INPOP planetary ephemerides: from 2003 to 2015

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Planetary ephemerides

Theory of planetary (and usually Moon) motions

What for ?

- celestial mechanics and reference frames
- tests of fundamental physics
- planetology: physics of asteroids, Moon
- solar physics

- preparation of space missions
- paleoclimatology and geological time scales

- other topics: preparation of stellar occultations, public outreach

3 generations of planetary ephemerides

	Gaillot 1913		DE200/VSOP 1983		INPOP10a 2011	
	angle	distance Earth-	angle	distance Earth-	angle	distance Earth-
	"	km	"	km	"	km
Mercury	1	450	0.050	5	0.050	0.002
Venus	0.5	100	0.050	2	0.001	0.004
Mars	0.5	150	0.050	0.050	0.001	0.002
Jupiter	0.5	1400	0.1	10	0.010	2
Saturn	0.5	3000	0.1	600	0.010	0.015
Uranus	1	12700	0.2	2540	0.100	1270
Neptune	1	22000	0.2	4400	0.100	2200
Pluto	1	24000	0.2	4800	0.100	2400

How did Intégrateur Numérique Planétaire de l'Observatoire de Paris start in 2003 ?

JPL numerical DE

- End of 70's: First positions of planets deduced from sc ⇒ DE200 numerical integration
- DE403 (1995), DE403 (1995)
- Mars orbiter accuracy ⇒ Up to 300 asteroids to be included (Standish and Fienga, 2002)

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BDL analytical VSOP

- Analytical planetary ephemerides : VSOP (Bretagnon 1982, Bretagnon 1987)
- 1998: First attempt of VSOP direct fit to observations (Fienga 1999)
- 2002: $\Rightarrow$ Extension of VSOP series / MGS, Viking Pathfinder observations (Fienga and Simon 2002)
- VSOP2013 (Simon et al. 2013): $\approx 2.4 \times 10^6$ terms @ first order of mass

How did INPOP start ?

Long term ephemerides

Phil. Trans. R. Soc. Lond. A (1999) **357**, 1735–1759

The limits of Earth orbital calcula geological time-scale use

BY JACQUES LASKAR

Astronomie et Systèmes Dynamiques, CNRS–Bureau des
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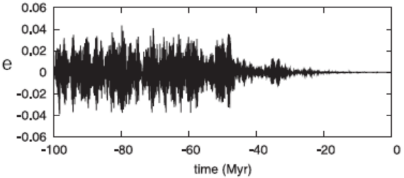
The orbital motion of the planets in the Solar System is chaotic. As a result, initially close orbits diverge exponentially with a characteristic Lyapunov time of 5 Ma. This sensitivity to initial conditions will limit the possibility of obtaining an accurate solution for the orbital and precessional motion of the Earth over more than 35–50 Ma. The principal sources of uncertainty in the model are reviewed here. It appears that at present the largest source of error could reside in the lack of knowledge of the value of the precession due to the oblateness (J_2) of the Sun. Nevertheless, for the calibration of geological time-scale, this limitation can be overcome to some extent if one considers in the geological data the signature of the outer orbital motion which is predictable on a much longer time-scale. Mc be possible to observe in the geological records the trace of trai $(s_4 - s_3) - 2(g_4 - g_3)$ secular resonance to the $(s_4 - s_3) - (g_4 - g_3$ detection and dating of these passages should induce extremely hig the dynamical models for the orbital evolution of the Solar System.

Keywords: precession; obliquity; orbital evolution; chao Solar System; Milankovitch cycles; palaeoclimates

J. Laskar et al.: Insolation quantities of the Earth
A&A 428, 261–285 (2004)

Table 8. Main sources of uncertainty in the orbital solution (from Laskar 1999). For each limiting factor, an analytical estimate of the time of validity of the solution T_V is given (in Myr), taking into account the exponential growth of the error.

Limiting factor	T_V
Uncertainty on the masses and intial conditions	38 Myr
Contribution of the main Galilean satellites	35 Myr
Uncertainty in the Earth–Moon system evolution	40 Myr
Effect of the main asteroids	32 Myr
Mass loss of the Sun	50 Myr
Uncertainty of 2×10^{-7} on the J_2 of the Sun	26 Myr



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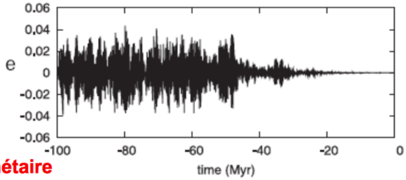


In 2003, start of the Intégrateur Numérique Planétaire de l'Observatoire de Paris, INPOP

