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The graph of the Walras correspondence
The production economies case*

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This paper shows topological properties of the graph of the Walras correspondence such as connectedness and simple connectedness for economies with production.

1. Introduction

As in the exchange economy model, the Walras correspondence associates the set equilibria to an economy (parametrized by the initial endowments of the consumers). The purpose of this paper is to generalize Balasko's (1975) results on the topological properties of this correspondence to economies with production.

The model and the main assumptions are given in section 2. In section 3, we prove that the graph of the Walras correspondence has the same topological structure as the set of production equilibria. In section 4, we study a subset of the graph of the Walras correspondence defined by a fixed total supply. In section 5 we prove the connectedness and the simple connectedness of the set of the production equilibria under convexity assumptions on the production sector.

Let ω and ω' be two m-tuples of initial endowments. Let e and e' be two equilibria associated with ω and ω', respectively. The connectedness means that there is a continuous modification $\omega(t)$, $t \in [0,1]$ from the m-tuple ω to ω' such that for every t there is an equilibrium $e(t)$ associated with $\omega(t)$ and $e(t)$ is a continuous function such that $e(0)=e$ and $e(1)=e'$.

Simple connectedness means that there is always a continuous deformation of a continuous trajectory linking (e,ω) to (e',ω') to another one linking the same points.

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2. The model[1]

Let $\mathscr{E}$ be an economy with l goods ($h=1,\dots,l$), m consumers ($i=1,\dots,m$), and n firms ($j=1,\dots,n$), and let $\omega=(\omega_1,\dots,\omega_m)\in\mathbb{R}^{lm}$ be the vector of initial endowments of the consumers. Let $S=\{p\in\mathbb{R}^l_{++}:\sum_{h=1}^l p_h=1\}$ be the set of normalized prices, the technology of the jth firm is described by $Y_j\subset\mathbb{R}^l$, and we assume that for every efficient production plan $y_j\in\partial Y_j$, the firm choses a price p in $\varphi_j(y_j)$, where $\varphi_j:\partial Y_j\to \mathrm{cl}(S)$ is a correspondence[2] called jth pricing rule.

The behavior of the ith consumer is described by a correspondence $D_i:S\times R\to\mathbb{R}^l$, which associates, to a price vector $p\in S$ and a wealth w_i, the set $D_i(p,w_i)$ of possible consumption plans for the ith consumer. The income w_i of this consumer is defined for production plans (y_j) and a price vector p by $w_i=p\omega_i+r_i((y_j),p)$ where r_i is a real valued function defined on $\prod_{j=1}^n Y_j\times S$.

Now we posit the following standard assumptions which describe the general framework of the paper: Let $k\geqq -1$ and $s=\max(0,k)$.

Assumption (D). For all i, D_i is a C^k-correspondence[2] satisfying the Walras law (i.e. for every (p,w) in $S\times R$ and every x in $D_i(p,w)$ we have $px=w$).

Assumption (P). For all j, Y_j is a nonempty closed set satisfying the free disposal assumption (i.e. $Y_j-\mathbb{R}^l_+\subset Y_j$).

We recall, following Bonnisseau and Cornet (1988), that if Y_j satisfies Assumption (P) then the restriction of $\mathrm{proj}_{e^\perp}$ to ∂Y_j is an homeomorphism. Consequently $\varphi_j\circ(\mathrm{proj}_{e^\perp}|\partial Y_j)^{-1}\circ\mathrm{proj}_{e^\perp}$ extends φ_j to $\mathbb{R}^l$, we denote also by φ_j this extension.

Assumption (PR). For all j, φ_j is a C^k-correspondence.

