
\text{Mercury} & 1.4 \pm 1.8 & \\
\text{Venus} & 0.2 \pm 1.5 & \\
\text{EMB} & 0.0 \pm 10.0 & 0.0 \pm 0.9 \\
\text{Mars} & 0.05 \pm 0.13 & \\
\text{Jupiter} & -40 \pm 42 & \\
\text{Saturn} & -0.1 \pm 0.4 & \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\dot{\varpi}_{sup} & \text{INPOP08} & \text{INPOP10a} & \text{P09} & \text{P10} \\
\text{mas.cy}^{-1} & & & & \\
\hline
\text{Mercury} & -10 \pm 30 & 1.2 \pm 1.6 & -3.6 \pm 5 & -4 \pm 5 \\
\text{Venus} & -4 \pm 6 & 0.2 \pm 1.5 & -0.4 \pm 0.5 & \\
\text{EMB} & 0.0 \pm 0.2 & -0.2 \pm 0.9 & -0.2 \pm 0.4 & \\
\text{Mars} & 0.4 \pm 0.6 & -0.04 \pm 0.15 & 0.1 \pm 0.5 & \\
\text{Jupiter} & 142 \pm 156 & -41 \pm 42 & \\
\text{Saturn} & -10 \pm 8 & 0.15 \pm 0.65 & -6 \pm 2 & -10 \pm 15 \\
\hline
\end{array}
\]

$\Rightarrow$ Limits for MOND (Blanchet and Novak 2011)
1- Specific INPOP developments for testing gravity

1- Variations/Estimations of PPN $\beta$, $\gamma$, Sun $J_2^\odot$ (Fienga et al. 2011)

2- Simulation of a Pioneer anomaly type of acceleration $\rightarrow \ddot{x}_{\text{constant}}$ (Fienga et al. 2011)

3- Supplementary advance of perihelia $\dot{\varpi}$ and nodes $\dot{\Omega} \rightarrow$ MOND constraints

4- Equivalence Principle $\odot$ astronomical scale

$\rightarrow \ddot{x}_j = \frac{m_j^G}{m_j^T} F(x_i, \dot{x}_i, m_i^G, ...) = (1 + \eta) F(x_i, \dot{x}_i, m_i^G, ...)$

With $\mu_\odot = GM_\odot$, $\mu_j = GM_j$ for planet $j$,

5- Estimation of $\frac{\dot{M}_\odot}{M_\odot}$ and $\frac{\dot{G}}{G}$ with $\frac{\mu_\odot}{\mu_\odot} = \frac{\dot{G}}{G} + \frac{\dot{M}_\odot}{M_\odot}$ and $\frac{\mu_j}{\mu_j} = \frac{\dot{G}}{G}$
Tests of gravity

\[ \ddot{x}_{\text{Planet}} = \sum_{A \neq B} \mu_B \frac{r_{AB}}{\|r_{AB}\|^3} + \ddot{x}_{\text{GR}}(\beta, \gamma, c^{-4}) + \ddot{x}_{\text{AST,300}} + \ddot{x}_{J_2^\odot} \]

With \( \mu_\odot = GM_\odot, \mu_j = GM_j \) for planet \( j \)

- \( \frac{\dot{M}_\odot}{M_\odot} \) and \( \frac{\dot{G}}{G} \) with \( \frac{\dot{\mu}_\odot}{\mu_\odot} = \frac{\dot{G}}{G} + \frac{\dot{M}_\odot}{M_\odot} \) and \( \frac{\dot{\mu}_j}{\mu_j} = \frac{\dot{G}}{G} \)

- For all \( t_i \), \( M_\odot(t_i) \) and \( G(t_i) \rightarrow \mu(t_i), \mu_\odot(t_i) \rightarrow \ddot{x}_{\text{Planet}}, \ddot{x}_{\text{Ast}}, \ddot{x}_{\text{Moon}} \)

What values of \( \frac{\dot{\mu}_\odot}{\mu_\odot} \) (and then \( \frac{\dot{M}_\odot}{M_\odot} \) or \( \frac{\dot{G}}{G} \)) BUT also PPN \( \beta, \gamma \) and \( J_2^\odot \) are acceptable / INPOP13c accuracy?
2 approaches based on INPOP13c