INPOP Evolution since 2003

INPOP08	4Dplanetary ephemerides: TT-TDB	TT-TDB 1st release
(Fienga et al. 2009)	a priori sigma fit	www.imcce.fr/inpop
	Fitted to planetary data and LLR	30 GM_{ast},3ρ
		AU, $J_2^{\odot}$,EMRAT
INPOP10a	289 asteroids, no mean density, ring	Long-term La2010
(Fienga et al. 2011)	Direct fit with constraints	145 GM_{ast},GM_{ring}
	Improvement of outer planet orbits	$GM_{\odot}$, $J_2^{\odot}$,EMRAT,
	Fixed AU, β, γ, $\dot{\varpi}$,$\dot{\Omega}$	Tests of GR
INPOP10e	Direct fit with constraints + a priori sigma	GAIA last release
(Fienga et al. 2013)	Solar corona studies and corrections	152 GM_{ast},GM_{ring}
(Verma et al. 2013)	Test of ICRF link with pulsar surveys	$GM_{\odot}$, $J_2^{\odot}$,EMRAT
	Use of raw MGS tracking data (GINS)	
INPOP13a, 13c	MESSENGER independant	Tests of GR
(Verma et al 2014)	orbit determination	150 GM_{ast},GM_{ring}
(Fienga et al 2015)	β, γ, $(\dot{G}/G)$	$GM_{\odot}$, $J_2^{\odot}$,EMRAT

INPOP today, INPOP13c

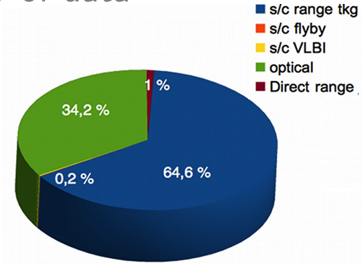
- Numerical integration of the (Einstein-Imfeld-Hoffmann, c^{-4} PPN approximation) equations of motion.

$$\ddot{x}_{Planet} = \sum_{A \neq B} \mu_B \frac{r_{AB}}{\|r_{AB}\|^3} + \ddot{x}_{GR}(\beta, \gamma, c^{-4}) + \ddot{x}_{AST,300} + \ddot{x}_{J_2^{\odot}}$$

- Adams-Cowell in extended precision
- 8 planets + Pluto + Moon + asteroids (point-mass, ring), GR, $J_2^{\odot}$, Earth rotation (Euler angles, specific INPOP)
- Moon: orbit and librations
- Simultaneous numerical integration TT-TDB, TCG-TCB
- Fit to observations in ICRF over 1 cy (1914-2014)
- GAIA ESA planetary ephemerides
- Asteroid physics, Tests of gravity, solar physics

More than 1 century of data

- Mercury: independent analysis of MESSENGER data (Verma et al. 2014)
- Saturn: Cassini VLBI and radio tracking
- Jupiter: Galileo VLBI and 5 s/c flybys
- Venus, Mars: VEX, MEX, MGS, MRO, MO, ...



		α	δ	ρ
S/C VLBI 1990-2010	V, Ma, **J**, S	1/10 mas	1/10 mas	
S/C Flybys 1976-2014	Me, **J**, S, U, N	0.1/1 mas	0.1/1 mas	1/30 m
S/C Range 1976-2014	Me, V, Ma			2/30 m
Direct range 1965-1997	Me,V			1 km
Optical 1914-2014	**J**, S, U, N, P	300 mas	300 mas	
LLR 1980-2014	Moon			1cm

INPOP and space missions

Providing ephemerides to ESA: GAIA

- Specific GR development for GAIA (TT-TDB, TCG-TCB)
- Navigation and data analysis ephemerides
- INPOP10e (Fienga et al. 2013)
- Expected new INPOP release for Gaia in 2020

INPOP and space missions

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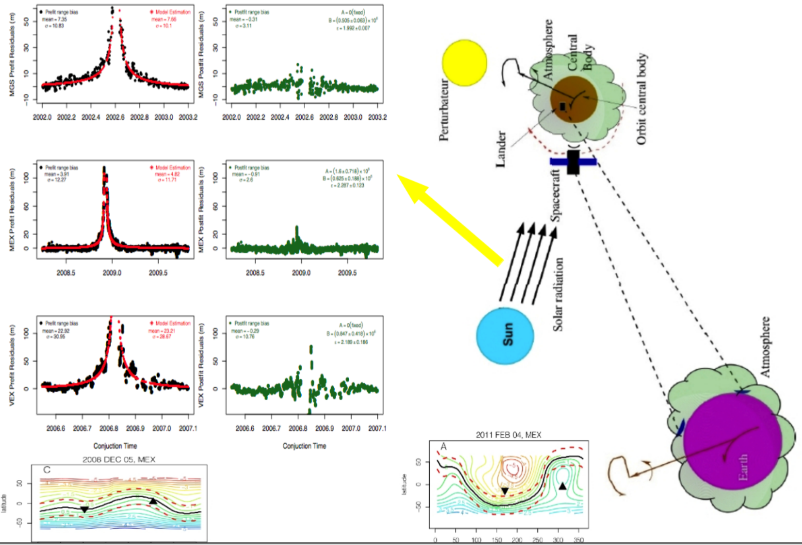
Using S/C navigation for INPOP construction

- since INPOP10e, direct analysis of S/C tracking data for constraining planet orbits
- MGS: INPOP10e + new constraints on solar plasma (Verma et al. 2013)
- MESSENGER: accurate orbit determination ⇒ INPOP13a (Verma et al. 2014)
- MESSENGER: New constraints over β, γ, $J_2^{\odot}$, $\frac{\dot{G}}{G}$
- Cassini, Juno, Bepi-Colombo, JUICE