[1]If $x=(x_h)$, $y=(y_h)$ are vectors in $\mathbb{R}^l$, we let $xy=\sum_{h=1}^l x_h y_h$ be the scalar product of $\mathbb{R}^l$, and $\|x\|=(xx)^{\frac{1}{2}}$ be the Euclidean norm. The notation $x\geqq y$ (resp. $x\gg y$) means $x_h\geqq y_h$ (resp. $x_h>y_h$) for all h, we let $\mathbb{R}^l_+=\{x\in\mathbb{R}^l:x\geqq 0\}$ and $\mathbb{R}^l_{++}=\{x\in\mathbb{R}^l:x\gg 0\}$. We denote by e, the vector in $\mathbb{R}^l$ with every coordinate equal to 1. For $A\subset\mathbb{R}^l$, we denote by $\mathrm{cl}\,A$, $\mathrm{int}\,A$, ∂A, $\mathrm{co}\,A$, and A° respectively, the closure, the interior, the boundary, the convex hull and the negative polar of A and, for $B\subset\mathbb{R}^l$, and real numbers λ, μ, we let $\lambda A+\mu B=\{\lambda a+\mu b:a\in A,b\in B\}$. If A is nonempty, and $x\in\mathbb{R}^l$, we let $d_A(x)=\inf\{\|x-a\|:a\in A\}$ and for $r\geqq 0$, $B(A,r)=\{x\in\mathbb{R}^l:d_A(x)\leqq r\}$. If A is a subspace of $\mathbb{R}^l$ we denote by proj_A the orthogonal projection on A.

[2]Given two topological spaces X and Y, a correspondence ϕ from X to Y, associates with each element x in X, a subset $\phi(x)$ of Y; it is said to be upper semi-continuous if ϕ is locally bounded and if the graph of ϕ, i.e. $\{(x,y)\in X\times Y:y\in\phi(x)\}$, is closed. We denote by $D(\phi)$ the domain of ϕ, i.e. $D(\phi)=\{x\in X:\phi(x)\neq\emptyset\}$. A correspondence F is called C^k, for $k\geqq 0$, on a set Ω, if defines a C^k-function on Ω and is called C^{-1} if is upper semi-continuous on Ω with nonempty convex compact values.

Assumption (*R*). For all i, r_i is a C^s-function and we have: for all $((y_j),p)\in\prod_{j=1}^n Y_j\times S$, $\sum_{i=1}^m r_i((y_j),p)=p\sum_{j=1}^n y_j$.

3. The Walras correspondence and the set of production equilibria

Let us now define the following sets:

$$A=\mathbb{R}^{lm}\times\prod_{j=1}^n Y_j\times S\times\mathbb{R}^{lm},$$

$$\mathrm{EP}=\left\{((y_j),p)\in\prod_{j=1}^n Y_j\times S: p\in\bigcap_{j=1}^n \varphi_j(y_j)\right\},$$

$$\mathrm{EG}=\left\{((x_i,(y_i),p,(\omega_i))\in A:((y_j),p)\in\mathrm{EP},(x_i)\in \prod_{i=1}^m D_i(p,p\omega_i+r_i((y_j),p)),\ \sum_{i=1}^m x_i=\sum_{j=1}^n y_j+\sum_{i=1}^m \omega_i\right\},$$

and

$$\mathrm{EE}=\left\{((x_i),(y_j),p,(\omega_i))\in A:((y_j),p)\in \mathrm{EP},(x_i)\in\prod_{i=1}^m D_i(p,p\omega_i),\ \sum_{i=1}^m x_i=\sum_{i=1}^m \omega_i\right\}.$$

Note that, the space EG is canonically homeomorphic to the graph of the Walras correspondence and EP is the set of the production equilibria of the economy $\mathscr{E}$.

Lemma 1. Under the assumption (*R*) *there exist C^s-functions $R_i:\prod_{j=1}^n Y_j\times S\to\mathbb{R}^l$ such that for all i and for all $((y_j),p)\in\prod_{j=1}^n Y_j\times S$, $r_i((y_j,p)=pR_i((y_j),p)$ and $\sum_{i=1}^m R_i((y_j),p)=\sum_{j=1}^n y_j$.*

