A mechanism for transport in nearly MBL systems

François Huveneers

DAUPHINE UNIVERSITÉ PARIS | CEREMADE UMR CNRS 7534

with

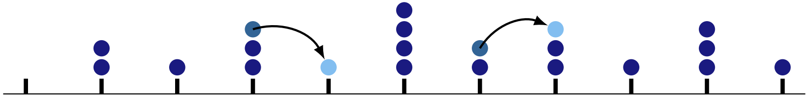
Wojciech De Roeck, Markus Müller, Mauro Schiulaz

Plan of the talk

1. MBL in translation invariant systems?
(no quenched disorder, “quantum glass”)
2. Strictly speaking, no:
delocalization through mobile ergodic spots
3. Are there many-body mobility edges
in quenched disordered systems?

Clean Many-Body systems with "frustrations"

Replace quenched disorder by state-dependent disorder



Typical classical configuration in a high temperature T state

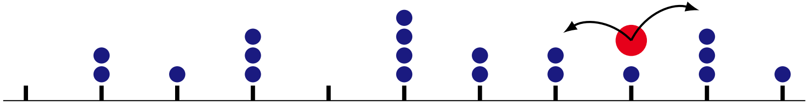
Example of Hamiltonian: Bose-Hubbard

$$H = \sum_x \left\{ \mathsf{U}(a_x^+ a_x)^2 + \mathsf{J}\,(a_x^+ a_{x+1} + \mathrm{h.c.}) \right\}, \qquad \mathsf{J}/\mathsf{U} \ll 1, \;\; T/\mathsf{U} \gg 1$$

Could this system be possibly MBL at $\mathsf{J} \neq 0$?

MBL in Bose-Hubbard?

First intuition: think all particles but one are frozen



blue particles are frozen, red particle moves

Bose-Hubbard for the red particle becomes

$$H = \sum_x \left\{ \mathsf{U}\mathsf{N}_x \, |x\rangle\langle x| \; + \; \mathsf{J}\sqrt{(\mathsf{N}_x+1)(\mathsf{N}_{x+1}+1)}\,(|x\rangle\langle x+1| + \text{h.c.}) \right\}$$

with N_x: number of blue particles at x.

Anderson localization with discrete disorder for $\mathsf{J}/\mathsf{U} \ll 1$, $T/\mathsf{U} \gg 1$.

Where do $\mathsf{J}/\mathsf{U} \ll 1$ and $T/\mathsf{U} \gg 1$ come from?

Gibbs state $\frac{1}{Z}e^{-H/T}$ implies $\mathsf{N}_x \sim \sqrt{T/\mathsf{U}}$.

First order transitions in J/U:

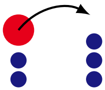
- Perturbative regime (small hopping): $\mathsf{J} \ll \mathsf{U}$ since

$$\mathsf{J}\sqrt{\mathsf{N}_x\mathsf{N}_{x+1}} \sim \mathsf{J}\sqrt{T/\mathsf{U}} \ \ll \ \Delta E_0 \sim \mathsf{U}\sqrt{T/\mathsf{U}}$$

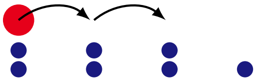
- Rare resonances: $T \gg \mathsf{U}$ since

$$\mathsf{P}(\text{resonance}) = \mathsf{P}(\mathsf{U}\mathsf{N}_x = \mathsf{U}\mathsf{N}_{x+1}) \sim 1/\sqrt{T/U} \ \ll \ 1$$

No resonance: $\Delta E_0 \neq 0$

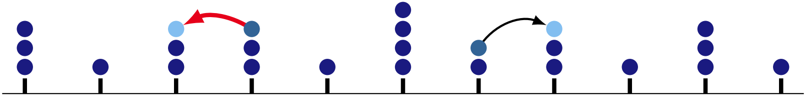


Resonance: $\Delta E_0 = 0$

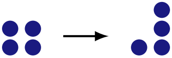


The analysis carries over to many-body physics

First order analysis in J/U stays the same:



- No resonance:



$$2^2 + 2^2 \neq 1^2 + 3^2$$

- Resonance:

$$2^2 + 3^2 = 3^2 + 2^2$$

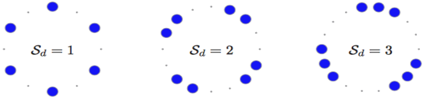
Asymptotic localization for a related model (De Roeck and Huveneers ’13):
$(a_x^+ a_x)^2 \to (a_x^+ a_x)^{2+\epsilon}$: temperature becomes the only relevant parameter,

$$\text{Essentially: } \kappa(\mathsf{U}, \mathsf{J}, T) \;\leq\; \mathrm{C}_n/T^n \quad \text{for all } \; n > 1 \quad \text{as} \quad T \to \infty$$

Several (recent) works with a similar idea

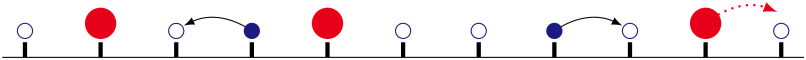
- Kagan and Maksimov (1984): Long range interactions
  Rare resonances, frustrating effect on the dynamics

- Carleo et al. (2012): Bose-Hubbard, starting with doubly occupied sites



  Relaxation to equilibrium is much slowed down as $\mathsf{J}/\mathsf{U} \to 0$.

- Schiulaz and Müller (2013): “Quantum glass”
  Light particles act as a (dynamical) random environment on heavy ones



  See also Grover and Fisher (2013)

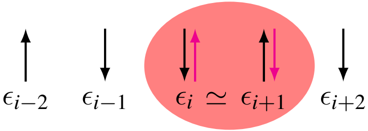
- Papic, Stoudenmire and Abanin (2015, see later),
- Pino, Altshuler and Ioffe (2015), ...

The analysis was a bit too crude until now

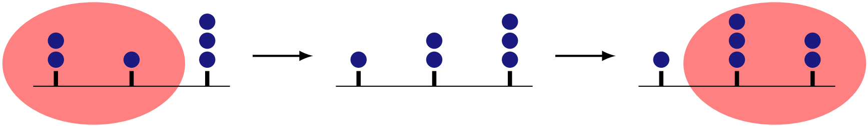
Several caveats. In particular, resonances may travel along the system

Quenched disorder, deep localized phase: location of the resonances is fixed

$$H = \sum_i \left\{ \epsilon_i \sigma_i^{(z)} + \mathsf{J}(\uparrow \downarrow)_i \right\}$$



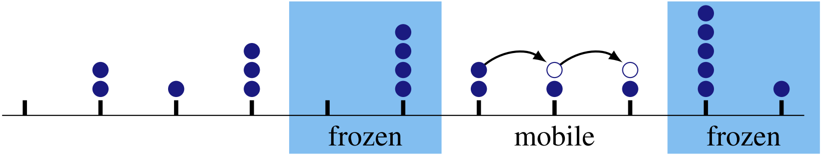
Translation invariant: location of the resonances depends on the state



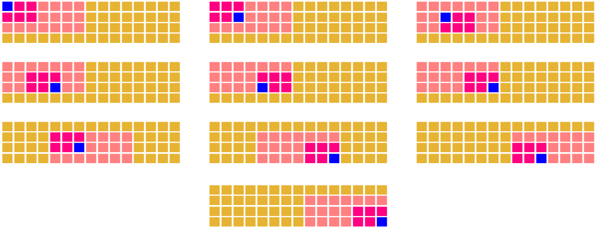
Resonant spot has changed place

We could play this game further

Strictly in $d = 1$, mobile resonant spots get eventually stuck



In $d = 2$, some spots become truly mobile



■ = 3 particles, ■ = 2 particles, ■ = 1 particle, . . . , ■ = 0 particle

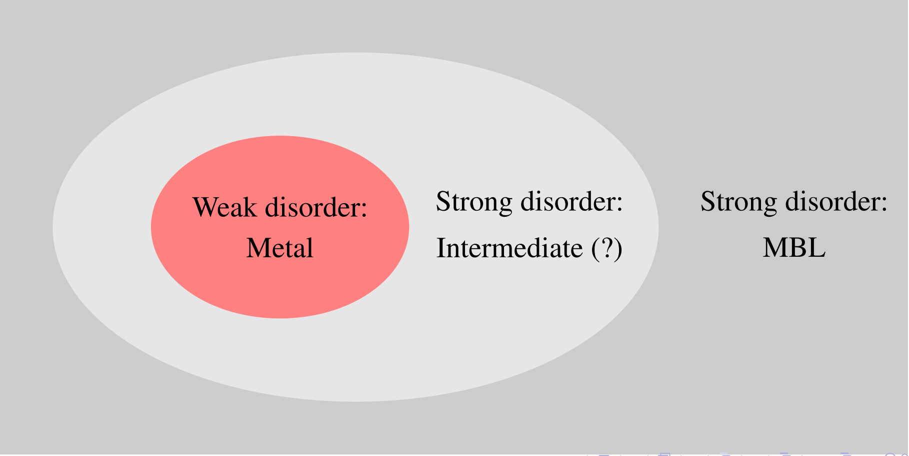
The first order analysis depends very much on fine details. This is not robust.