INPOP and the solar physics (Verma et al. 2013)



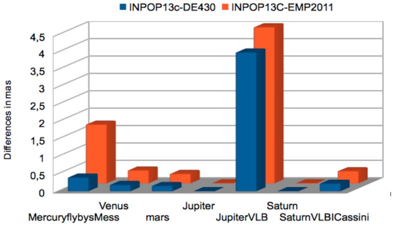
INPOP13 accuracy versus to DE, EPM (I)

Name	# Perturbers		# fitted masses		Ring		$GM_\odot$
	TNO	Main belt	TNO	Main belt	TNO	Main belt	
INPOP13c	0	150	0	150	0	F	F
DE430	0	343	0	343	0	0	F
EPM2011	21	301	0	21 + 3 density classes	F	F	

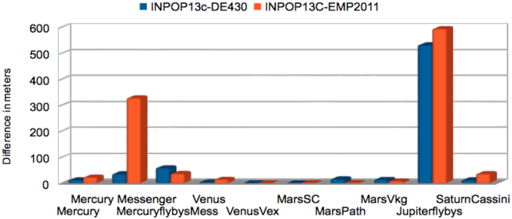
- Differences in dynamical modeling and adjustments
- INPOP13c data sets ≈ DE430 data sets BUT
  - 2 specific data analysis for MESSENGER
  - new JPL analysis for CASSINI → Independent analysis in progress
- EPM2011 older data sets without MESSENGER data and recently published Cassini VLBI

INPOP13 accuracy versus to DE, EPM (II)

Differences in angles [mas]



Differences in range [m]



- (INPOP13 - DE430) <= 50% of (O-C)
- Limit for considering an ephemeris significantly different from INPOP13c or DE430

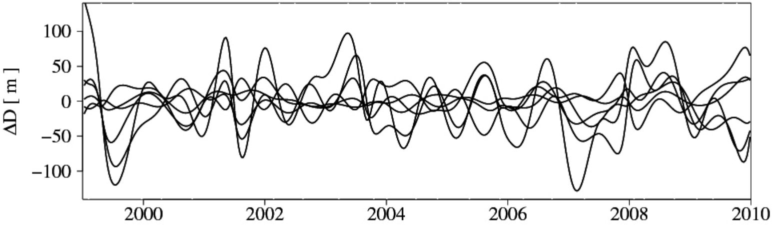
INPOP and the asteroids

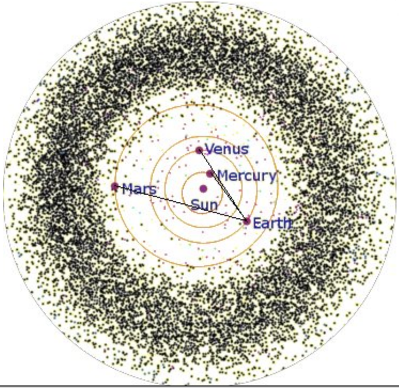
From the INPOP point-of-view:

- Mars is crucial for INPOP
- 35% INPOP data from Mars orbiters
- big amount of perturbers with unknown masses !
- limiting factor for PE extrapolation ...
- ... but also for other applications !

From the planetology point-of-view:

- very few asteroids with known masses and densities
- constraints on asteroid formation

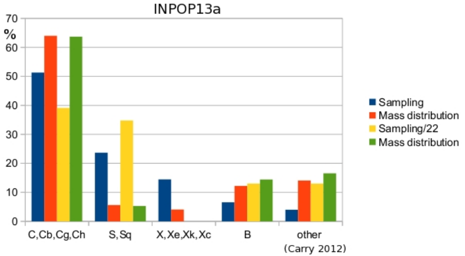


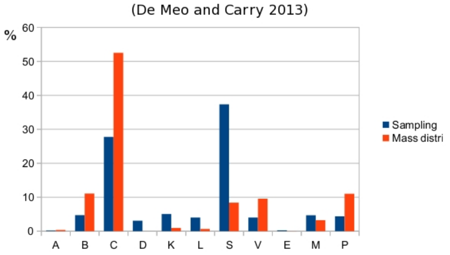


- How to distangle these accelerations ?
- How to identify the perturbers ?
- PhD thesis (P. Kuchynka)