Proof. Let $((y_j),p)\in\prod_{j=1}^n Y_j\times S$, let $\alpha=p\sum_{j=1}^n y_j$ and let

$$R_i((y_j),p)=\frac{r_i((y_j),p)+(\alpha^2+1)/m}{\alpha^2+\alpha+1}\left(\sum_{j=1}^n y_j+(\alpha^2+1)e\right)$$
$$-[(\alpha^2+1)/m]e,$$

where $e=(1,\dots,1)$. Then the functions R_i satisfy the conditions of the above lemma. □

Let $\Phi:\mathbb{R}^{lm}\times\prod_{j=1}^{n}Y_j\times S\times\mathbb{R}^{lm}\to\mathbb{R}^{lm}\times\prod_{j=1}^{n}Y_j\times S\times\mathbb{R}^{lm}$ be the map defined by $((x_i),(y_j),p,(\omega_i))\to((x_i),(y_j),p,(\omega_i+R_i((y_j),p))$.

Proposition 3.1. The map Φ is a C^s-homeomorphism and $\Phi(\mathrm{EG})=\mathrm{EE}$.

The proof of this result is left to the reader.

For $x\in\mathbb{R}^l$ we denote by $\bar{x}$ the vector of $\mathbb{R}^{l-1}$ defined by $\bar{x}=(x_1,\dots,x_{l-1})$ and let $\eta:\mathbb{R}^{ln}\times\prod_{j=1}^{n}Y_j\times S\times\mathbb{R}^{lm}\to\prod_{j=1}^{n}Y_j\times S\times\mathbb{R}^m\times\mathbb{R}^{(l-1)(m-1)}\times\mathbb{R}^l\times\mathbb{R}^{lm}$ be the correspondence defined by $((x_i),(y_j),p,(\omega_i))\to((y_j),p,(p\omega_i),(\bar{\omega}_i-\bar{x}_i)_{i=1}^{m-1},\sum_{i=1}^{m}\bar{\omega}_i-\sum_{i=1}^{m}\bar{x}_i,(x_i-D_i(p,p\omega_i)))$.

Theorem 3.2. If the demand correspondences are C^s, then η is a C^s-homeomorphism and $\eta(\mathrm{EE})=\mathrm{EP}\times\mathbb{R}^m\times\mathbb{R}^{(l-1)(m-1)}\times\{0\}\times\{0\}$.

The proof of this result is left to the reader.

Corollary 3.3. The spaces EE *and* EG *are homeomorphic to the product of* EP *by an euclidian space. Consequently,* EE *and* EG *have the same topological structure as* EP.

Proof. This result is a direct consequence of Proposition 3.1. and Theorem 3.2. □

Remark. Note that, in general, EP is not connected. For example, let $Y_1=\{(y_1,y'_1)\in\mathbb{R}^2: y_1+y'_1\leqq 0\}$, $Y_2=\{(y_2,y'_2)\in\mathbb{R}^2: y_2^3+y'_2\leqq 0\}$, $\varphi_1(y_1,y'_1)=(\frac{1}{2},\frac{1}{2})$, for all $(y_1,y'_1)\in\partial Y_1$, and $\varphi_2(y_2,y'_2)=(3y_2^2/[1+3y_2^2],1/[1+3y_2^2]$, for all $(y_2,y'_2)\in\partial Y_2$. In fact φ_1 and φ_2 are the marginal pricing rules associated to Y_1 and Y_2, respectively. Furthermore, it is clear that

$$\mathrm{EP}=\left\{\left((y_1,-y_1)\left(\frac{1}{\sqrt{3}},-\frac{1}{3\sqrt{3}}\right)(\tfrac{1}{2},\tfrac{1}{2})\right): y_1\in R\right\}$$

$$\cup\left\{\left((y_1,-y_1)\left(\frac{1}{\sqrt{3}},-\frac{1}{3\sqrt{3}}\right)(\tfrac{1}{2},\tfrac{1}{2})\right): y_1\in\mathbb{R}\right\},$$

which is not connected.

Lemma 2. Let A be a connected space and $\gamma: A \to B$ be a C^{-1} correspondence, then the graph[2] of γ is connected.