Imperfect bath in quenched disordered systems

E.g.: spin chain for sites $s \in \mathbb{Z}^2$ with local interactions:

$$H = \sum_{s \in \mathbb{Z}^2} \left\{ \epsilon_s \sigma_s^{(z)} + \mathsf{J}(\uparrow\downarrow)_{s' \sim s} \right\}, \qquad \mathsf{J} \ll W.$$

Rare spot of size V where the disorder is weak: Proba($\bullet$) $\sim \mathrm{e}^{-\rho_W V}$



MBL survives to a fixed imperfect bath (see Imbrie '14)

– Region of size V with anomalously low disorder: imperfect bath, ETH

Level spacing: $\Delta E \sim 2^{-V}\epsilon_0$, extended eigenstates ψ in V

– Spins close to the bath, distance ℓ with $\ell \log(W/J) \lesssim V$: thermalization

$$\langle \psi, \uparrow | H | \psi', \downarrow \rangle \sim 2^{-V/2} \mathsf{J} (\mathsf{J}/W)^{\ell} \quad \gg \quad 2^{-V}\epsilon_0 \sim \Delta E \qquad (\star)$$

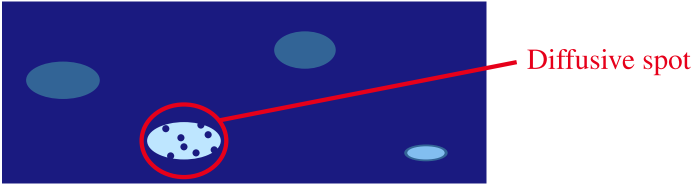
For intermediate ℓ, intermediate phase shows up (partial thermalization?)

– Spins far from the bath, $d \log(W/J) \gg V$: MBL survives

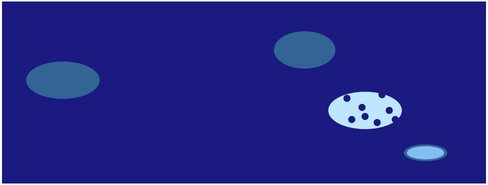
$\gg$ becomes $\ll$ in $(\star)$

In clean systems, localization is less robust

A resonant spot now appears because of a local fluctuation in the states



↓ Translation invariance ↓

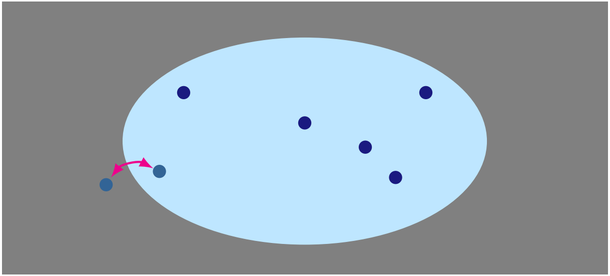


"Diffusive spots move by absorbing/expelling bosons"

To visualize better, we consider the Bose-Hubbard Hamiltonian in $d = 2$:

$$H = \sum_{s \in \mathbb{Z}^2} \left\{ \mathsf{U}(a_s^+ a_s)^2 + \mathsf{J}(a_s^+ a_{s'} + \text{h.c.}) \right\}, \qquad \mathsf{J} \ll \mathsf{U}, \quad \mathsf{U} \ll T$$

Bubbles with very low density are thermal (like hard core bosons in $d = 2$)



Absorbing/expelling a boson into/from the spot is a resonant process

Similar ideas by Huse/Nandkishore

Hybridization of states with thermal spot at different places

Level spacing in the bubble:

$$\Delta E \sim \frac{1}{\mathcal{N}}\epsilon_0 \qquad \text{with} \qquad \mathcal{N} = e^{s(\rho)V}$$

