From periodic to quasi-periodic: bifurcations of n-body relative equilibria and Horn polytopes

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Relative equilibria of n bodies in R^3 submitted to the Newton or similar attraction exist only for very special configurations, the so-called *central configurations*. The motions are periodic and necessarily take place in a fixed plane.

Things become richer if one allows the dimension d of the Euclidean ambient space to be greater than 3: then, a relative equilibrium is determined not only by its initial configuration but also by the choice of a hermitian structure on the space where the motion really takes place; moreover, for the more general *balanced configurations*, the motion is in general quasi-periodic.

A way to distinguish up to isometry the relative equilibria of a given configuration is to look for the frequency spectrum of their angular momentum bivector. Determining the set of these spectra, and in particular those for which a bifurcation from a periodic to a quasi-periodic relative equilibrium may occur by deformation of the configuration from central to balanced, is a purely algebraic problem whose solution is closely related to the classical problem of Horn.