Proof. Assume that A is connected and let U and V be open subsets of $A \times B$ such that $\mathrm{gr}(\gamma) \subset U \cup V$ and $U \cap V = \emptyset$. Let A_U and A_V be subsets of A defined by $A_U = \{a \in A : \gamma(a) \cap U \neq \emptyset\}$ and $A_V = \{a \in A : \gamma(a) \cap V \neq \emptyset\}$ and for $a \in A$ we denote by $U_a = \{b \in B : (a, b) \in U\}$ and $V_a = \{b \in B : (a, b) \in V\}$. It is easy to check that $\gamma(a) \subset U_a \cup V_a$ and $U_a \cap V_a = \emptyset$. Since $\gamma(a)$ is convex and consequently connected we have $U_a \cap \gamma(a) = \emptyset$ or $V_a \cap \gamma(a) = \emptyset$ which implies that $A_U \cap A_V = \emptyset$. The previous property implies that $A_U = \{a \in A : \gamma(a) \subset U\}$ and $A_V = \{a \in A : \gamma(a) \subset V\}$. Since γ is upper semi-continuous, A_U and A_V are open subsets of A such that $A = A_U \cup A_V$ and $A_U \cap A_V = \emptyset$. Since A is connected we have $A_U = \emptyset$ or $A_V = \emptyset$ and consequently $\mathrm{gr}(\gamma) \cap V = \emptyset$. Then $\mathrm{gr}(\gamma)$ is connected. □

Corollary 3.4. If $n = 1$ (i.e., if there is only one firm) then the graph of the Walras correspondence is connected. If we further assume that $k \geqq 0$ then the graph of the Walras correspondence is homeomorphic to an Euclidian space.

Proof. Following Corollary 3.3, EG is homeomorphic to the product of EP by an euclidian space. Furthermore, for $n = 1$, EP is canonically homeomorphic to the graph of φ_1. Since ∂Y_1 is homeomorphic to ${}_{e}{}^{\perp}$, the graph of φ_1 is connected by Lemma 2. Furthermore if $k \geqq 0$, φ_1 is a function and $\mathrm{gr}(\varphi_1)$ is homeomorphic to ∂Y_1 and consequently to $\mathbb{R}^{l-1}$. The graph of the Walras correspondence is then homeomorphic to $\mathbb{R}^{lm}$. □

Remark. Note that if $k = -1$, even for the marginal pricing rule, the graph of the Walras correspondence, in general, is not homeomorphic to an euclidian space. Indeed, we refer to Jouini (1992) to show that there exist production sets Y such that, for all $y \in \partial Y$, $\varphi(y) = \mathrm{cl}(S)$, where φ is the marginal pricing rule associated to Y. For such set, we have that EP is homeomorphic to $\mathbb{R}^{l-1} \times \mathrm{cl}(S)$ and EG is homeomorphic to $\mathbb{R}^{ml} \times \mathrm{cl}(S)$. Nevertheless we shall see in Proposition 5.4 that if Y is convex then EP is actually homeomorphic to $\mathbb{R}^{l-1}$.

4. A subset of the graph of the Walras correspondence

For the next, it is necessary to refine our description of the behavior of the consumers. We assume that the ith consumer has a utility function $u_i: \mathbb{R}^l \to \mathbb{R}$ and the $D_i(p, w_i)$ is defined as the set of the solutions of the following maximization program:

$$\max u_i(x_i), \quad \text{s.t.} \quad px_i \leqq w_i.$$

For $x_i \in \mathbb{R}^l$ we denote by $d_i(x_i)$ the matrix defined by

$$d_i(x_i)=\begin{pmatrix} (\partial^2 u_i/\partial x^h \partial x^k(x_i))_{h,k} & {}^t\nabla u_i(x_i) \\ \nabla u_i(x_i) & 0 \end{pmatrix},$$

and we assume the following.

Assumption (U). For all i, u_i is a C^∞-function such that for all $x_i \in \mathbb{R}^l$, $\nabla u_i(x_i) \in \mathbb{R}^l_{++}$ and $d_i(x_i)$ is definite negative.

