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Economic Theory

Equilibria in large economies with differentiated commodities and non-ordered preferences*

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Summary. The purpose of this paper is to provide an equilibrium existence result for economies with a measure space of agents, a finite set of producers and infinitely many differentiated commodities. The approach proposed in this paper, based on the *discretization* of measurable correspondences, allows us to extend the existence results in Ostroy and Zame (1994) and Podczeck (1997) to economies with a non-trivial production sector and with possibly non-ordered preferences. Moreover, our approach allows for more general consumption sets than the positive cone and following the direction introduced by Podczeck (1998), the *uniform substitutability* assumptions of Mas-Colell (1975), Jones (1983), and Ostroy and Zame (1994), are replaced by the weaker assumptions of *uniform properness*.

Keywords and Phrases: Measure space of agents, Differentiated commodities, Non-ordered preferences, Uniform properness, Discretization of measurable correspondences.

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1 Introduction

The purpose of this paper is to provide an equilibrium existence result for economies with a measure space of agents, a finite set of producers and infinitely many differentiated commodities. The approach proposed in this paper, based on the *discretization* of measurable correspondences, allows us to extend the existence results in Ostroy and Zame [32] and Podczeck [34] to economies with a non-trivial production sector and with possibly non-ordered preferences. Moreover, our approach

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allows for more general consumption sets than the positive cone and following the direction introduced by Podczeck in [35], the *uniform substitutability* assumptions of Mas-Colell [29], Jones [23] and Ostroy and Zame [32], are replaced by the weaker assumptions of *uniform properness*. Our *uniform properness* assumptions are inspired from those presented in Podczeck [33] and in Florenzano and Marakulin [16], and they generalize uniform properness assumptions introduced in Podczeck [35].

As in the models of Mas-Colell [29], Jones [23], Yannelis and Zame [46], and Ostroy and Zame [32], we specify the set of commodities as a compact metric space T. Commodity bundles are then modelled as measures on this space of commodities. As in Jones [23], Ostroy and Zame [32], and Podczeck [34], but different to Mas-Colell [29] and Cornet and Medecin [11], it is assumed that all commodities are divisible. The space of all commodity bundles is then $M(T)$, the space of all finite signed Borel measures on T.

In the framework of differentiated commodities there exist, among others, two approaches to model economies with infinitely many agents (or consumers). In Mas-Colell [29], Jones [23], and Podczeck [35], economies are described by distributions on the space of agents' characteristics. Following Ostroy and Zame [32], Podczeck [34], and Cornet and Medecin [11], we describe an economy as a mapping from a measure space of agents to the space of agents' characteristics. The measure space of agents is not supposed to be purely non-atomic, then we encompass the finite agents' set-up.

In our model, as in Ostroy-Zame [32], Podczeck [34], and Cornet and Medecin [11], aggregation of individual commodity bundles is formalized in terms of the Gelfand integral and not in terms of the Bochner integral. The Bochner integration is related to the norm topology whereas the Gelfand integration is related to the weak star topology. Since the positive cone of the commodity space, endowed with the weak star topology is separable, this allows us to avoid the problem of "many more commodities than agents" addressed[1] by Tourky and Yannelis [43] and Podczeck [36]. However, Tourky and Yannelis noted (Remark 10.2 in [43]) that the use of the Gelfand integral could lead interpretive problem in economics. But for the commodity differentiation, this interpretive problem does not arise.[2] Moreover Cornet and Medecin [11] provided an example of an allocation with a natural economic interpretation which is Gelfand measurable but not Bochner measurable.

As in Hildenbrand [20], we consider a private ownership economy with a finite set of firms. We mention the papers of Hart [18, 19] and Podczeck [35] on economies with infinitely many producers and differentiated commodities.

In the framework of economies with finitely many consumers, one of the most important generalization of the assumptions of Debreu [12], concern the transitivity and the completeness of preferences. Mas-Colell [28] and Gale and Mas-Colell [17] proved that the assumptions of completeness and transitivity of preferences are

[1] Note that in our context, contrary to [43] and [36], the positive cone of the commodity space has an empty interior.

[2] Indeed, the constant unit price $t \mapsto 1$ is strictly positive and then Proposition 10.2 in [43] applies in our setting.

superfluous for the existence of equilibrium in the model of Arrow–Debreu–McKenzie. We can find other generalizations in Shafer and Sonnenschein [40, 41]. In the framework of economies with infinitely many consumers and infinitely many differentiated commodities, we propose to establish the same generalization, i.e., to prove that the assumptions of completeness and transitivity of preferences are superfluous for the existence of equilibrium.

The existence proof in Mas-Colell [29], Jones [23], Ostroy and Zame [32], and Cornet and Medecin [11] consists of a limit argument based on equilibria in economies with finitely many commodities, by the discretization of the space of characteristics. When the consumption sets are more general than the positive cone, or with a non-trivial production sector, this approach leads *ad hoc* assumptions relative to the choice of discretization process (we refer to Jones [24] and Cornet and Medecin [11] for precisions).

For economies with finitely many agents, Aliprantis and Brown [2] (see also [3–5]) proposed that the appropriate setting for infinite dimensional analysis is that of a vector lattice dual system. Mas-Colell [30] then proved the existence of equilibrium for economies with a topological vector lattice commodity space. But this setting does not cover the duality pairing of the commodity differentiation. However, this work was quickly extended in various directions (see Richard [37], Mas-Colell and Richard [31], Yannelis and Zame [46], Podczeck [34], Tourky [42], and Florenzano and Marakulin [16]). In particular, the framework of locally-solid lattice commodity spaces was replaced by the more general setting (which in particular cover the commodity differentiation) of both lattice commodity and lattice price spaces. For economies with infinitely many agents, Rustichini and Yannelis in [38] and [39], proved the equivalence between the set of Core allocations and the set of Walrasian equilibria for Banach lattice commodity space. But for commodity differentiation with infinitely many agents, Podczeck in [35] is the first to focus on the lattice structure of both the commodity and price spaces to prove the existence of Walrasian equilibria. He succeeded to solve the equilibrium existence problem by using fixed point arguments in infinite dimensional spaces directly, rather than to proceed by finite dimensional approximations. However, in Podczeck [34], economies are described by distributions on the space of agents' characteristics and not as a mapping from a measure space of agents to the space of agents' characteristics.

Our approach also focuses on the lattice structure of the commodity and price spaces. In order to use the recent results establishing existence of equilibria for economies with finitely many agents (e.g. in Yannelis and Zame [46], Podczeck [33], Tourky [42], Florenzano and Marakulin [16], and many others papers [1,7,9,15,25]), our approach consists of a limit argument based on equilibria for economies with finitely many agents. If $\mathcal{E}$ is an economy with a measure space of agents, using the measurability assumptions on the characteristics (initial endowments, consumption sets, preference relations and share functions) we succeed to construct a sequence $(\mathcal{E}^n)_n$ of economies with an increasing but finite set of agents, *converging* to the economy $\mathcal{E}$. We then are able to apply a classical equilibria existence result to each economy $\mathcal{E}^n$, in order to obtain a sequence of quasi-equilibria which will converge to a quasi-equilibrium of the initial economy $\mathcal{E}$.