1 - Global fit including $\frac{\dot{\mu}_\odot}{\mu_\odot}$, PPN $\beta$, $\gamma$ and $J_2^\odot$

- Planet CI, 290 $GM_{ast}, GM_{ring}$, $GM_\odot$, EMRAT + GR
- Full data samples including Messenger and Cassini data
- Correlations between parameters and correlated datasets

2 - Monte Carlo simulation

- about 36000 runs
- optimized by a genetic algorithm

With $\frac{\dot{M}_\odot}{M_\odot} = (-0.92 \pm 0.46) \times 10^{-13} \text{yr}^{-1} \rightarrow \frac{\dot{G}}{G}$ (Pinto et al. 2013),(Pitjeva et al. 2012)
MC simulations

"Real" uncertainty/LS estimations + "my theory proposes this violation of GR. Is it compatible with INPOP?"

Grid of sensitivity for GRP determinations

(Fienga et al. 2009, 2011), (Verma et al. 2014)

- GRP: PPN $\beta, \gamma, \dot{\omega}, \dot{\Omega}, a_{\text{supp}}, \dot{G}/G, J^\odot_2$
- Construction of different INPOP for different values of GRP
- For each value of GRP, all parameters (IC planets, $GM_{\text{Ast}}, GM_\odot$) of INPOP are fitted.
- Iteration = all correlations are taken into account
- 2 selection criteria
  1. $C_1 = \text{Postfit residuals }/\text{INPOP} \rightarrow \text{GRP intervals with } \Delta \text{ residuals } < 25, 50\%$
  2. $C_2 = \Delta \chi^2 < 0.5, 1, 2, 3\%$

What values of GRP are acceptable at the level of data accuracy?
INPOP13a (Verma et al. 2014):

\[(\beta - 1) \times 10^5 = 0.2 \pm 2.5 , \ (\gamma - 1) \times 10^5 = -0.3 \pm 2.5 , \ J_2^\odot \times 10^7 = 2.40 \pm 0.20\]
Simple Genetic Algorithm with mutation (SGAM)

- 1 individual = INPOP \((\mu/\mu, J_2^\circ, \beta, \gamma)\)
- 1 chromosome = a set of \((\mu/\mu, J_2^\circ, \beta, \gamma)\)
- 2 crossovers + 1/10 mutation (= new random value each over 10)
- 2 selection criteria : \((O-C) < 25, \text{ 50\% and } \Delta \chi^2 < 3\% \ (H3)\)
  \(\Delta \chi^2 < 2\% \ (H2), \Delta \chi^2 < 1\% \ (H1), \Delta \chi^2 < 0.5\% \ \text{(Hiter)}\)
- 35 800 runs with 4000 MC simulation as population 0

set i \[\left[ \left( \frac{\mu}{\mu} \right)_i, (J_2^\circ)_i, \beta_i, \gamma_i \right]\]
set j \[\left[ \left( \frac{\mu}{\mu} \right)_j, (J_2^\circ)_j, \beta_j, \gamma_j \right]\]

1 crossover \[\left[ \left( \frac{\mu}{\mu} \right)_i, (J_2^\circ)_j, \beta_j, \gamma_j \right] \]
[\left[ \left( \frac{\mu}{\mu} \right)_j, (J_2^\circ)_i, \beta_i, \gamma_i \right]\]

2 crossovers \[\left[ \left( \frac{\mu}{\mu} \right)_i, (J_2^\circ)_j, \beta_i, \gamma_j \right] \]
[\left[ \left( \frac{\mu}{\mu} \right)_j, (J_2^\circ)_i, \beta_j, \gamma_i \right]\]
35 800 runs with SGAM

- @ PSL mesocentre: NEC 1472 kernels on 92 nodes
- 2 nodes allocated for INPOP
- 12 runs (= 12 x 4 iterations) @ 1hr / node
- 4000 MC simulation = population 0 @ SGAM
- 30 generations
- C1 convergency reached @ 18th (12125 runs)
- C2 convergency reached @ 25th

![Graphs of C1 and C2 variations over total number of runs]
Improvement of gaussianity