Take two bubble states ψ and Ψ with

ψ: bubble without the extra particle

Ψ: bubble with the extra particle

Assuming ETH in the bubble:

$$|\langle\Psi|H|\psi\rangle|^2 = \langle H\Psi\big(\big|\psi\rangle\langle\psi\big|\big)H\Psi\rangle \sim \mathrm{Tr}_\rho(|H\Psi\rangle\langle H\Psi|) \sim \frac{\epsilon_0^2}{\mathcal{N}}$$

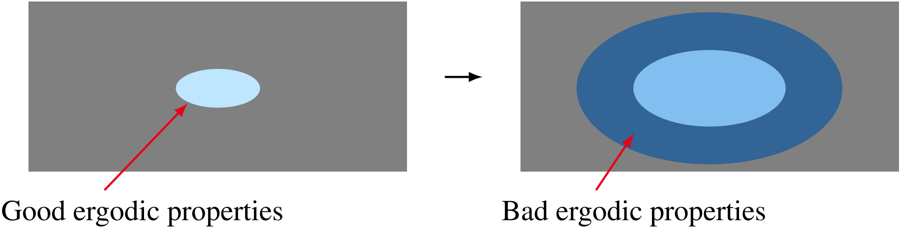
Take ψ whatever and select Ψ to minimize the energy difference:

$$|\langle\Psi|H|\psi\rangle| \sim \frac{\epsilon_0}{\sqrt{\mathcal{N}}} \gg \frac{\epsilon_0}{\mathcal{N}} \sim \Delta E$$

The bubble can absorb a particle, it acts as a bath.

Could the bubble be just evanescent?

Entropically, the bubble is likely to get stuck quickly:



- Gibbs state $\frac{1}{Z}e^{-H/T}$ is invariant under dynamics.
  One keeps finding good thermal bubbles all the time
- Micro-reversibility of hamiltonian dynamics: if dynamics outside the bubbles is localized, bubble eventually go back to initial state
- As such, effect also found in stochastic dynamics for glasses (KCM)

This phenomenon should slow down (a lot) thermalization, not suppress it.

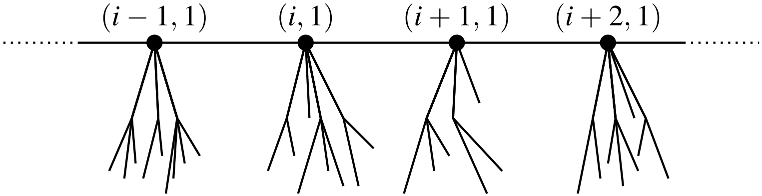
Could $\langle \Psi | H | \psi \rangle$ be suppressed by orthogonality catastrophe?

Neglecting terms in the Hamiltonian can have a drastic effect.

E.g.: quenched disordered single particle Hamiltonian

$$H = \sum_{trees} H_{tree} + \mathsf{J} \sum_i \left\{ |i, 1\rangle\langle i+1, 1| + \text{h.c.} \right\}$$

A finite tree (N branches) hanging at each node



Dressing the sites matters

Surprising effect when the energy bandwidth $\mathcal{W}$ in each tree is large. Take

- Inside the tree ($\mathsf{J} = 0$): ergodic
- For nodes not dressed by trees ($N = 1$): delocalized ($\mathsf{J}/W > 1$, $d = 3$)

Diagonalize inside each tree: states $|i, \alpha\rangle$.

- Coupling between states of neighboring trees: $\langle i, \alpha|V|i+1, \beta\rangle \sim \mathsf{J}/N$
- Level spacing: $\Delta E \sim \mathcal{W}/N$ with $\mathcal{W}$ bandwidth of each tree

We may now have localization: $\mathsf{J}/N \ll \mathcal{W}/N$ for large $\mathcal{W}$ (though $\mathsf{J} \gg W$!)

Many-body case: intermediate states of the bubble could be like trees.
But it does not seem possible for this mechanism to be at work:

- Available energy width: $\mathcal{W} \sim V\epsilon_0$ ($V \sim$ volume bubble)
- Effective coupling/disorder: $\mathsf{J}/W \sim \mathrm{e}^{cV}$, $c > 0$.