Our approach of discretization provides an *economic interpretation* of the measurability assumptions. Indeed, if the characteristics of the economy (initial endowment, consumption sets, preferences and share functions) are measurable, then we will be able to approach the large economy by a sequence of finite economies.

The paper is organized as follows. In Section 2, we set out the main definitions and notations. In Section 3 we define the model of large economies with differentiated commodities, we introduce the concepts of equilibria, we give the list of assumptions that economies will be required to satisfy and finally, we present the existence result (Theorem 3.14). The Section 4 is devoted to the mathematical *discretization* of measurable correspondences. The proof of the main theorem (Theorem 3.14) is then given in Section 5.

2 Notations and definitions

Consider (E,τ) a topological vector space. If $X \subset E$ is a subset, then the τ-interior of X is denoted by τ-int X, the τ-closure of X is denoted by τ-cl X. The convex hull of X is denoted by co X and the τ-closed convex hull of X is denoted by τ-$\overline{\text{co}}\, X$. We let $A(X) = \{v \in E\colon X + \{v\} \subset X\}$ be the asymptotic cone of X and we let $A_\tau(X)$ be the set of elements $x \in E$ such that $x = \tau\text{-}\lim_{n\to\infty} \lambda_n x_n$ where $(\lambda_n)_n$ is a real sequence decreasing to 0 and $(x_n)_n$ is a sequence in X. Note that we always have $A(X) \subset A_\tau(X)$, and if X is τ-closed convex, then $A(X) = A_\tau(X)$. If $(C_n)_n$ is a sequence of subsets of E, the τ-*sequential upper limit* of $(C_n)_n$, is denoted by τ-ls C_n and is defined by

$$\tau\text{-ls}\, C_n := \{x \in E\colon x = \tau\text{-}\lim x_k\,, \quad x_k \in C_{n(k)}\}$$

where $(C_{n(k)})_k$ is a subsequence of $(C_n)_n$. Let T be any compact metric space. The set of all continuous functions on T is denoted by $C(T)$ and the set of all finite signed Borel measures on T is denoted by $M(T)$. Note that $C(T)$ and $M(T)$, endowed with their natural positive cones $C(T)_+$ and $M(T)_+$, are vector lattices. Given elements x, y of $C(T)$ or $M(T)$, x^+, x^-, $|x|$, $x \vee y$, and $x \wedge y$ have the usual lattice theoretical meaning. A subset $Z \subset M(T)$ is a lattice if for every $z \in Z$, z^+ and z^- still lie in Z. If $p \in C(T)$, then $\|p\|_\infty$ denotes the sup-norm of p. If $x \in M(T)$, then $\|x\|$ denotes the variation norm of x, that is $\|x\| = |x|(T) = x^+(T) + x^-(T)$. Following the Riesz representation theorem, $M(T)$ is the topological dual of $(C(T), \|.\|_\infty)$. The natural dual pairing $\langle C(T), M(T)\rangle$ is defined by

$$\forall (p,x) \in C(T) \times M(T), \quad \langle p, x\rangle = \int_T p(t)dx(t).$$

If $p \in C(T)$, then $p > 0$ means that [$p \in C(T)_+$ and $p \neq 0$], and $p \gg 0$ means that for each $t \in T$, $p(t) > 0$. If X is a subset of $M(T)$ and p belongs to $C(T)$, then $\sup\{\langle p, x\rangle : x \in X\}$ is denoted by $\sup \langle p, X\rangle$. If $x \in M(T)$, then $x \gg 0$ means that for every $p \in C(T)$, if $p > 0$ then $\langle p, x\rangle > 0$. The polar V° of a subset $V \subset M(T)$ is the subset of $C(T)$ defined by $V^\circ := \{p \in C(T)\colon \forall x \in V,\ |\langle p, x\rangle| \leqslant 1\}$. By the support of $x \in M(T)$, denoted by supp x, we mean the smallest closed subset F of T such that $|x|(T \setminus F) = 0$. Note that $x \in M(T)$

satisfies $x \gg 0$ if and only if [$x > 0$ and $\operatorname{supp} x = T$]. Given any $t \in T$, we write δ_t for the Dirac measure at t, and we denote by 1_K the unit constant function on T, i.e. $1_K(t) = 1$ for each $t \in T$. The weak topology $\sigma(M(T), C(T))$ on $M(T)$ is denoted by w^* and we denote by bw^* the strongest topology on $M(T)$ agreeing with the w^*-topology on every w^*-compact set. The Borel σ-algebra of $(M(T), w^*)$ and of $(M(T), bw^*)$ coincide and is denoted by $\mathcal{B}$.

We consider $(A, \mathcal{A}, \mu)$ a finite measure space, i.e., A is a set, $\mathcal{A}$ is a σ-algebra of subsets of A and μ is a finite measure on $\mathcal{A}$. The measure space $(A, \mathcal{A}, \mu)$ is complete if $\mathcal{A}$ contains all μ-negligible[3] subsets of A. A mapping f from A to $M(T)$ is *measurable* if for every $B \in \mathcal{B}$, $f^{-1}(B) \in \mathcal{A}$. Note that f is measurable if and only if it is *Gelfand measurable*, i.e., for each $p \in C(T)$, the real valued function $a \mapsto \langle p, f(a) \rangle$ is measurable. A measurable mapping f from A to $M(T)$ is *Gelfand integrable* if for each $p \in C(T)$, the real valued function $a \mapsto \langle p, f(a) \rangle$ is integrable. Then there exists a unique element $x \in M(T)$, satisfying for each $p \in C(T)$, $\langle p, x \rangle = \int_A \langle p, f(a) \rangle \, d\mu(a)$. The element x is denoted by $\int_A f(a) d\mu(a)$. A measurable mapping f from A to $M(T)$ is *norm integrable* if $\|f(.)\| : a \mapsto \|f(a)\|$ is integrable. Note that norm integrability implies Gelfand integrability and if f has its values in $M(T)_+$ then the converse is true. A sequence $(f_n)_n$ of measurable mappings from A to $M(T)$ is *integrably bounded* if there exists an integrable function h from A to $\mathbb{R}_+$ such that for a.e. $a \in A$, for every n, $\|f_n(a)\| \leqslant h(a)$. If $F : A \twoheadrightarrow M(T)$ is a correspondence then $f : A \to M(T)$ is a measurable selection of F if f is measurable and satisfies for almost every $a \in A$, $f(a) \in F(a)$. The set of measurable selections of F is denoted by $S(F)$ and the set of Gelfand integrable selections of F is denoted by $S^1(F)$.

Let X be a space and $P \subset X \times X$ be a binary relation on X. The relation P is irreflexive if $(x, x) \notin P$, for every $x \in X$. The relation P is transitive if [$(x, y) \in P$ and $(y, z) \in P$] implies $(x, z) \in P$, for every $(x, y, z) \in X^3$. The relation P is negatively transitive if [$(x, y) \notin P$ and $(y, z) \notin P$] implies $(x, z) \notin P$, for every $(x, y, z) \in X^3$. The relation P is a partial order if it is irreflexive and transitive. The relation P is an order if it is irreflexive, transitive and negatively transitive. When P is an order, it is usually denoted by $\succ$ and $X^2 \setminus P$ is denoted by $\preceq$. Note that when P is an order, then $\preceq$ is transitive, reflexive ($x \preceq x$ for every $x \in X$) and complete (for every $(x, y) \in X^2$ either $x \preceq y$ or $y \preceq x$).