$\Delta(O-C) < 50\%$

$(\beta - 1)x1e5$

$(\gamma - 1)x1e5$

$((d\mu/dt)/\mu)x1e13$

$J2 \times 1e7$

$H3$

$(\beta - 1)x1e5$

$(\gamma - 1)x1e5$

$((ds/dt)/s)x1e11$

$J2 \times 1e7$

→ Proper definition of mean and 1-$\sigma$
Decrease of $\sigma$ of the GRP distribution with the number of runs

C1

C2
PPN $\beta, \gamma, \mu/\mu, J_2^\odot$ after 30 generations

<table>
<thead>
<tr>
<th>Method</th>
<th>PPN $\beta - 1 \times 10^{-5}$</th>
<th>PPN $\gamma - 1 \times 10^{-5}$</th>
<th>$\dot{G}/G \times 10^{13} \text{ yr}^{-1}$</th>
<th>$J_2^\odot \times 10^7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>$-4.4 \pm 5.5$</td>
<td>$-0.81 \pm 4.5$</td>
<td>$0.42 \pm 0.75$</td>
<td>$2.27 \pm 0.3$</td>
</tr>
<tr>
<td>MC + SGAM C1 50 %</td>
<td>$-0.5 \pm 6.3$</td>
<td>$-1.2 \pm 4.4$</td>
<td>$0.36 \pm 1.22$</td>
<td>$2.26 \pm 0.11$</td>
</tr>
<tr>
<td>MC + SGAM C1 25 %</td>
<td>$-1.6 \pm 4.5$</td>
<td>$-0.75 \pm 3.2$</td>
<td>$0.41 \pm 1.00$</td>
<td>$2.28 \pm 0.08$</td>
</tr>
<tr>
<td>MC + SGAM C2 (H3)</td>
<td>$-0.01 \pm 7.10$</td>
<td>$-1.7 \pm 5.2$</td>
<td>$0.55 \pm 1.22$</td>
<td>$2.22 \pm 0.14$</td>
</tr>
<tr>
<td>MC + SGAM C2 (H2)</td>
<td>$0.05 \pm 7.12$</td>
<td>$-1.62 \pm 5.17$</td>
<td>$0.53 \pm 1.20$</td>
<td>$2.221 \pm 0.137$</td>
</tr>
<tr>
<td>MC + SGAM C2 (H1)</td>
<td>$0.11 \pm 7.07$</td>
<td>$-1.62 \pm 5.10$</td>
<td>$0.52 \pm 1.18$</td>
<td>$2.220 \pm 0.135$</td>
</tr>
<tr>
<td>MC + SGAM C2 (Hiter)</td>
<td>$0.34 \pm 6.91$</td>
<td>$-1.62 \pm 5.12$</td>
<td>$0.51 \pm 1.18$</td>
<td>$2.218 \pm 0.135$</td>
</tr>
<tr>
<td>MC + SGAM C1,C2</td>
<td>$-0.25 \pm 6.7$</td>
<td>$-1.5 \pm 4.8$</td>
<td>$0.49 \pm 1.20$</td>
<td>$2.24 \pm 0.125$</td>
</tr>
</tbody>
</table>

$\dot{G}/G \pm 1 \times 10^{-13} \text{ yr}^{-1}$

$\beta - 1 \pm 7 \times 10^{-5}$

$\gamma - 1 \pm 5 \times 10^{-5}$

EP $\eta = 4\beta - \gamma - 3 \pm 2 \times 10^{-4}$

$J_2^\odot = (2.24 \pm 0.15) \times 10^{-7}$
## PPN $\beta, \gamma, \mu/\mu, J_2^\odot$