- LS with constraints and a-priori sigmas
- More fitted masses

Distribution of asteroid masses



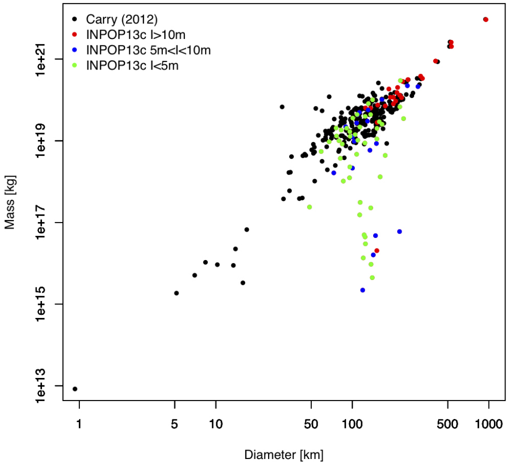


- About 150 asteroid masses are regularly estimated with INPOP constrained by previous estimations
- About few tens asteroid masses are known with good accuracy from direct methods (s/c flybys, binary)
- Consistent with previous studies with a densification of the number of estimated masses: mass vs #
- INPOP13c $\rho_S = 3.0 \pm 1.2$ $\rho_C = 2.4 \pm 1.1$ g.cm^{-3}

	(DeMeo and Carry 2013)	INPOP13a
C-class	1.33 ± 0.58	2.00 ± 1.18
S-class	2.72 ± 0.54	3.22 ± 1.30
X-class		4.2 ± 3.0
B-class	2.38 ± 0.45	2.34 ± 0.27

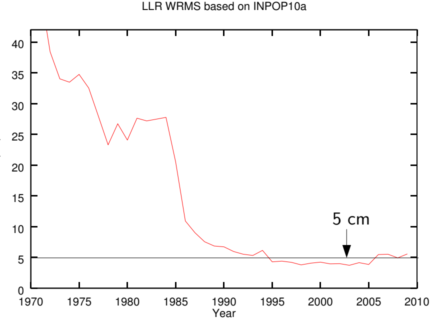
INPOP13c: 35% of C-class with 23% TM & 41% of S-class with 17% TM

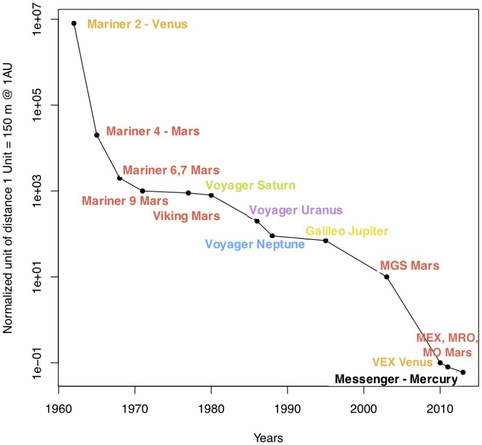
Asteroid densities



The Solar system and the tests of gravity

With such accuracy, the solar system is still the ideal lab for testing gravity





INPOP and gravity tests

In Planetary and Lunar ephemerides (like INPOP), GR plays a role in

$$\Delta t_{SHAP} = (1+\gamma) GM_{\odot}(t) ln \frac{l_0 + l_1 + t}{l_0 + l_1 - t}$$

$$\Delta \dot{\varpi}_{PLA} = \frac{(2\gamma - \beta + 2) GM_{\odot}(t)}{a(1-e^2)c^2} + \Delta \dot{\varpi}_{J_2^{\odot}}(J_2^{\odot}, a^2) + \Delta \dot{\varpi}_{AST}$$

$$\Delta \dot{\varpi}_{Moon} = \frac{(2\gamma - \beta + 2) GM_{\odot}(t)}{a(1-e^2)c^2} + \Delta \dot{\varpi}_{GEO} + \Delta \dot{\varpi}_{SEL} + \Delta \dot{\varpi}_{S,PLA}$$

GR tests are then limited by

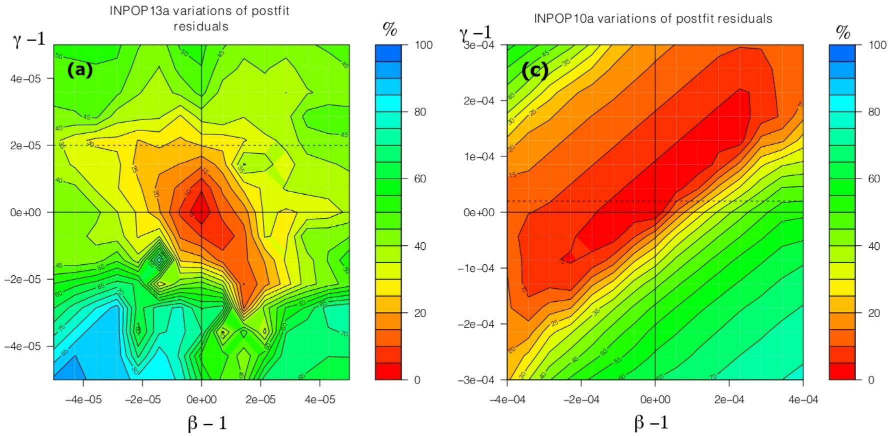
- Contributions by $J_2^{\odot}$, Asteroids, $2\gamma - \beta + 2$
- Lunar and Earth physics

BUT

- Decorrelation with all the planets
- Benefit of PE global fit versus single space mission

Specific INPOP developments for testing gravity

1- Variations/Estimations of PPN β, γ, Sun $J_2^{\odot}$ INPOP13a (Verma et al. 2014)