3 The model and the result

3.1 The model

We consider a compact metric space T, a complete finite measure space $(A, \mathcal{A}, \mu)$ and a finite set J. Moreover, we consider, for each $j \in J$, an integrable positive function θ_j from A to $\mathbb{R}_+$, satisfying $\int_A \theta_j = 1$, and a set $Y_j \subset M(T)$, a Gelfand integrable mapping e from A to $M(T)$, a correspondence X from A into $M(T)$ and a correspondence of preference relations P in X, i.e., P is a correspondence

[3] A set N is μ-negligible if there exists $E \in A$ such that $N \subset E$ and $\mu(E) = 0$.

from A into $M(T) \times M(T)$ such that for every $a \in A$, $P(a) \subset X(a) \times X(a)$ and $P(a)$ is irreflexive. An economy $\mathcal{E}$ with differentiated commodities is a list

$$\mathcal{E} = ((A, \mathcal{A}, \mu), \langle C(T), M(T) \rangle, (X, P, e), (Y_j, \theta_j)_{j \in J}) .$$

The commodity space of $\mathcal{E}$ is represented by $M(T)$. Each point of T has the interpretation of representing a complete description of all characteristics of a certain commodity. Let $x \in M(T)$ be a commodity bundle, then for each Borel set $B \subset T$, $x(B)$ specifies the total amount of commodities having their characteristics in B. Note that since we let every element of $M(T)$ represent a possible commodity bundle, we assume, as in the models of Jones [23,24] and Ostroy and Zame [32] but different to those of Mas-Colell [29] and Cornet and Medecin [11], that all commodities are perfectly divisible. The natural dual pairing $\langle C(T), M(T) \rangle$ is interpreted as the *price-commodity* pairing. If $p \in C(T)$, then for each $t \in T$, $p(t)$ is interpreted as the value (or price) of one unit of the commodity with characteristic t.

The set of agents (or consumers) is represented by A, the set $\mathcal{A}$ represents the set of admissible coalitions, and the number $\mu(E)$ represents the fraction of consumers which are in the coalition $E \in \mathcal{A}$. For each agent $a \in A$, the consumption set is represented by $X(a) \subset M(T)$ and the preferences are represented by the binary relation $P(a)$. We define the correspondence[4] $P_a : X(a) \twoheadrightarrow X(a)$ by $P_a(x) = \{x' \in X(a) \colon (x, x') \in P(a)\}$. In particular, if $x \in X(a)$ is a consumption bundle, $P_a(x)$ is the set of consumption bundles strictly preferred to x by the agent a. The set of consumption allocations (or plans) of the economy is the set $S^1(X)$ of Gelfand integrable selections of X. The aggregate consumption set X_Σ is defined by

$$X_\Sigma := \int_A X(a) d\mu(a) := \left\{ v \in M(T) \colon \exists x \in S^1(X),\ v = \int_A x(a) d\mu(a) \right\} .$$

The initial endowment of the consumer $a \in A$ is represented by the commodity bundle $e(a) \in M(T)$. We denote by $\omega := \int_A e d\mu$ the aggregate initial endowment.

The production sector of the economy $\mathcal{E}$ is represented by a finite set J of firms with production sets $(Y_j)_{j \in J}$, where for every $j \in J$, $Y_j \subset M(T)$. The profit made by the firm $j \in J$ is distributed among the consumers following the share function θ_j. For each $j \in J$, the function $\theta_j : A \to \mathbb{R}_+$ satisfies $\int_A \theta_j d\mu = 1$. The set of production allocations (or plans) of the economy is the set $S^1(Y) = \prod_{j \in J} Y_j$. The aggregate production set Y_Σ is defined by $Y_\Sigma := \sum_{j \in J} Y_j$.

3.2 The equilibrium concepts

We present hereafter the usual concepts of Walrasian quasi-equilibrium and Walrasian equilibrium for an economy.

Definition 3.1. *An element (x^*, y^*, p^*) of $S^1(X) \times S^1(Y) \times C(T)$ with $p^* \neq 0$ is a Walrasian equilibrium of an economy $\mathcal{E}$, if the following properties are satisfied.*

[4] Note that the binary relation $P(a)$ coincide with the graph of the correspondence P_a.

(a) For almost every $a \in A$,

$$\langle p^*, x^*(a)\rangle = \langle p^*, e(a)\rangle + \sum_{j\in J} \theta_j(a) \left\langle p^*, y_j^*\right\rangle$$

and

$$x \in P_a(x^*(a)) \Longrightarrow \langle p^*, x\rangle > \langle p^*, x^*(a)\rangle .$$

(b) For every $j \in J$,

$$y \in Y_j \Longrightarrow \langle p^*, y\rangle \leqslant \left\langle p^*, y_j^*\right\rangle .$$

(c)

$$\int_A x^*(a)d\mu = \int_A e(a)d\mu + \sum_{j\in J} y_j^*.$$

An element $(x^*, y^*, p^*) \in S^1(X) \times S^1(Y) \times C(T)$ with $p^* \neq 0$ is a Walrasian quasi-equilibrium of an economy $\mathcal{E}$, if the conditions (b) and (c) together with

(a') for almost every $a \in A$,

$$\langle p^*, x^*(a)\rangle = \langle p^*, e(a)\rangle + \sum_{j\in J} \theta_j(a) \left\langle p^*, y_j^*\right\rangle$$

and

$$x \in P_a(x^*(a)) \Longrightarrow \langle p^*, x\rangle \geqslant \langle p^*, x^*(a)\rangle$$

are satisfied.

A Walrasian equilibrium of a production economy $\mathcal{E}$ is clearly a Walrasian quasi-equilibrium of $\mathcal{E}$. We provide in the following remark, a classical condition on $\mathcal{E}$ under which a Walrasian quasi-equilibrium is in fact a Walrasian equilibrium.

Remark 3.2. Let (x^*, y^*, p^*) be a quasi-equilibrium of an economy $\mathcal{E}$. If for almost every agent $a \in A$, $X(a)$ is convex, the strict-preferred set $P_a(x^*(a))$ is w^*-open in $X(a)$ and

$$\inf \langle p^*, X(a)\rangle < \langle p^*, e(a)\rangle + \sum_{j\in J} \theta_j(a) \sup \langle p^*, Y_j\rangle , \tag{3.1}$$

then (x^*, y^*, p^*) is a Walrasian equilibrium of $\mathcal{E}$. In particular, if $p^* \gg 0$ then the condition (3.1) is automatically valid if for almost every agent $a \in A$,

$$\left[\{e(a)\} + \sum_{j\in J} \theta_j(a) Y_j - X(a)\right] \cap M(T)_+ \neq \emptyset.$$

3.3 The assumptions

We present the list of assumptions that the economy $\mathcal{E}$ will be required to satisfy.