The previous conditions are the first and second order sufficient condition for monotonicity and strict quasi-concavity of the preferences of the consumers. The following result is due to Balasko (1987),

Proposition 4.1. Under the Assumption (U), the map $\theta: S \times \mathbb{R}^m \to \mathbb{R}^l \times \mathbb{R}^{m-1}$ defined by $(p,(w_i)) \to (\sum_{i=1}^m D_i(p,w_i),(u_i(D_i(p,w_i)))_{i=1}^{m-1})$ is a C^∞-diffeomorphism.

For $r \in \mathbb{R}^l$ we define the following sets:

$$\mathrm{EG}_r=\left\{((x_i),(y_j),p,(\omega_i)) \in \mathrm{EG}: \sum_{i=1}^m \omega_i + \sum_{j=1}^n y_j = r\right\},$$

and

$$\mathrm{EE}_r=\left\{((x_i),(y_j),p,(\omega_i)) \in \mathrm{EE}: \sum_{i=1}^m \omega_i = r\right\}.$$

The space EG_r is the set of economies with associated equilibria such that the total supply is equal to r. It is interesting to study the existence of trajectories between two equilibria and of deformations of these trajectories when the total supply is kept fixed.

Proposition 4.2. The map Φ induces a C^s-homeomorphism between EG_r and EE_r. Furthermore

$$\eta(\mathrm{EE}_r)=B(r)\times \mathbb{R}^{(l-1)(m-1)} \times \{0\} \times \{0\},$$

where $B(r)=\{((y_j),p,(w_i)):((y_j),p)\in \mathrm{EP}, \sum_{i=1}^m D_i(p,w_i)=r\}$.

The proof of this last result is left to the reader.

Consequently, for $k \geqq 0$ and following Theorem 3.2, EG_r is C^k-homeomorphic to the product of an Euclidian space with the set $B(r)$.

Let us now introduce the following maps: $\Delta: \prod_{j=1}^n \partial Y_j \times S \times \mathbb{R}^m \to (e\perp)^n \times S \times \mathbb{R}^m$ defined by $\Delta((y_j),p,(w_i))=((\varphi_j(y_j)-p),p,(w_i))$ and $\Gamma:(e\perp)^n \times S \times \mathbb{R}^m \to (e\perp)^n \times \mathbb{R}^l \times \mathbb{R}^{m-1}$ defined by $\Gamma((z_j),p,(w_i))=((z_j), \sum_{i=1}^m D_i(p,w_i), (u_i(D_i(p,w_i)))_{i=1}^{m-1})$.

Following Proposition 4.1 and under Assumption (U), it is clear that Γ is a C^∞-diffeomorphism and that

$$B(r)=\Delta^{-1}(\Gamma^{-1}(\{0\}\times\{r\}\times\mathbb{R}^{m-1})).$$

Theorem 4.3. Under Assumption (U), if $k\geqq 0$ and if for $j=1,\dots,n$, φ_j is one-to-one with $S\subset\varphi_j(\partial Y_j)$ then EG$_r$ *is homeomorphic to* $\mathbb{R}^{l(m-1)}$.

Proof. Under the above assumptions it is easy to show that Δ is one-to-one with $\{0\}\times S\times\mathbb{R}^m\subset\Delta(\prod_{j=1}^n\partial Y_j\times S\times\mathbb{R}^m$. Consequently, $\Gamma\circ\Delta$ is a C^k-homeomorphism between $B(r)$ and $\{0\}\times\{r\}\times\mathbb{R}^{m-1}$. Hence $B(r)$ is C^k-homeomorphic to $\mathbb{R}^{m-1}$ and EG$_r$ is C^k-homeomorphic to $\mathbb{R}^{m-1}\times\mathbb{R}^{(l-1)(m-1)}=\mathbb{R}^{l(m-1)}$.