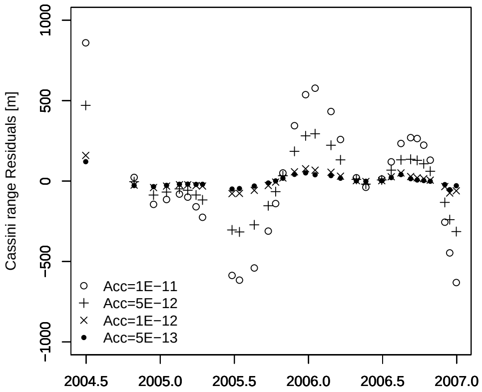


$$(\beta-1)\times10^5 = 0.2\pm2.5\ ,\ (\gamma-1)\times10^5 = -0.3\pm2.5\ ,\ J_2^{\odot}\times10^7 = 2.40\pm0.20$$

Specific INPOP developments for testing gravity

1- Variations/Estimations of PPN β, γ, Sun $J_2^{\odot}$

2- Simulation of a Pioneer anomaly type of acceleration $\rightarrow \ddot{x}_{constant}$ (Fienga et al. 2011)



Specific INPOP developments for testing gravity

1- Variations/Estimations of PPN β, γ, Sun $J_2^{\odot}$ (Fienga et al. 2011)

2- Simulation of a Pioneer anomaly type of acceleration $\rightarrow$ $\ddot{x}_{constant}$ (Fienga et al. 2011)

3- Supplementary advance of perihelia $\dot{\varpi}$ and nodes $\dot{\Omega}$ $\rightarrow$ $R(\varpi(t_i), \Omega(t_i))$

$\dot{\Omega}_{\mathrm{sup}}$ mas.cy^{-1}	INPOP08	INPOP10a
Mercury		1.4 ± 1.8
Venus	200 ± 100	0.2 ± 1.5
EMB	0.0 ± 10.0	0.0 ± 0.9
Mars	0.0 ± 2	-0.05 ± 0.13
Jupiter	-200 ± 100	-40 ± 42
Saturn	-200 ± 100	-0.1 ± 0.4

$\dot{\varpi}_{\mathrm{sup}}$ mas.cy^{-1}	INPOP08	INPOP10a	P09	P10
Mercury	-10 ± 30	1.2 ± 1.6	-3.6 ± 5	-4 ± 5
Venus	-4 ± 6	0.2 ± 1.5	-0.4 ± 0.5	
EMB	0.0 ± 0.2	-0.2 ± 0.9	-0.2 ± 0.4	
Mars	0.4 ± 0.6	-0.04 ± 0.15	0.1 ± 0.5	
Jupiter	142 ± 156	-41 ± 42		
Saturn	-10 ± 8	0.15± 0.65	-6 ± 2	-10 ± 15

$\Rightarrow$ Limits for MOND (Blanchet and Novak 2011)

1- Specific INPOP developments for testing gravity

1- Variations/Estimations of PPN β, γ, Sun $J_2^{\odot}$ (Fienga et al. 2011)

2- Simulation of a Pioneer anomaly type of acceleration $\rightarrow \ddot{x}_{constant}$ (Fienga et al. 2011)

3- Supplementary advance of perihelia $\dot{\varpi}$ and nodes $\dot{\Omega} \rightarrow$ MOND constraints

4- Equivalence Principle @ astronomical scale
$\rightarrow \ddot{x}_j = \frac{m_j^G}{m_j^I} F(x_i, \dot{x}_i, m_i^G, ...) = (1+\eta) F(x_i, \dot{x}_i, m_i^G, ...)$

With $\mu_\odot = GM_\odot$, $\mu_j = GM_j$ for planet j,
5- Estimation of $\frac{\dot{M}_\odot}{M_\odot}$ and $\frac{\dot{G}}{G}$ with $\frac{\dot{\mu_\odot}}{\mu_\odot} = \frac{\dot{G}}{G} + \frac{\dot{M}_\odot}{M_\odot}$ and $\frac{\dot{\mu_j}}{\mu_j} = \frac{\dot{G}}{G}$

Tests of gravity

$$\ddot{x}_{Planet} = \sum_{A \neq B} \mu_B \frac{r_{AB}}{\|r_{AB}\|^3} + \ddot{x}_{GR}(\beta, \gamma, c^{-4}) + \ddot{x}_{AST,300} + \ddot{x}_{J_2^{\odot}}$$

With $\mu_\odot = GM_\odot$, $\mu_j = GM_j$ for planet j

- $\frac{\dot{M}_\odot}{M_\odot}$ and $\frac{\dot{G}}{G}$ with $\frac{\dot{\mu_\odot}}{\mu_\odot} = \frac{\dot{G}}{G} + \frac{\dot{M}_\odot}{M_\odot}$ and $\frac{\dot{\mu_j}}{\mu_j} = \frac{\dot{G}}{G}$
- $\forall t_i, M_\odot(t_i)$ and $G(t_i) \rightarrow \mu(t_i), \mu_\odot(t_i) \rightarrow \ddot{x}_{Planet}, \ddot{x}_{Ast}, \ddot{x}_{Moon}$

What values of $\frac{\dot{\mu_\odot}}{\mu_\odot}$ (and then $\frac{\dot{M}_\odot}{M_\odot}$ or $\frac{\dot{G}}{G}$) BUT also PPN β, γ and $J_2^{\odot}$ are acceptable / INPOP13c accuracy ?