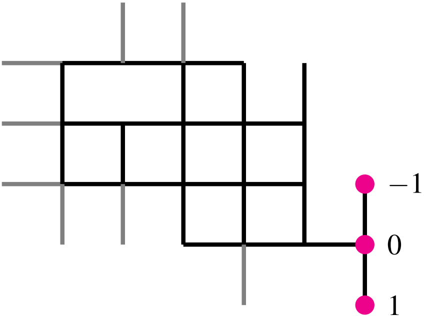
see also *From Anderson to Zeno* by Huse et al. '14.

Do resonant transitions imply delocalization?

Not always: quantum percolation in 1-body physics

Giant percolation cluster $\mathcal{C}$ in $\mathbb{Z}^d$ ($p > p_c$: classical percolation threshold)

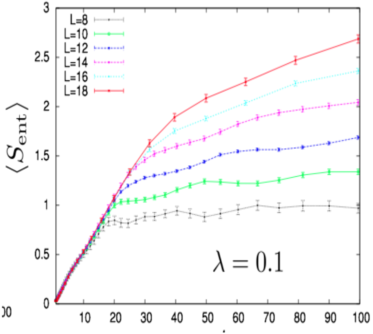
$$H = \sum_{x,y \in \mathcal{C}, x \sim y} |x\rangle\langle y| \quad \text{adjacency matrix on } \mathcal{C}$$

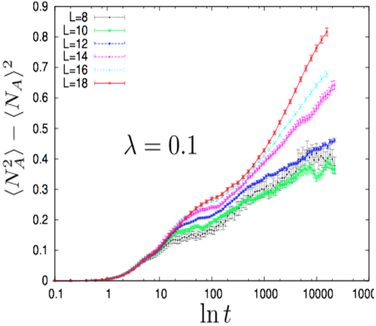


$H\psi = 0$ "Tuning fork"

Very particular example (no mobility edges), also delocalized states ($p > p_c$)

Some numerical results by Papiè, Stoudenmire and Abanin



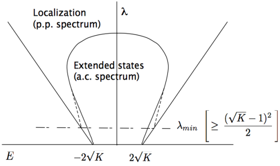


$$H = -\lambda \sum_i b_i^\dagger b_{i+1} + h.c. + n_i \sum_{r>0} \frac{U}{r^2} n_{i+r} \prod_{k=1}^{r-1} (1 - n_{i+k}),$$

Nice, but to be frank, hard to invoke bubbles to understand this

Mobility edges in quenched disordered MBL systems

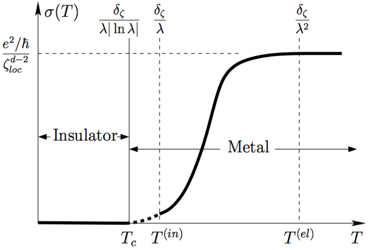
Single body Anderson localization: $d \geq 3$, intermediate J/W or continuum



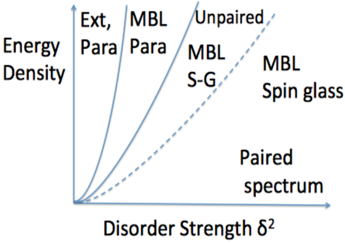
Aizenman and Warzel,

Bethe Lattice, $H_\lambda(\omega) = T + \lambda V(\omega)$

Many-body physics: transition in function of the temperature/energy density



Basko, Aleiner, Althsuler



Huse et al.

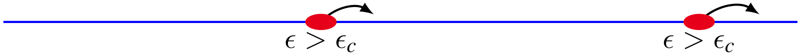
Thermal bubbles could destroy Many-Body mobility edges

Quenched disorder spin chain with mobility edges:

$$\epsilon < \epsilon_c\text{: localized,} \qquad \epsilon > \epsilon_c\text{: delocalized}$$

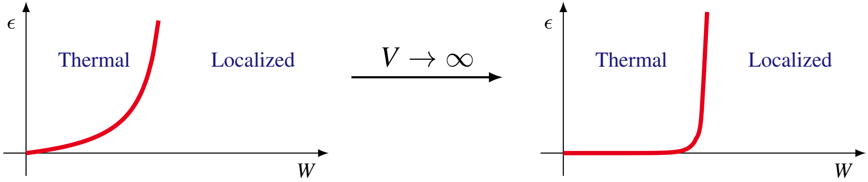
Prepare the system in a localized state ψ:

- overall $E(\psi)/V < \epsilon_c$: negligible overlap with thermal states in the limit
- locally: thermal islands



- Location of the islands is not determined by the disorder: they do travel

In the true thermodynamic limit, we would expect

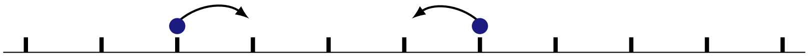


Bubble issue not covered by Imbrie, not addressed in BAA.