Assumption (C). [Consumption Side] *For almost every agent $a \in A$, the consumption set $X(a)$ is w^*-closed, convex and comprehensive;*[5] *for each bundle $x \in X(a)$, $P_a(x)$ is w^*-open in $X(a)$, $P_a^{-1}(x) := \{y \in X(a) \colon x \in P_a(y)\}$ is w^*-open in $X(a)$, $x \notin \operatorname{co} P_a(x)$, and if a belongs to the non-atomic*[6] *part of $(A, \mathcal{A}, \mu)$, then $X(a) \setminus P_a^{-1}(x)$ is convex.*

Remark 3.3. Note that when $P(a)$ is ordered, then, following the notations of Section 2, $X(a) \setminus P_a^{-1}(x) = \{y \in X(a) \colon y \succeq_a x\}$ and assuming that for every $x \in X(a)$, $\{y \in X(a) \colon y \succeq_a x\}$ is convex implies that for $x \in X(a)$, $x \notin \operatorname{co} P_a(x)$. It follows that Assumption C is implied by Assumptions E1–E3 and S1 in Podczeck [34] and by Assumptions P1–P4 for economically thick markets of Ostroy and Zame [32].

Assumption (M). [Measurability] *The correspondence X is graph measurable, i.e.,*

$$\{(a, x) \in A \times M(T) \colon x \in X(a)\} \in \mathcal{A} \otimes \mathcal{B}$$

and the correspondence of preferences P is lower graph measurable, i.e.,

$$\forall y \in S(X), \quad \{(a, x) \in A \times M(T) \colon (x, y(a)) \in P(a)\} \in \mathcal{A} \otimes \mathcal{B}.$$

Remark 3.4. In Podczeck [34], the correspondences X and P are supposed to be graph measurable. It can be proved (see [26]) that in the framework of [34], graph measurability of the correspondence of preference relations implies lower graph measurability, in particular Assumption M is valid. In Ostroy and Zame [32], it is assumed that preferences are Aumann measurable. It can be proved (see [26]) that in the context of Ostroy and Zame [32], Aumann measurability of the correspondence of preference relations imply lower graph measurability. In particular Assumption M is valid.

Assumption (P) [Production side] *The aggregate production set Y_Σ is a bw^*-closed and convex subset of $M(T)$.*

Remark 3.5. For economies with finitely many commodities, Hildenbrand [20] already used Assumption P. For economies with finitely many consumers, Jones [24] supposed that Y_Σ is w^*-closed and convex. This is equivalent to Assumption P since the bw^*-topology is locally convex and compatible with the duality $\langle M(T), C(T) \rangle$.

[5] That is $X(a) + M(T)_+ \subset X(a)$.

[6] An element $E \in \mathcal{A}$ is an atom of $(A, \mathcal{A}, \mu)$ if $\mu(E) \neq 0$ and $[B \in \mathcal{A}$ and $B \subset E]$ implies $\mu(B) = 0$ or $\mu(E \setminus B) = 0$.

Assumption (S) [Survival] *For almost every $a \in A$,*

$$e(a) \in X(a) - \sum_{j \in J} \theta_j(a) Y_j.$$

Remark 3.6. Assumption S means that we have compatibility between individual needs and resources. In the literature of economies with differentiated commodities, this assumption is automatically valid since initial endowments are supposed to lie in the consumption set and since inaction is supposed to be a possible production plan. Note that Theorem 3.14 will be proved under the following weaker Survival assumption: for a.e. $a \in A$, $e(a) \in X(a) - \sum_{j \in J} \theta_j(a) \overline{\text{co}}\, Y_j$.

Assumption (MON) [Monotonicity] *For almost every agent $a \in A$, the preference relation $P(a)$ is monotone, i.e.,*

$$\forall x \in X(a), \quad \{x\} + M(T)_+ \subset P_a(x) \cup \{x\}.$$

Remark 3.7. Theorem 3.14 will be proved under the following weaker assumption: for a.e. $a \in A$, for every $x \in X(a)$, for every $m \in M(T)_+$, there exists $\alpha > 0$ such that $x + \alpha m \in P_a(x) \cup \{x\}$.

Assumption (E) [Endowments] *There exists $\overline{v} \in X_\Sigma$ and $\overline{u} \in Y_\Sigma$ such that $\omega + \overline{u} - \overline{v} \gg 0$.*

Remark 3.8. That is, there exists an aggregate production plan $\overline{u} \in Y_\Sigma$ such that together with the aggregate initial endowment, all commodities are available in the aggregate consumption set. Usually in the literature of differentiated commodities, the consumption sets are suppose to coincide with the positive cone. It follows that if $\omega \gg 0$ and $0 \in Y_\Sigma$ (e.g. in [23,24,32,34]) or if $\omega + \overline{u} \gg 0$ (in [35]), then Assumption E is valid.

Assumption (B) [Bounded] *The correspondence X of consumption sets is norm integrably bounded from below, the initial endowment mapping e is norm integrable and the aggregate set of free production $Y_\Sigma \cap M(T)_+$ is norm bounded.*

Remark 3.9. A correspondence X is norm integrably bounded from below if there exists a norm integrable mapping $\underline{x} : A \to M(T)$ such that for a.e. $a \in A$, $X(a) \subset \{\underline{x}(a)\} + M(T)_+$. Usually in the literature the consumption sets are supposed to coincide with the positive cone $M(T)_+$ and initial endowments are suppose to be Gelfand integrable and to lie in the positive cone. Note that if $\underline{x}$ is a Gelfand integrable mapping from A to $-M(T)_+$ and e is a Gelfand integrable mapping such that for every $a \in A$, $e(a) \geqslant \underline{x}(a)$ then $\underline{x}$ and e are norm integrable. Hildenbrand in [20], and Podczeck in [35] assumed that there is no free production, i.e., $Y_\Sigma \cap M(T)_+ = \{0\}$.

Assumption (WSS) [Weak Strong Survival] *For almost every agent $a \in A$, there exists $0 \neq m \in M(T)_+$ such that*

$$e(a) + m \in X(a) - \sum_{j \in J} \theta_j(a) Y_j.$$

Remark 3.10. Under Assumption C and WSS, each quasi-equilibrium (x^*, y^*, p^*) with $p^* \gg 0$ is in fact a Walrasian equilibrium. This assumption may be replaced by standard irreducibility conditions adapted to our context (see Podczeck [36]). Note that Theorem 3.14 will be proved under the following weaker assumption: for a.e. $a \in A$, there exists $m > 0$ such that $e(a)+m \in X(a) - \sum_{j \in J} \theta_j(a) \overline{\text{co}}\, Y_j - A(Y_\Sigma)$.

Assumption (UP) [Uniform Properness] *There exists a cone Γ bw^*-open, satisfying $\Gamma \cap M(T)_+ \neq \emptyset$ and such that for almost every $a \in A$, for every $j \in J$, for every $x \in X(a)$ and every $y \in Y_j$,*

(a) there exists a subset A^a_x of $M(T)$, radial[7] at x, such that

$$(\{x\} + \Gamma) \cap \{z \in M(T) \colon z \geqslant x \wedge e(a)\} \cap A^a_x \subset \overline{co}\, P_a(x);$$

(b) there exists a subset A^j_y of $M(T)$, radial at y, such that

$$(\{y\} - \Gamma) \cap \{z \in M(T) \colon z \leqslant y \vee 0\} \cap A^j_y \subset \overline{co}\, Y_j.$$

Remark 3.11. This assumption is borrowed from the F-properness assumption introduced by Podczeck [33] for pure exchange economies with finitely many agents and adapted to production economies by Florenzano and Marakulin [16]. Assumption UP is close to the uniform properness assumptions developed in Mas-Colell [30] and Richard [37] for economies with finitely many consumers. For refinements about the properness conditions used in the literature, we refer to Aliprantis, Tourky and Yannelis [6].