2 approaches based on INPOP13c

1 - Global fit including $\frac{\dot{\mu_\odot}}{\mu_\odot}$, PPN β, γ and $J_2^\odot$

- Planet CI, 290 GM_{ast},GM_{ring}, $GM\odot$, EMRAT + GR
- Full data samples including Messenger and Cassini data
- Correlations between parameters and correlated datasets

2 - Monte Carlo simulation

- about 36000 runs
- optimized by a genetic algorithm

With $\frac{\dot{M}_\odot}{M_\odot} = (-0.92 \pm 0.46) \times 10^{-13}\,\mathrm{yr}^{-1} \rightarrow \frac{\dot{G}}{G}$ (Pinto et al. 2013),(Pitjeva et al. 2012)

MC simulations

"Real" uncertainty/LS estimations + "my theory proposes this violation of GR. Is it compatible with INPOP ?"

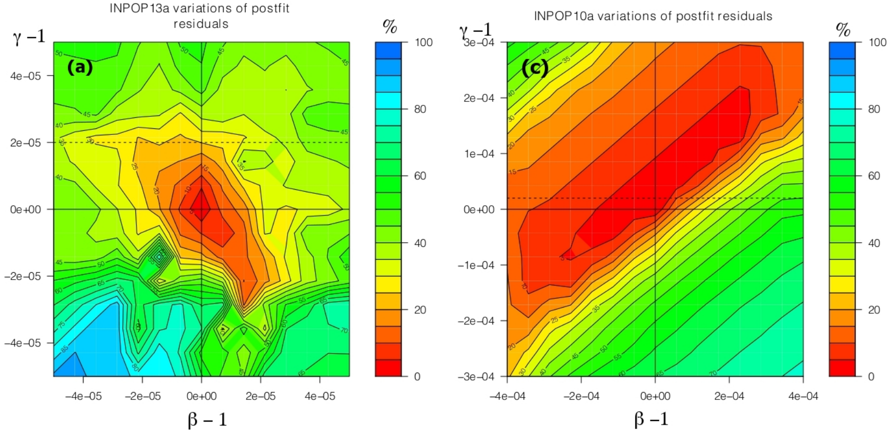
Grid of sensitivity for GRP determinations

(Fienga et al. 2009, 2011), (Verma et al. 2014)

- GRP: PPN β,γ, $\dot{\varpi}$, $\dot{\Omega}$, a_{supp}, $\dot{G}/G$, $J_2^{\odot}$
- Construction of different INPOP for different values of GRP
- For each value of GRP , all parameters (IC planets, GM_{Ast}, $GM_{\odot}$) of INPOP are fitted.
- Iteration = all correlations are taken into account
- 2 selection criteria
  1. C1 = Postfit residuals /INPOP $\rightarrow$ GRP intervals with Δ residuals $< 25, 50\%$
  2. C2 = $\Delta\chi^2 < 0.5, 1, 2, 3\%$

What values of GRP are acceptable at the level of data accuracy ?

INPOP13a and tests of GR: PPN β and γ



INPOP13a (Verma et al. 2014):

$$(\beta - 1) \times 10^5 = 0.2 \pm 2.5 \ , \ (\gamma - 1) \times 10^5 = -0.3 \pm 2.5 \ , \ J_2^{\odot} \times 10^7 = 2.40 \pm 0.20$$

Simple Genetic Algorithm with mutation (SGAM)

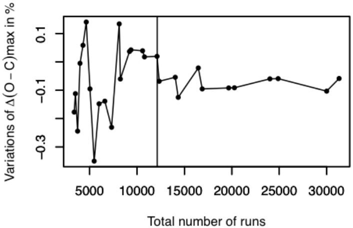
- 1 individual = INPOP ($\dot{\mu}/\mu$, $J_2^{\odot}$, β, γ)
- 1 chromosome = a set of ($\dot{\mu}/\mu$, $J_2^{\odot}$, β, γ)
- 2 crossovers + 1/10 mutation (= new random value each over 10)
- 2 selection criteria : (O-C) $<$ 25, 50% and $\Delta\chi^2 < 3\%$ (H3)
  $\Delta\chi^2 < 2\%$ (H2), $\Delta\chi^2 < 1\%$ (H1), $\Delta\chi^2 < 0.5\%$ (Hiter)
- 35 800 runs with 4000 MC simulation as population 0

set i	$[(\dot{\mu}/\mu)_i, (J_2^{\odot})_i, \beta_i, \gamma_i]$
set j	$[(\dot{\mu}/\mu)_j, (J_2^{\odot})_j, \beta_j, \gamma_j]$
1 crossover	$[(\dot{\mu}/\mu)_i, (J_2^{\odot})_i, \beta_j, \gamma_j]$
	$[(\dot{\mu}/\mu)_j, (J_2^{\odot})_j, \beta_i, \gamma_i]$
2 crossovers	$[(\dot{\mu}/\mu)_i, (J_2^{\odot})_j, \beta_i, \gamma_j]$
	$[(\dot{\mu}/\mu)_j, (J_2^{\odot})_i, \beta_j, \gamma_i]$

