Mechanical Caucus Race

"They began running when they liked."

a report on a surprising new phenomenon
and a versatile principle for getting out more than we put in

on the occasion of Alain Chenciner's 傘寿
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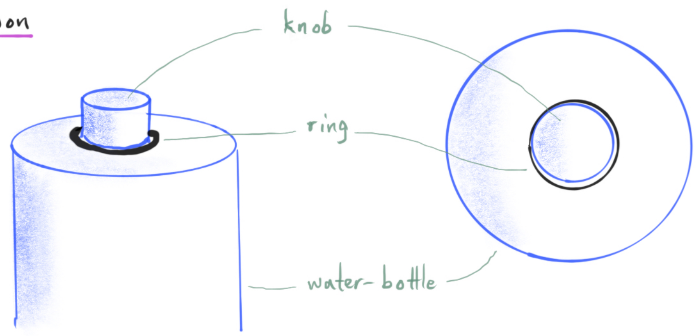
Phenomenon



The ring is loosely fitted around the knob.

On a frictional floor, rotate the bottle (hence the knob) at ω. The ring should just follow at ω.

But no: in 2019 Cole Graham discovered

the ring spins $\Omega >$ a few times ω

(Recall in applied math 'a few' is a technical term meaning half an order of magnitude, i.e. a few $:= \sqrt{10} \approx 3$ or 4.)

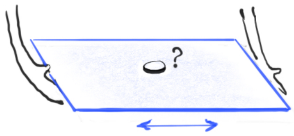
T^2 couldn't help generalizing:

> Design mechanisms where a substrate is driven gently yet a superstructure goes wild.

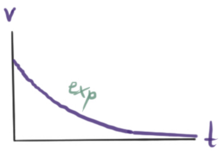
Ex Place a coin on a tray. Shake the tray symmetrically sideways.

Surely the coin shakes symmetrically?

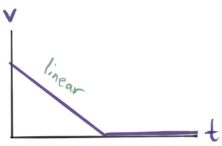
But no: depending on the initial condition, the coin can drift asymmetrically.



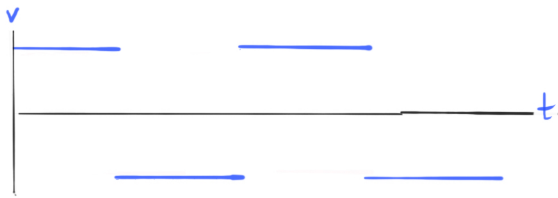
Key Drag in viscous flow $\dot{v} \propto -v$



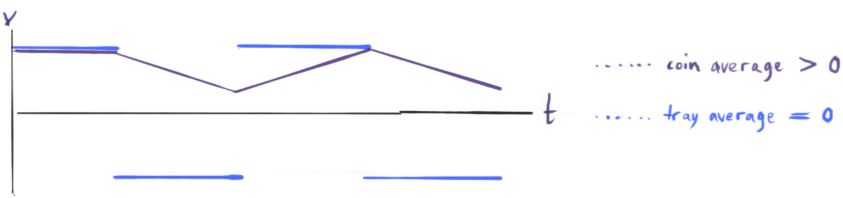
in dry friction $\dot{v} \propto -\text{const}$



Suppose we shake the tray



The coin can be made to respond

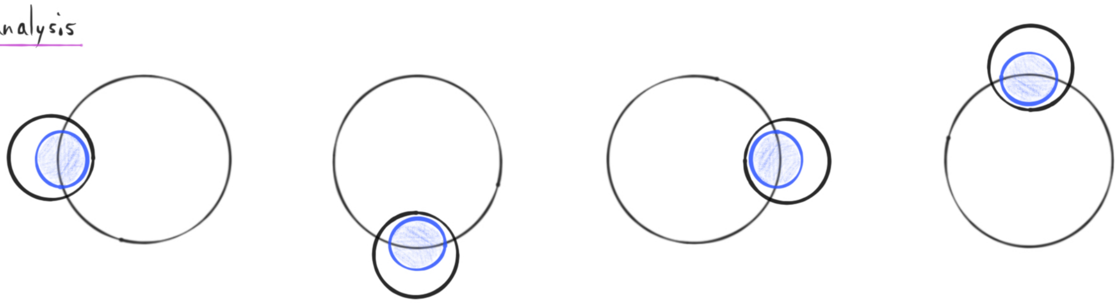


We call this mechanism stick-slip.