5. Profit maximization

Let Y_j be a production set, a pricing rule φ_j is called profit maximization rule if for all $y_j\in\partial Y_j$ we have $\varphi_j(y_j)=\{p\in \mathrm{cl}(S):py_j\geqq py'_j$, for all $y'_j\in Y_j\}$. It is well known that if Y_j is a convex set then the profit maximization rule satisfies the assumption (PR) and that if Y_j is differentially strictly convex (i.e., the set Y_j satisfies the second order sufficient conditions for strict convexity, more precisely Y_j is convex and the Gaussian curvature of ∂Y_j is everywhere different from 0) then the profit maximization rule defines a C^1-diffeomorphism between ∂Y_j and $\varphi_j(\partial Y_j)$.

Proposition 5.1. Under Assumption (U), if all the producers maximize their profits on differentially strictly convex production sets with $S\subset\varphi_j(\partial Y_j)$, $j=1,\dots,n$, then EG$_r$ *is C^s-homeomorphic to $\mathbb{R}^{l(m-1)}$ and* EG *is C^s-homeomorphic to $\mathbb{R}^{lm}$.*

Proof. Under the previous assumptions φ_j is C^1 for $j=1,\dots,n$ and D_i is C^∞ for $i=1,\dots,m$, consequently we have $k\geqq 0$. Furthermore, Δ is one-to-one and $\{0\}\times S\times\mathbb{R}^m\subset\Delta(\prod_{j=1}^n\partial Y_j\times S\times\mathbb{R}^m)$ then, following Theorem 4.3, EG$_r$ is C^s-homeomorphic to $\mathbb{R}^{l(m-1)}$. Finally it is clear that EG$=(\Gamma\circ\Delta)^{-1}(\{0\}\times\mathbb{R}^l\times\mathbb{R}^{m-1})\times\mathbb{R}^{(l-1)(m-1)}$ which implies that EG is C^s-homeomorphic to $\mathbb{R}^{lm}$. □

It is easy to show that the previous assumption $S\subset\varphi(\partial Y)$ is satisfied if there exist two vectors a and b such that $a-\mathbb{R}^l_+\subset Y\subset b-\mathbb{R}^l_+$.

Proposition 5.2. If all the producers, except at most one of them ($j=1$), maximize their profits on convex production sets with $\varphi_1(\partial Y_1)\subset\mathrm{int}(\varphi_j(\partial Y_j))$, $j=2,\dots,n$, then EP *is connected.*

Proof. Let A be the graph of the correspondence φ_1 (i.e., $A=\{(y_1,p)\in\partial Y_1\times \mathrm{cl}(S):p\in\varphi_1(y_1)\}$) and $\gamma:A\to\prod_{j=2}^{n}\partial Y_j$ be the correspondence defined by $(y_1,p)\to(\eta_2(p),\dots,\eta_n(p))$, where for $p\in\mathrm{cl}(S)$, $\eta_j(p)=\{y_j\in Y_j:py_j=\max pY_j\}$. By Lemma 2, A is connected. Since $\varphi_1(\partial Y_1)\subset\mathrm{int}(\varphi(\partial Y_j))$, $j=2,\dots,n$, it is easy to show that $\mathrm{int}(D(\gamma))\neq\emptyset$. Furthermore γ is a maximal monotone operator consequently, by Brezis (1973, proposition 2.9) $\mathrm{int}(D(\gamma))=\mathrm{int}(\mathrm{cl}(D(\gamma)))$ and γ is locally bounded on $\mathrm{int}(D(\gamma))$ and consequently on A. If we further remark that γ is convex valued and has a closed graph we obtain then that γ is a C^{-1}-correspondence on A. By Lemma 2, EP, which is canonically homeomorphic to the graph of γ, is connected. □

In the next we denote by Y the set defined by $Y=Y_1+\cdots+Y_n$ and by $A(Y)$ the asymptotic cone of Y, i.e. $A(Y)=\bigcap_{\rho\geqq 0}\{\lambda y:y\in Y,\ \lambda\geqq 0,\ \|y\|\geqq\rho\}$.