Remark 3.12. Assumption UP is weaker than Assumptions C3 and P4 in Podczeck [35], since the radial sets A^a_x and A^j_y are supposed to coincide with $M(T)$. Hence following Propositions 3.2.1 and 3.3.1 in [35], Assumption UP is weaker than usual assumptions about marginal rates of substitution in models of commodity differentiation, e.g. in Jones [23,24], Ostroy and Zame [32], and Podczeck [34].

Remark 3.13. Following the proof of the existence theorem, we can replace the condition (b) by the following condition (b').

(b') For every $u \in Y_\Sigma$, there exists a subset A'_u of $M(T)$, radial at u, such that

$$(\{u\} - \Gamma) \cap \{z \in M(T) \colon z \leqslant u \vee 0\} \cap A'_u \subset Y_\Sigma.$$

[7] A subset $R \subset M(T)$ is radial at $x \in R$ if for every $v \in M(T)$, there exists $\lambda > 0$ such that the segment $[x, x + \lambda v]$ still lie in R.

3.4 Existence result

We shall now state the main result of the paper.

Theorem 3.14. *If $\mathcal{E}$ is an economy satisfying Assumptions* **C**, **M**, **P**, **S**, **MON**, **E**, **B** *and* **UP**, *then there exists a quasi-equilibrium (x^*, y^*, p^*), with $p^* \gg 0$. If moreover $\mathcal{E}$ satisfies* **WSS**, *then (x^*, y^*, p^*) is a Walrasian equilibrium.*

Remark 3.15. This existence result extends to economies with a non-trivial production sector and with possibly non-ordered preferences, existence results (Theorem 1.a and 3.a) in Ostroy and Zame [32], and (in the framework of convex preferences) in Podczeck [34] (Theorem 5.3). Theorem 3.14 allows for more general consumption sets than the positive cone and the Uniform Properness Assumption is weaker than usual assumptions about marginal rates of substitution in models of commodity differentiation, e.g. in Jones [23,24], Ostroy and Zame [32] and Podczeck [34].

Remark 3.16. In this model, aggregation of individual commodity bundles is formalized in terms of the Gelfand integral and not the Bochner integral. This allows us to avoid the problem of "many more commodities than agents" addressed by Tourky and Yannelis [43] and Podczeck [36].

Remark 3.17. As it is frequently done in the literature, instead of Assumption E, we can assume that the aggregate endowment is a uniform properness vector of the economy, or more generally:

Assumption (E') *There exists $\overline{v} \in X_\Sigma$ and $\overline{u} \in Y_\Sigma$ such that $\omega + \overline{u} - \overline{v} \in \Gamma \cap M(T)_+$.*

4 Discretization of measurable correspondences

We consider $(A, \mathcal{A}, \mu)$ a finite complete measure space and (D, d) a separable metric space.

4.1 Notations and definitions

A mapping $f : A \to D$ is *measurable* if for every open set $G \subset D$, $f^{-1}(G) \in \mathcal{A}$ where $f^{-1}(G) := \{a \in A \colon f(a) \in G\}$. A correspondence $F : A \twoheadrightarrow D$ is *graph measurable* if $G_F := \{(a, x) \in A \times D \colon x \in F(a)\} \in \mathcal{A} \otimes \mathcal{B}(D)$, where $\mathcal{B}(D)$ is the σ-algebra of Borelian subsets of D.

Definition 4.1. *A partition $\sigma = (A_i)_{i \in I}$ of A is a measurable partition if the set I is finite and if for every $i \in I$, the set A_i is non-empty and belongs to $\mathcal{A}$. A finite subset A^σ of A is subordinated to the partition σ if there exists a family $(a_i)_{i \in I} \in \prod_{i \in I} A_i$ such that $A^\sigma = \{a_i \colon i \in I\}$.*

Given a couple (σ, A^σ) where $\sigma = (A_i)_{i \in I}$ is a measurable partition of A, and $A^\sigma = \{a_i \colon i \in I\}$ is a finite set subordinated to σ, we consider $\phi(\sigma, A^\sigma)$ the mapping which maps each measurable mapping f to a simple measurable mapping $\phi(\sigma, A^\sigma)(f)$, defined by

$$\phi(\sigma, A^\sigma)(f) := \sum_{i \in I} f(a_i)\chi_{A_i},$$

where χ_{A_i} is the characteristic[8] mapping associated to A_i.

Definition 4.2. *A mapping $s : A \to D$ is called a simple mapping subordinated to f if there exists a couple (σ, A^σ) where σ is a measurable partition of A, and A^σ is a finite set subordinated to σ, such that $s = \phi(\sigma, A^\sigma)(f)$.*

Given a couple (σ, A^σ) where $\sigma = (A_i)_{i\in I}$ is a measurable partition of A, and $A^\sigma = \{a_i \colon i \in I\}$ is a finite set subordinated to σ, we consider $\psi(\sigma, A^\sigma)$, the mapping which maps each measurable correspondence $F : A \twoheadrightarrow D$ to a simple measurable correspondence $\psi(\sigma, A^\sigma)(F)$, defined by

$$\psi(\sigma, A^\sigma)(F) := \sum_{i \in I} F(a_i)\chi_{A_i}.$$

Definition 4.3. *A correspondence $S : A \to D$ is called a simple correspondence subordinated to a correspondence F if there exists a couple (σ, A^σ) where σ is a measurable partition of A, and A^σ is a finite set subordinated to σ, such that $S = \psi(\sigma, A^\sigma)(F)$.*

Remark 4.4. If f is a mapping from A to D, let $\{f\}$ be the correspondence from A into D, defined for every $a \in A$ by $\{f\}(a) := \{f(a)\}$. We check that

$$\psi(\sigma, A^\sigma)(F) = \{\phi(\sigma, A^\sigma)(f)\}.$$

The space of all non-empty subsets of D is denoted by $\mathcal{P}^*(D)$. We let τ_{W_d} be the Wisjman topology on $\mathcal{P}^*(D)$, i.e., the weak topology on $\mathcal{P}^*(D)$ generated by the family of distance functions $(d(x, .))_{x\in D}$.

4.2 Approximation of measurable correspondences

Hereafter we assert that for a countable set of graph measurable correspondences, there exists a sequence of measurable partitions *approximating* each correspondence. The proof of the following theorem is given in Martins-da-Rocha [26].

Theorem 4.5. *Let $\mathcal{F}$ be a countable set of graph measurable correspondences with non-empty values from A into D and let $\mathcal{G}$ be a finite set of integrable functions from A into $\mathbb{R}$. There exists a sequence $(\sigma^n)_n$ of finer and finer measurable partitions $\sigma^n = (A_i^n)_{i\in I^n}$ of A, satisfying the following properties.*

[8] That is, for every $a \in A$, $\chi_{A_i}(a) = 1$ if $a \in A_i$ and $\chi_{A_i}(a) = 0$ elsewhere.