Attempted analysis of ring on knob — perhaps rotational stick-slip ?

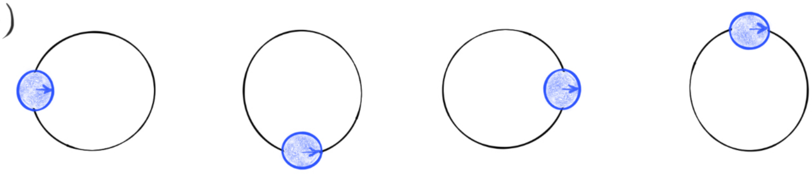
But in 2022 Ian Madden's data showed that stick-slip accounted for only a small part of the ring's spin.

Correct analysis

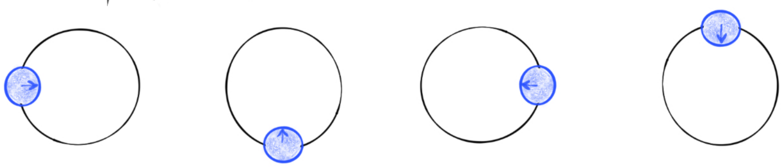


In the first instance, consider pure swirling by the knob. It causes the ring to spin centrifugally.

Scholium swirl ($\approx$ touiller)



is not to be confused with spin ($\approx$ tourner)



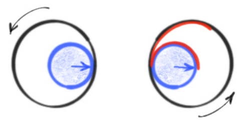
Why swirl ?

Because the bottle's bottom rubs against the floor and rattles the bottle.

As the knob swirls, by how much does the ring spin ?

Another key Let R = radius of ring $>$ radius of knob $= r$

θ = angle of ring $<$ angle of knob $= \Theta$



Smear the knob with paint. As the ring rolls, it too gets colored by paint, and

$$1 = \frac{\text{paint on ring}}{\text{paint on knob}} = \frac{R\theta}{r\Theta} \quad \Rightarrow \quad \theta = \Theta \frac{r}{R}$$

This θ is how much the ring's spin lags behind the knob's swirl Θ. Therefore the ring's spin entrained by the knob's swirl is

$$\dot{\Theta}\left(1 - \frac{r}{R}\right)$$

Check When $r = R$ (ring tightly stuck on knob), the ring gains no spin.
When $r = 0$ (knob reduced to needle), the ring spins a full circle.

We call this mechanism hula-hoop.

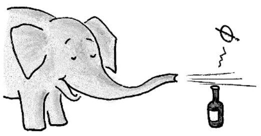
Altogether we should observe

$$\underset{\text{ring spin}}{\Omega} \approx \underset{\text{knob spin}}{\omega} + \underset{\text{hula-hoop}}{\dot{\Theta}\left(1 - \frac{r}{R}\right)} + \underset{\text{stick-slip}}{\dot{\varphi}}$$

Estimates

$\frac{r}{R}$: I measured $r \approx 13$ mm, $R \approx 13.5$ mm $\Rightarrow \frac{r}{R} \approx 0.96$

$\dot{\Theta}$: I guessed middle C (i.e. C_4) ≈ 261.63 Hz



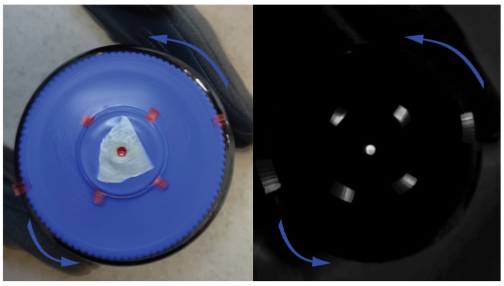
$\approx 40 \frac{\text{swirls}}{\text{sec}}$ because I'd hear a few rattles per half-swirl