Baby version: Shepelyansky's two particles '94

Just two interacting particles on a 1-d disordered lattice:

$$H = \sum_i \epsilon_i n_i + \mathsf{U} \sum_i n_i(n_i - 1) + \mathsf{J} \sum_i \{a_i a_{i+1}^+ + \text{h.c.}\}$$



Coherent propagation of the bound state:

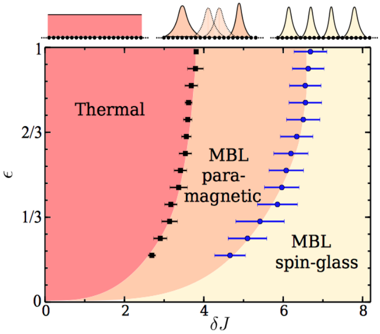
- $l_1 > 1$: localization length of free particles ($\mathsf{U} = 0$)
- l_c: localization length of the bound state ($\mathsf{U} \neq 0$)

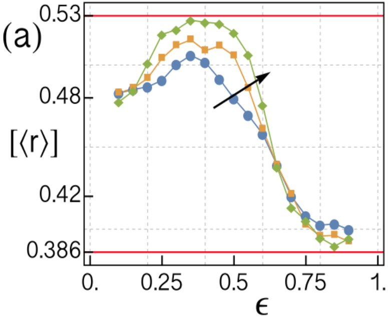
$$\frac{l_c}{l_1} \sim l_1 (\mathsf{U}/\mathsf{J})^2$$

Interaction strongly enhances localization length (still localized since $d = 1$)

Recent numerics do not support our view

Disordered spin chains:





- Left: from Kjäll, Bardarson and Pollmann,
  $H = -\sum_i J_i \sigma_i^z \sigma_{i+1}^z + J_2 \sum_i \sigma_i^z \sigma_{i+2}^z + h \sum_i \sigma_i^x$, $\langle J_i \rangle = 1, h/2 = J_2 = 0.3$
- Right: from Mondragon-Shem, Pal, Hughes and Laumann,
  $H = \sum_i t(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+) + U S_i^z S_{i+1}^z + W w_i S_i^z$, $(U, W, S^z) = (7, 6, 0)$

How could we solve the conflict? (work in progress)

Some possible issues:

- Thermalization by thermal bubbles is due to rare fluctuations: finite size effects may be huge
- Coexistence of localized and delocalized states possible at finite volume
- Thermal phase at finite volume may not yet be "thermal enough" for bubbles to appear

Coexistence of states at finite volume L? Yes: Shepelyansky in 3-d

- Bound states are delocalized
- Particles farther appart than $\log L$ are localized
- Statistically, localized states dominate, but we expect the opposite when

$$2 \text{ particles} \longrightarrow \text{density of particles}$$

We the Hamiltonian by Kjäll et al. and prepared the system in a product state

- thermal state on the left, ground state on the right
- overall below mobility edge

Very hard to see a clean effect

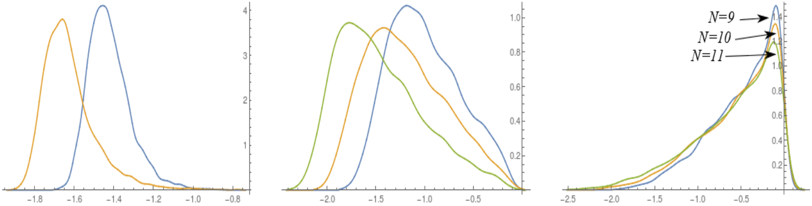
Two unfortunate facts:

- Very narrow δJ-region where the mobility edge could be seen ($\delta J \sim 3$)
- Still a lot of signatures of localization in that region

Participation Ratio (PR): Pick a realization of the disorder and and eigenstate ψ at random with $0.9 < \epsilon_\psi < 1$ (high energy). Pick a site i at random. Define

$$PR_i = \log \sum_{\psi'} |\langle \psi' | \sigma_i^z | \psi \rangle|^4$$

Localization: $PR \sim 0$, Delocalization: $PR \sim -L$



$\delta J = 0.1$ $\delta J = 1$ $\delta J = 3$