Proposition 5.3. If all the producers maximize their profits on strictly convex production sets and if $A(Y)\cap -A(Y)=\{0\}$ (irreversibility assumption) then EP *is contractible (and consequently simply connected).*

Proof. It is clear that EP is canonically homeomorphic to the graph of the following correspondence $\delta:\mathrm{cl}(S)\to\prod_{j=1}^{n}\partial Y_j$ defined by $\delta(p)=(\eta_1(p),\dots,\eta_n(p))$. Let $\eta:\mathrm{cl}(S)\to\partial Y$ be the correspondence defined by $p\to\{y\in Y:py=\max pY\}$. We can show easily that δ and η have the same domain. Since Y is convex, it is clear that $\mathrm{int}(A(Y)^{\circ})\cap\mathrm{cl}(S)\subset D(\eta)$. By the irreversibility assumption we have $\mathrm{int}(A(Y)^{\circ})\neq\emptyset$, and then $\mathrm{int}(D(\eta))\neq\emptyset$. Since η is clearly a maximal monotone operator, by the same argument used in the proof of Proposition 5.2 we have, $\mathrm{int}(D(\eta))=\mathrm{int}(\mathrm{cl}(D(\eta)))$ and η is locally bounded on $\mathrm{int}(D(\eta))$, furthermore this last set is convex.

Since $\mathrm{int}(D(\delta))$ is a nonempty open convex set and $D(\delta)\subset\mathrm{cl}(\mathrm{int}(D(\delta)))$, we can choose p_0 in $\mathrm{int}(D(\delta))$ and we have for all $p\in D(\delta)$ and all $t\in[0,1[$, $(1-t)p_0+tp\in\mathrm{int}(D(\delta))$. Thus let $H:D(\delta)\times[0,1]\to D(\delta)$ be the map defined by $(p,t)\to(1-t)p_0+tp$. It is clear that H is continuous and that $H(D(\delta)\times\{0\})=\{p_0\}$ *and* $H(D(\delta)\times\{1\})=D(\delta)$. Consequently $D(\delta)$ is contractible.

Since production sets are strictly convex, δ is a single valued function defined on a contractible set and consequently $\mathrm{gr}(\gamma)$ is contractible. □

In the following result we consider constant-returns production structure as in Kehoe (1982) and Mas-Colell (1985). This type of specification includes decreasing returns as a special case.

Proposition 5.4. If all the production sets are convex cones of vertex 0 *and if $Y\cap -Y=\{0\}$ (irreversibility assumption) then* EP *is connected.*

Proof. Under the irreversibility assumption we have $\mathrm{int}(Y^{\circ})\neq\emptyset$ and consequently $\bigcap_{j=1}^{n} \operatorname{int}(Y_j^\circ) \neq \emptyset$. With the notations of the Proposition 5.2 we then have $\operatorname{int}(D(\gamma) \neq \emptyset$. The proof then follows as in Proposition 5.2. □

Proposition 5.5. If $n=1$ and if this producer maximizes its profit on a convex production set then EP *is homeomorphic to* $\mathbb{R}^{l-1}$.

Proof. We denote by Y the set Y_1. Let $\mu:\mathrm{EP}\to\partial(Y+B(0,1))$ be the map defined by $(y,p)\to p/\|p\|+y$ and let π be the projection on the convex set Y. For $z\in\partial(Y+B(0,1))$ and $(y,p)\in\mathrm{EP}$, it is easy to show that

$$\left(\pi(z),\frac{z-\pi(z)}{(z-\pi(z))e}\right)\in\mathrm{EP},\quad \pi(\mu(y,p))=y,\quad\text{and}\quad \frac{\mu(y,p)-y}{(\mu(y,p)-y)e}=p.$$

Consequently, μ is an homemorphism between EP and $\partial(Y+B(0,1))$. Furthermore the set $Y+B(0,1)$ satisfies the assumption (P) then, following Bonnisseau and Cornet (1988), $\partial(Y+B(0,1))$ is homeomorphic to $\mathbb{R}^{l-1}$. □

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