Geometric quantization and semi-classical limits (common work with Paul-Emile Paradan).

Let G be a compact connected Lie group with lie algebra $\mathfrak{g}$. Let M be a compact spin manifold with a G-action, and $\mathcal{L}$ be a G-equivariant line bundle on M. A particular case is when M is Hamiltonian and $\mathcal{L}$ the Kostant line bundle. Consider an integer k, and let $Q_G(M, \mathcal{L}^k)$ be the equivariant index of the Dirac operator on M twisted by $\mathcal{L}^k$.

Let $m_G(\lambda, k)$ be the multiplicity in $Q_G(M, \mathcal{L}^k)$ of the irreducible representation of G attached to the admissible coadjoint orbit $G\lambda$. We prove that the distribution $(\Theta_k, \varphi) = k^{\dim(G/T)/2} \sum_\lambda m_G(\lambda, k)(\beta_{\lambda/k}, \varphi)$ has an asymptotic expansion when k tends to infinity of the form $\Theta_k \equiv k^{\dim M/2} \sum_{n=0}^{\infty} k^{-n}\theta_n$. Here φ is a test function on $\mathfrak{g}^*$ and (β_ξ, φ) is the integral of φ on the coadjoint orbit $G\xi$ with respect to the canonical Liouville measure. The dominant term θ_0 is the analog of the Duistermaat-Heckman measure in the spin context. We compute explicitly the distribution θ_n in terms of the graded equivariant $\hat{A}$ class of M and the equivariant curvature of $\mathcal{L}$. A particular case is the asymptotic formula for Riemann sums over (Delzant) polytopes given by Guillemin-Sternberg.