(a) Let $(A^n)_n$ be a sequence of finite sets A^n subordinated to the measurable partition σ^n and let $F \in \mathcal{F}$. For every $n \in \mathbb{N}$, we define the simple correspondence $F^n := \psi(\sigma^n, A^n)(F)$ subordinated to F. Then for each $a \in A$, $F(a)$ is the Wijsman limit of the sequence $(F^n(a))_n$, i.e.,

$$\forall a \in A, \quad \forall x \in A, \quad \lim_{n\to\infty} d(x, F^n(a)) = d(x, F(a)).$$

(b) There exists a sequence $(A^n)_n$ of finite sets A^n subordinated to the measurable partition σ^n, such that for each n, if we let $f^n := \phi(\sigma^n, A^n)(f)$ be the simple function subordinated to each $f \in \mathcal{G}$, then

$$\forall f \in \mathcal{G}, \quad \forall a \in A, \quad |f^n(a)| \leqslant 1 + \sum_{g\in\mathcal{G}} |g(a)|.$$

In particular, for each $f \in \mathcal{G}$,

$$\lim_{n\to\infty} \int_A |f^n(a) - f(a)| d\mu(a) = 0.$$

Remark 4.6. The property (a) implies in particular that if $(x^n)_n$ is a sequence of D, d-converging to $x \in D$, then

$$\forall a \in A, \quad \lim_{n\to\infty} d(x^n, F^n(a)) = d(x, F(a)).$$

It follows that if F is non-empty closed valued, then property (a) implies that

$$\forall a \in A, \quad \mathrm{ls}\, F^n(a) \subset F(a).$$

5 Proof of Theorem 3.14

Let $\mathcal{E}$ be an economy satisfying Assumptions C, M, P, S', MON, E, B and UP, where Assumption S' defined by

$$\text{for a.e. } a \in A, \quad e(a) \in X(a) - \sum_{j\in J} \theta_j(a)\overline{\mathrm{co}}\, Y_j,$$

replaces the stronger survival Assumption S. Without any loss of generality[9] we can suppose that for every $a \in A$, $\underline{x}(a) = 0$ and for every $j \in J$, $0 \in Y_j$.

Following Podczeck [33] and Holmes [22], the w^* and bw^* topologies coincide on $D := M(T)_+$. Moreover this topology is separable and completely metrizable. We let d be a metric on D satisfying these properties. Applying Proposition B.2, there exists a sequence $(f_k)_k$ of measurable selections of X such that for every $a \in A, X(a) := d\text{-cl}\,\{f_k(a) \colon k \in \mathbb{N}\}$. For every k, we let R_k be the correspondence

[9] Following Assumption S, for each $j \in J$ there exists $\tilde{y}_j \in Y_j$. Consider now the economy $\tilde{\mathcal{E}}$ where for each $a \in A$, $\tilde{X}(a) = X(a) - \{\underline{x}(a)\}$, for each $j \in J$, $\tilde{Y}_j = Y_j - \{\tilde{y}_j\}$ and $\tilde{e}(a) = e(a) - \underline{x}(a) + \sum_{j\in J} \theta_j(a)\tilde{y}_j$.

from A into $M(T)$, defined by $R_k(a) = \{x \in X(a) \colon f_k(a) \notin P_a(x)\}$. Then for almost every agent $a \in A$, for every $x \in X(a)$,

$$d(x, R_k(a)) > 0 \Leftrightarrow f_k(a) \in P_a(x).$$

If f is a mapping from A to D, then we let $\{f(.)\}$ be the correspondence from A into D defined for every $a \in A$, by $\{f(.)\}(a) := \{f(a)\}$. Note that if f is measurable then f is Gelfand integrable if and only if $\|f(.)\| : a \mapsto \|f(a)\|$ from A to $\mathbb{R}_+$ is integrable.

Let $\mathcal{G} := \{\|e(.)\|, \theta_j \colon j \in J\}$ and $\mathcal{F} := \{\{e(.)\}, \{f_k(.)\}, R_k \colon k \in \mathbb{N}\}$. Applying Theorem 4.5, there exists a sequence $(\sigma^n)_n$ of measurable partitions $\sigma^n = (A_i^n)_{i \in S^n}$ of $(A, \mathcal{A})$, and a sequence $(A^n)_n$ of finite sets $A^n = \{a_i^n \colon i \in S^n\}$ subordinated to the measurable partition σ^n, satisfying the following properties.[10]

Fact 5.1 *For every $a \in A$,*

(i) for every $j \in J$ and for every $k \in \mathbb{N}$,

$$\lim_{n\to\infty} e^n(a) = e(a), \quad \lim_{n\to\infty} \theta_j^n(a) = \theta_j(a) \qquad \lim_{n\to\infty} f_k^n(a) = f_k(a);$$

(ii) for every sequence $(x^n)_n$ of D, d-converging to $x \in D$,

$$\lim_{n\to\infty} d(x^n, X^n(a)) = d(x, X(a))$$

and for every k,

$$\lim_{n\to\infty} d(x^n, R_k^n(a)) = d(x, R_k(a));$$

(iii) if we pose $g(a) := \sum_{j\in J} \theta_j(a) + \|e(a)\|$ then g is an integrable function satisfying

$$\forall n \in \mathbb{N}, \quad \max\{\theta_j^n(a), \|e^n(a)\| : j \in J\} \leqslant 1 + g(a)$$

and if we pose for each $n \in \mathbb{N}$, $\omega^n := \int_A e^n$ and $\vartheta_j^n := \int_A \theta_j^n$, then

$$\lim_{n\to\infty} \omega^n = \omega \qquad \forall j \in J, \quad \lim_{n\to\infty} \vartheta_j^n = 1.$$

5.1 Approximating sequence of economies

We propose to construct a sequence $(\mathcal{E}^n)_n$ of economies with finitely many consumers and differentiated commodities, *converging* to $\mathcal{E}$. For each n, we let $\vartheta^n := \max\{\vartheta_j^n \colon j \in J\}$. Applying Fact 5.1, $\lim_{n\to\infty} \vartheta^n = 1$, thus, without any loss of generality, we can suppose that for all n, $1/2 \leqslant \vartheta^n \leqslant 2$. For each n, we denote by $\mathcal{E}^n$ the following economy with finitely many consumers and differentiated commodities:

$$\mathcal{E}^n = \left(\langle C(T), M(T) \rangle, (X_i^n, P_i^n, e_i^n)_{i \in I^n \cup \{\infty\}}, \left(Y_j^n, \theta_j^n\right)_{j\in J} \right),$$

[10] Following notations of Section 4, if f is mapping from A to D, then for each n, $\{f(.)\}^n = \{f^n(.)\}$.

where $I^n := \{i \in S^n \colon \mu(A_i^n) \neq 0\}$. For every $j \in J$, the production set is defined by $Y_j^n := \vartheta^n Y_j$ and the shares are defined by

$$\forall i \in I^n, \quad \theta_{ij}^n := \frac{1}{\vartheta^n}\mu\left(A_i^n\right)\theta_j\left(a_i^n\right) \qquad \theta_{\infty j}^n := \frac{\vartheta^n - \vartheta_j^n}{\vartheta^n}.$$

The characteristics of each consumer $i \in I^n$ are defined by

$$X_i^n = \mu\left(A_i^n\right)X\left(a_i^n\right),\ e_i^n = \mu\left(A_i^n\right)e\left(a_i^n\right) \text{ and } P_i^n = \mu\left(A_i^n\right)P\left(a_i^n\right).$$

The characteristics of the consumer ∞ are defined by $X_\infty^n := D$, $e_\infty^n := 0$ and $P_\infty^n := \{(x,y) \in D^2 \colon y - x \in \Gamma\}$. We assert that we can apply to each economy $\mathcal{E}^n$, a quasi-equilibrium existence result (Lemma A.1 in Appendix) for economies with finitely many agents.