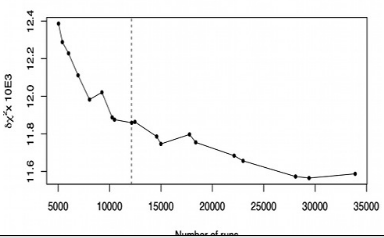
35 800 runs with SGAM

- @ PSL mesocentre : NEC 1472 kernels on 92 nodes
- 2 nodes allocated for INPOP
- 12 runs (= 12 x 4 iterations) @ 1hr / node
- 4000 MC simulation = population 0 @ SGAM
- 30 generations
- C1 convergency reached @ 18th (12125 runs)
- C2 convergency reached @ 25th

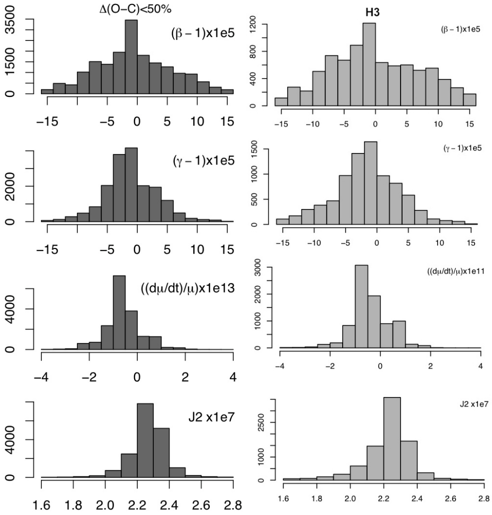
C1



C2



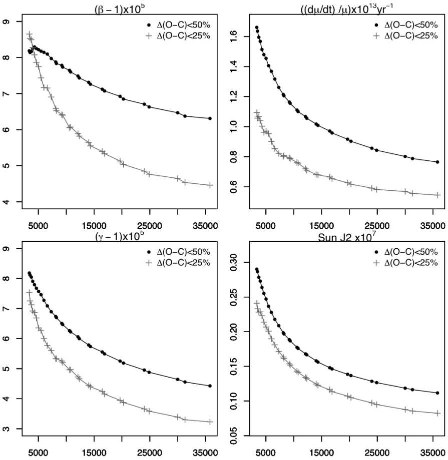
Improvement of gaussianity



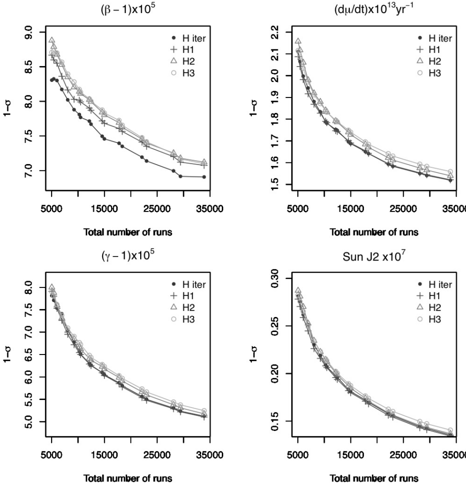
→ Proper definition of mean and 1-σ

Decrease of σ of the GRP distribution with the number of runs

C1



C2



PPN β, γ, $\dot{\mu}/\mu$, $J_2^{\odot}$ after 30 generations

Method	PPN $\beta - 1$ $\times 10^{-5}$	PPN $\gamma - 1$ $\times 10^{-5}$	$\dot{G}/G$ $\times 10^{13}$ yr^{-1}	$J_2^{\odot}$ $\times 10^{7}$
LS	-4.4 $\pm$ 5.5	-0.81 $\pm$ 4.5	0.42 $\pm$ 0.75	2.27 $\pm$ 0.3
MC + SGAM C1 50 %	-0.5 $\pm$ 6.3	-1.2 $\pm$ 4.4	0.36 $\pm$ 1.22	2.26 $\pm$ 0.11
MC + SGAM C1 25 %	-1.6 $\pm$ 4.5	-0.75 $\pm$ 3.2	0.41 $\pm$ 1.00	2.28 $\pm$ 0.08
MC + SGAM C2 (H3)	-0.01 $\pm$ 7.10	-1.7 $\pm$ 5.2	0.55 $\pm$ 1.22	2.22 $\pm$ 0.14
MC + SGAM C2 (H2)	0.05 $\pm$ 7.12	-1.62 $\pm$ 5.17	0.53 $\pm$ 1.20	2.221 $\pm$ 0.137
MC + SGAM C2 (H1)	0.11 $\pm$ 7.07	-1.62 $\pm$ 5.10	0.52 $\pm$ 1.18	2.220 $\pm$ 0.135
MC + SGAM C2 (Hiter)	0.34 $\pm$ 6.91	-1.62 $\pm$ 5.12	0.51 $\pm$ 1.18	2.218 $\pm$ 0.135
MC + SGAM C1,C2	-0.25 $\pm$ 6.7	-1.5 $\pm$ 4.8	0.49 $\pm$ 1.20	2.24 $\pm$ 0.125