$\Rightarrow \dot{\Theta} \approx 80\pi \frac{\text{rad}}{\text{sec}}$

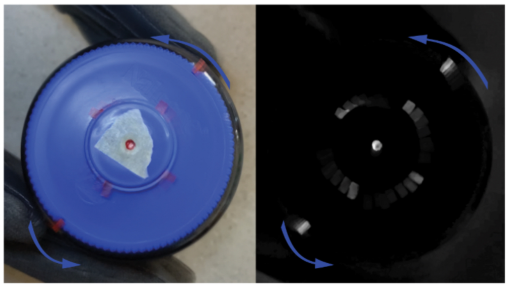
This was confirmed by a high-speed camera.

ω : This I control, half $\frac{\text{turn}}{\text{sec}}$ $\Rightarrow \omega \approx \pi \frac{\text{rad}}{\text{sec}}$ on average

$\dot{\varphi}$: at most amplitude of fluctuations of ω $\Rightarrow \dot{\varphi} \approx \frac{\omega}{\text{a few}} \approx 1 \frac{\text{rad}}{\text{sec}}$



no rattle : pure stick-slip



rattle : stick-slip plus hula-hoop

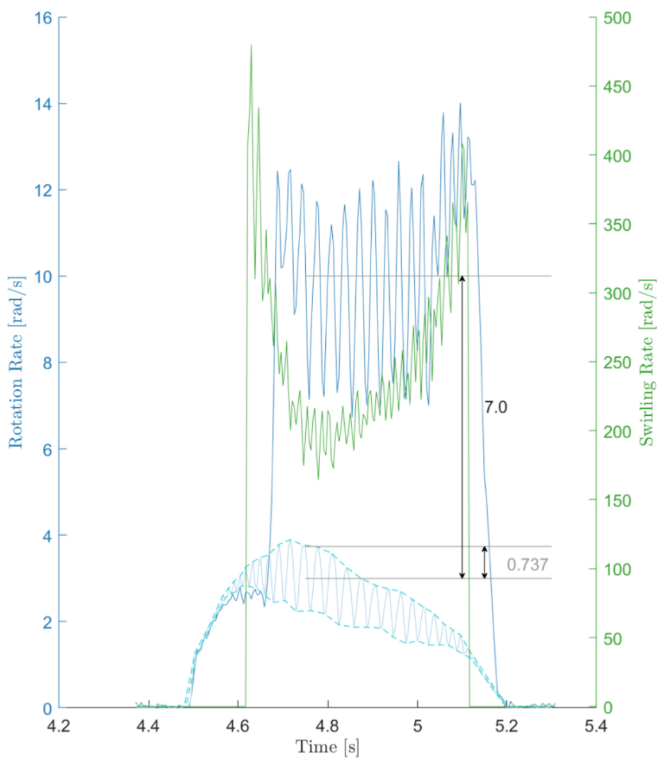
Thus the model predicts

hula-hoop $\dot{\Theta}\left(1 - \frac{r}{R}\right) \approx 10 \frac{\text{rad}}{\text{sec}}$

stick-slip $\dot{\varphi} \approx \frac{1}{10}\dot{\Theta}$

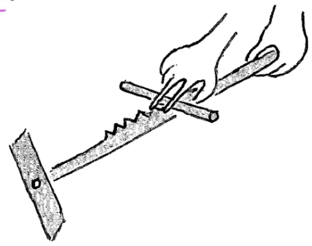
ring faster than knob by $\frac{\Omega - \omega}{\omega} \approx$ a few times

while the data reveal → LOOK!



... in superb agreement for so artisanal a model.

Application



A propeller is loosely nailed to the head of a rod ; the rod is lined with notches.
Rub on the notches with a transverse stick.
This makes the rod vibrate, and there's no reason for the propeller to gain any spin.

Yet I can control the propeller to spin in either way , at will.

Many claim that the propeller begins to spin in one of the ways by chance , and rubbing supplies the resonant power.
This is incorrect : I can reverse the spin , at will.

Many feel that rubbing forward or backward differentiates the spins,
This too is incorrect : I rub symmetrically , forward and backward.

Hmm ?