Claim 5.1. For every n, $\mathcal{E}^n$ satisfies the assumptions of Lemma A.1.

Proof. Indeed, the only assumption whose verification is not trivial is the boundedness of the set $\mathcal{A}_X(\mathcal{E}^n)$ of realizable consumption allocations. We recall that:

$$\mathcal{A}_X(\mathcal{E}^n) = \left\{x \in \prod_{i \in I^n \cup \{\infty\}} X_i^n \colon \sum_{i \in I^n} x_i + x_\infty - \omega^n \in \vartheta^n Y_\Sigma\right\}.$$

It follows that

$$x \in \mathcal{A}_X(\mathcal{E}^n) \Longrightarrow \sum_{i \in I^n} x_i + x_\infty \in D \cap Z\,, \quad \text{where} \quad Z := \bigcup_n \left(\{\omega^n\} + \vartheta^n Y_\Sigma\right).$$

Since $0 \in Y_\Sigma$ and Y_Σ is convex, $\bigcup_n \vartheta^n Y_\Sigma \subset 2Y_\Sigma$ and $A_{w^*}(Z) \subset A_{w^*}(Y_\Sigma)$. Now since $0 \in Y_\Sigma$ and Y_Σ is convex and w^*-closed, $A_{w^*}(Y_\Sigma) \subset Y_\Sigma$. Therefore (since $A_{w^*}(Y_\Sigma)$ is a cone) Assumption B implies $A_{w^*}(Y_\Sigma) \cap M(T)_+ = \{0\}$. Applying Proposition B.1, we get that for every $x \in \mathcal{A}_X(\mathcal{E}^n)$, $\sum_{i \in I^n} x_i$ lie in a bounded set. For each $i \in I^n$, $x_i \geqslant 0$ and $\left\|\sum_{i \in I^n} x_i\right\| = \sum_{i \in I^n}\|x_i\|$. Hence $\mathcal{A}_X(\mathcal{E}^n)$ is bounded. $\square$

Let $v \in \Gamma \cap M(T)_+$ be a properness vector and let V be a bw^*-open convex and symmetric subset of $M(T)$ such that $\{v\} + V \subset \Gamma$. Applying Claim 5.1 and Lemma A.1, there exists a quasi-equilibrium

$$\left(\left(x_i^n\right)_{i \in I^n \cup \{\infty\}}, \left(z_j^n\right)_{j \in J}, p^n\right) \in \prod_{i \in I^n \cup \{\infty\}} X_i^n \times \prod_{j \in J} Y_j^n \times C(T)$$

for the economy $\mathcal{E}^n$, with $\langle p^n, v\rangle = 1$. Moreover, there exist individual prices $p_i^n \in V^\circ$, $i \in I^n \cup \{\infty\}$, such that $p^n = \sup\{p_i^n \colon i \in I^n \cup \{\infty\}\}$. Following Proposition B.4, there exists a set K compact in $(C(T), \|.\|_\infty)$ such that for all n, $p^n \in K$. For every $j \in J$, let $y_j^n := \frac{1}{\vartheta^n} z_j^n \in Y_j$. Let us then define $x^n : A \to D$, by:

$$x^n := \sum_{i \in I^n} \frac{1}{\mu(A_i^n)} x_i^n \chi_{A_i^n}.$$

We have defined a Gelfand integrable mapping $x^n : A \to D$ such that:

$$\forall a \in \bigcup_{i \in I^n} A_i^n, \quad \langle p^n, x^n(a) \rangle = \langle p^n, e(a) \rangle + \sum_{j \in J} \theta_j^n(a) \vartheta^n \left\langle p^n, y_j^n \right\rangle \tag{5.1}$$

$$\langle p^n, x_\infty^n \rangle = \sum_{j \in J} (\vartheta^n - \vartheta_j^n) \left\langle p^n, y_j^n \right\rangle \tag{5.2}$$

$$\forall a \in \bigcup_{i \in I^n} A_i^n, \quad \langle p^n, P_a^n(x^n(a)) \rangle \geqslant \langle p^n, x^n(a) \rangle \tag{5.3}$$

$$\langle p^n, P_\infty^n(x_\infty^n) \rangle \geqslant \langle p^n, x_\infty^n \rangle \tag{5.4}$$

$$\forall j \in J, \quad \left\langle p^n, y_j^n \right\rangle \geqslant \langle p^n, Y_j \rangle \tag{5.5}$$

$$\int_A x^n(a) d\mu(a) + x_\infty^n = \omega^n + \vartheta^n \sum_{j \in J} y_j^n. \tag{5.6}$$

We let A_0 be the following measurable set $A_0 := \bigcup_{n \in \mathbb{N}} A \setminus (\cup_{i \in I^n} A_i^n)$. Note that $\mu(A_0) = 0$.

5.2 Convergence of $(x^n, y^n, p^n)_{n \in \mathbb{N}}$

Since for every $n \in \mathbb{N}$, $p^n \in K$, we can suppose (extracting a subsequence if necessary) that $(p^n)_n$ is a $\|.\|_\infty$-convergent sequence to $p^* \in K \subset C(T)$. Since, for every n, $\langle p^n, v \rangle = 1$ then $\langle p^*, v \rangle = 1$. Let us remark that following (5.3) and Assumption MON, we have for every n, $p^n \geqslant 0$, and thus $p^* \geqslant 0$. We let $G := \{-\omega^n / \vartheta^n \colon n \in \mathbb{N}\}$ and $u^n := \sum_{j \in J} y_j^n$. Following (5.6), we have for every n,

$$u^n \in (G + M(T)_+) \cap Y_\Sigma.$$

Since G is bounded, $A_{w^*}(G + M(T)_+) = M(T)_+$. Applying Proposition B.1 and Assumption B, we can conclude that the sequence $(u^n)_n$ is $\|.\|$-bounded. We can suppose (extracting a subsequence if necessary) that $(u^n)_n$ is a sequence w^*-converging to $u^* \in Y_\Sigma$. It follows that there exists $y^* \in S^1(Y)$ such that $u^* = \sum_{j \in J} y_j^*$.