$\dot{G}/G \pm 1 \times 10^{-13}$ yr^{-1}

$\beta - 1 \pm 7 \times 10^{-5}$

$\gamma - 1 \pm 5 \times 10^{-5}$

EP $\eta = 4\beta - \gamma - 3 \pm 2\times 10^{-4}$

$J_2^{\odot} = (2.24 \pm 0.15) \times 10^{-7}$

PPN β, γ, $\dot{\mu}/\mu$, $J_2^{\odot}$

Method	PPN $\beta - 1$ $\times 10^{-5}$	PPN $\gamma - 1$ $\times 10^{-5}$	$\dot{G}/G$ $\times 10^{13}$ yr^{-1}	$J_2^{\odot}$ $\times 10^{7}$
LS	-4.4 $\pm$ 5.5	-0.81 $\pm$ 4.5	0.42 $\pm$ 0.75	2.27 $\pm$ 0.3
MC + SGAM C1	-0.5 $\pm$ 6.3	-1.2 $\pm$ 4.4	0.4 $\pm$ 1.22	2.26 $\pm$ 0.11
MC + SGAM C2	-0.01 $\pm$ 7.1	-1.7 $\pm$ 5.2	0.5 $\pm$ 1.20	2.22 $\pm$ 0.14
MC + SGAM C1,C2	-0.25 $\pm$ 6.7	-1.5 $\pm$ 4.8	0.5 $\pm$ 1.2	2.24 $\pm$ 0.125
K11-DE	4 $\pm$ 24	18 $\pm$ 26	1.0 $\pm$ 2.06	fixed to 1.8
P13-EMP	-2 $\pm$ 3	4 $\pm$ 6	0.29 $\pm$ 0.89	2.0 $\pm$ 0.2
INPOP13a	0.2 $\pm$ 2.5	-0.3 $\pm$ 2.5	0.0	2.4 $\pm$ 0.2
F13-DE	0.0	0.0	0.0	2.1 $\pm$ 0.70
W09-LLR	12 $\pm$ 11	fixed	0.0	fixed
M05-LLR	15 $\pm$ 18	fixed	6 $\pm$ 8	fixed
B03-Cass	0.0	2.1 $\pm$ 2.3	0.0	
L11-VLB	0.0	-8 $\pm$ 12	0.0	fixed
Planck +WP+BAO			-1.42$\pm$ 2.48	
Heliosismo.				2.206 $\pm$ 0.05

$\dot{G}/G \pm 1 \times 10^{-13}$ yr^{-1} $\quad \beta - 1 \pm 7 \times 10^{-5}$ $\quad \gamma - 1 \pm 5 \times 10^{-5}$

$J_2^{\odot} = (2.24 \pm 0.15) \times 10^{-7}$

Conclusions

$\beta - 1 \pm 7 \times 10^{-5}$ $\gamma - 1 \pm 5 \times 10^{-5}$

EP $\eta = 4\beta - \gamma - 3 \pm 2 \times 10^{-4}$

$\dot{G}/G \pm 10^{-13}$ yr^{-1} $J_2^{\odot} = (2.24 \pm 0.15) \times 10^{-7}$

Short term improvement:

- Saturn: Full reprocessing of Cassini 10 years data
- Jupiter: JUNO mission in 2015/2016
- Mars: MO,MEX,MRO
- Moon: new IR LLR data

In the coming years, GAIA, Bepi-Colombo, JUICE (3GM team)

Conclusions

$\beta - 1 \pm 7 \times 10^{-5}$ $\gamma - 1 \pm 5 \times 10^{-5}$

EP $\eta = 4\beta - \gamma - 3 \pm 2 \times 10^{-4}$

$\dot{G}/G \pm 10^{-13}$ yr^{-1} $J_2^{\odot} = (2.24 \pm 0.15) \times 10^{-7}$

Short term improvement:

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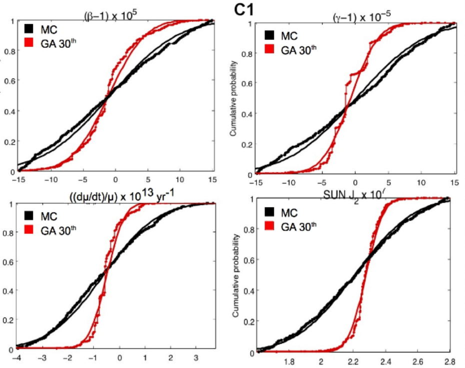
In the coming years, GAIA, Bepi-Colombo, JUICE (3GM team)

More constraints on long-term ephemerides !

The end

Improvement of gaussianity

C1



C2