<table>
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<tr>
<th>Method</th>
<th>PPN $\beta - 1 \times 10^{-5}$</th>
<th>PPN $\gamma - 1 \times 10^{-5}$</th>
<th>$\dot{G}/G \times 10^{13}$ yr$^{-1}$</th>
<th>$J_2^\odot \times 10^7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>-4.4 ± 5.5</td>
<td>-0.81 ± 4.5</td>
<td>0.42 ± 0.75</td>
<td>2.27 ± 0.3</td>
</tr>
<tr>
<td>MC + SGAM C1</td>
<td>-0.5 ± 6.3</td>
<td>-1.2 ± 4.4</td>
<td>0.4 ± 1.22</td>
<td>2.26 ± 0.11</td>
</tr>
<tr>
<td>MC + SGAM C2</td>
<td>-0.01 ± 7.1</td>
<td>-1.7 ± 5.2</td>
<td>0.5 ± 1.20</td>
<td>2.22 ± 0.14</td>
</tr>
<tr>
<td>MC + SGAM C1,C2</td>
<td>-0.25 ± 6.7</td>
<td>-1.5 ± 4.8</td>
<td>0.5 ± 1.2</td>
<td>2.24 ± 0.125</td>
</tr>
<tr>
<td>K11-DE</td>
<td>4 ± 24</td>
<td>18 ± 26</td>
<td>1.0 ± 2.06</td>
<td>fixed to 1.8</td>
</tr>
<tr>
<td>P13-EMP</td>
<td>-2 ± 3</td>
<td>4 ± 6</td>
<td>0.29 ± 0.89</td>
<td>2.0 ± 0.2</td>
</tr>
<tr>
<td>INPOP13a</td>
<td>0.2 ± 2.5</td>
<td>-0.3 ± 2.5</td>
<td>0.0</td>
<td>2.4 ± 0.2</td>
</tr>
<tr>
<td>F13-DE</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>2.1 ± 0.70</td>
</tr>
<tr>
<td>W09-LLR</td>
<td>12 ± 11</td>
<td>fixed</td>
<td>0.0</td>
<td>fixed</td>
</tr>
<tr>
<td>M05-LLR</td>
<td>15 ± 18</td>
<td>fixed</td>
<td>6 ± 8</td>
<td>fixed</td>
</tr>
<tr>
<td>B03-Cass</td>
<td>0.0</td>
<td>2.1 ± 2.3</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>L11-VLB</td>
<td>0.0</td>
<td>-8 ± 12</td>
<td>0.0</td>
<td>fixed</td>
</tr>
<tr>
<td>Planck +WP+BAO</td>
<td></td>
<td></td>
<td>-1.42 ± 2.48</td>
<td></td>
</tr>
<tr>
<td>Heliosismo.</td>
<td></td>
<td></td>
<td>2.206 ± 0.05</td>
<td></td>
</tr>
</tbody>
</table>

\[
\dot{G}/G \pm 1 \times 10^{-13}$ yr$^{-1}$ \hspace{1cm} \beta - 1 \pm 7 \times 10^{-5} \hspace{1cm} \gamma - 1 \pm 5 \times 10^{-5} \\
J_2^\odot = (2.24 \pm 0.15) \times 10^{-7}
\]
Conclusions

\[ \beta - 1 \pm 7 \times 10^{-5} \quad \gamma - 1 \pm 5 \times 10^{-5} \]

\[ \dot{G}/G \pm 10^{-13} \text{ yr}^{-1} \quad \text{EP} \, \eta = 4\beta - \gamma - 3 \pm 2 \times 10^{-4} \]

\[ J_2^\odot = (2.24 \pm 0.15) \times 10^{-7} \]

Short term improvement:

- Saturn: Full reprocessing of Cassini 10 years data
- Mars: MO, MEX, MRO
- Moon: new IR LLR data

In the coming years, GAIA, Bepi-Colombo, JUICE (3GM team)
### Conclusions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
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<td>( \beta - 1 \pm 7 \times 10^{-5} )</td>
<td>( \gamma - 1 \pm 5 \times 10^{-5} )</td>
</tr>
<tr>
<td>( \dot{G}/G \pm 10^{-13} \text{ yr}^{-1} )</td>
<td>( J_{2}^{\odot} = (2.24 \pm 0.15) \times 10^{-7} )</td>
</tr>
</tbody>
</table>

\[ \text{EP } \eta = 4\beta - \gamma - 3 \pm 2 \times 10^{-4} \]

Short term improvement:
- **Saturn**: Full reprocessing of Cassini 10 years data
- **Jupiter**: JUNO mission in 2015/2016
- **Mars**: MO,MEX,MRO
- **Moon**: new IR LLR data

In the coming years, **GAIA**, Bepi-Colombo, JUICE (3GM team)

**More constraints on long-term ephemerides!**
The end
Improvement of gaussianity

C1

C2