Claim 5.2. It can be assumed that for every $j \in J$,

$$\lim_{n \to \infty} \left\langle p^n, y_j^n \right\rangle = \left\langle p^*, y_j^* \right\rangle \quad \text{and} \quad \left\langle p^*, y_j^* \right\rangle = \sup \langle p^*, Y_j \rangle .$$

Proof. The sequence $(p^n)_n$ is $\|.\|_\infty$-convergent to p^* and the sequence $(u^n)_n$ is w^*-convergent to u^*, it follows that

$$\lim_{n\to\infty} \langle p^n, u^n \rangle = \langle p^*, u^* \rangle .$$

Since $(\langle p^n, u^n\rangle)_n$ converges, the sequence $(\sum_{j\in J} \langle p^n, y_j^n\rangle)_n$ is bounded. For every $j \in J$, $0 \in Y_j$, hence for every n, $\langle p^n, y_j^n\rangle \geqslant 0$. It follows that for each $j \in J$, the sequence $\left(\langle p^n, y_j^n\rangle\right)_n$ is bounded. Then passing to a subsequence if necessary, we can suppose that for each $j \in J$, the sequence $\left(\langle p^n, y_j^n\rangle\right)_n$ converges to some $\alpha_j \geqslant 0$. We easily check that:

$$\sum_{j\in J} \alpha_j = \sum_{j\in J} \langle p^*, y_j^* \rangle .$$

Following (5.5), we have that for every n, $\langle p^n, u^n\rangle = \sup \langle p^n, Y_\Sigma\rangle$. Passing to the limit, we get that $\langle p^*, u^*\rangle = \sup \langle p^*, Y_\Sigma\rangle$. It is now routine to prove that:

$$\forall j \in J, \quad \langle p^*, y_j^* \rangle = \sup \langle p^*, Y_j \rangle .$$

Moreover, since for every n, for each $j \in J$, $\langle p^n, y_j^n\rangle = \sup \langle p^n, Y_j\rangle$, we easily check that for each $j \in J$, $\alpha_j \geqslant \langle p^*, Y_j\rangle$. It follows that for each $j \in J$, $\alpha_j = \langle p^*, y_j^*\rangle$. □

Following Claim 5.2 , the production plan $y^* \in S^1(Y)$ satisfies the condition (b) of the definition of a quasi-equilibrium for the economy $\mathcal{E}$.

Claim 5.3. $p^* \gg 0$.

Proof. We already proved that $p^* \geqslant 0$. Suppose that there exists $t \in T$ such that $p^*(t) = 0$. We let $\overline{x} \in S^1(X)$ be such that $\overline{v} = \int_A \overline{x} d\mu$ and we let $\overline{y} \in S^1(Y)$ be such that $\overline{v} = \sum_{j\in J} \overline{y}_j$, where $\overline{u}$ and $\overline{v}$ are defined by Assumption E. We let $B \in \mathcal{A}$ be the following set:

$$B := \left\{ a \in A\colon \left\langle p^*, e(a) + \sum_{j\in J} \theta_j(a)\overline{y_j} - \overline{x}(a) \right\rangle > 0 \right\} .$$

Assumption E implies that $\langle p^*, \omega + \overline{u} - \overline{v}\rangle > 0$, hence $\mu(B) > 0$.

Claim 5.4. For a.e. $a \in B$, $\lim_{n\to\infty} \|x^n(a)\| = +\infty$.

Proof. Let $B' \subset B$ be a measurable subset of B, with $\mu(B \setminus B') = 0$, such that all *almost everywhere* assumptions and properties are satisfied for every $a \in B'$ and such that $B' \subset A \setminus A_0$. Let $a \in B'$. Suppose that there exists a subsequence, still denoted by $(x^n(a))_n$, w^*-converging to $m \in M(T)$. For every n, $x^n(a) \in X^n(a)$. It follows that for every n, $d(x^n(a), X^n(a)) = 0$. Now applying[11] Fact 5.1 and

[11] We recall that in D the w^*-topology and the bw^*-topology coincide with the metric d.

using the fact that $(x^n(a))_n$ converges to m, we get that $d(m, X(a)) = 0$. Since $X(a)$ is closed, it means that $m \in X(a)$. We will now prove that:

$$\forall z \in P_a(m), \quad \langle p^*, z \rangle \geqslant \langle p^*, m \rangle .$$

Let $z \in P_a(m)$. We have that $X(a) = d\text{-cl}\,\{f_k(a) \colon k \in \mathbb{N}\}$, thus there exists a subsequence still denoted by $(f_k(a))_k$, converging to z. But $P_a(m)$ is d-open in $X(a)$, thus there exists $k_0 \in \mathbb{N}$, such that for every $k \geqslant k_0$, $f_k(a) \in P_a(m)$. To prove that $\langle p^*, z\rangle \geqslant \langle p^*, m\rangle$, it is sufficient to prove that for every k large enough, $\langle p^*, f_k(a)\rangle \geqslant \langle p^*, m\rangle$. Now, let $k \geqslant k_0$. Since $(x^n(a))_n$ is d-convergent to m, applying Fact 5.1,

$$\lim_{n\to\infty} d(x^n(a), R_k^n(a)) = d(m, R_k(a)).$$

Since $f_k(a) \in P_a(m)$, $d(m, R_k(a)) > 0$. It follows that for every n large enough, $d(x^n(a), R_k^n(a)) > 0$. Since $x^n(a) \in X^n(a)$, it follows that for every n large enough, $f_k^n(a) \in P_a^n(x^n(a))$. Applying (5.3), we obtain that for every n large enough, $\langle p^n, f_k^n(a)\rangle \geqslant \langle p^n, x^n(a)\rangle$. Applying Fact 5.1, we get that $\langle p^*, f_k(a)\rangle \geqslant \langle p^*, m\rangle$.

Now we prove that for every $z \in P_a(m)$, $\langle p^*, z\rangle > \langle p^*, m\rangle$. Since $a \in B'$, we have that $\left\langle p^*, e(a) + \sum_{j\in J} \theta_j(a)\overline{y_j} - \overline{x}(a)\right\rangle > 0$ and thus

$$\inf \langle p^*, X(a)\rangle < \left\langle p^*, e(a) + \sum_{j\in J} \theta_j(a)\overline{y_j}\right\rangle \leqslant \left\langle p^*, e(a) + \sum_{j\in J} \theta_j(a) y_j^*\right\rangle .$$

Passing to the limit in (5.1), $\inf \langle p^*, X(a)\rangle < \langle p^*, m\rangle$ and the rest of the proof is routine.

Following Assumption MON, there exists $\alpha > 0$ such that $m + \alpha\delta_t \in P_a(m)$ thus, following the previous result, we have that $\langle p^*, m + \alpha\delta_t\rangle > \langle p^*, m\rangle$, i.e., $p^*(t) > 0$. Contradiction. It follows that the sequence $(x^n(a))_n$ has no w^*-convergent subsequence. Hence $\lim_{n\to\infty} \|x^n(a)\| = +\infty$. □

From (5.6), $\int_A x^n(a)d\mu(a) + x_\infty^n = \omega^n + \vartheta^n u^n$. But for almost every $a \in A$, for every n, $x^n(a) \geqslant 0$. It follows that $\|x^n(a)\| = \langle 1_K, x^n(a)\rangle$ and

$$\int_A \|x^n(a)\|\, d\mu(a) + \|x_\infty^n\| = \left\| \int_A x^n(a)d\mu(a) + x_\infty^n \right\| = \|\omega^n + \vartheta^n u^n\| .$$

Since $\lim_{n\to\infty} \omega^n + \vartheta^n u^n = \omega + u^*$, applying Fatou's lemma, we get a contradiction. □

The sequence $(p^n)_n$ is $\|.\|_\infty$-convergent to p^*, it follows that there exists $\eta > 0$, such that for every n large enough, $p^n \geqslant \eta 1_K$.

Claim 5.5. The sequence $(x^n)_n$ is integrably bounded and $(x_\infty^n)_n$ is w^*-convergent to 0.

Proof. We will first prove that $\lim_{n\to\infty} x^n_\infty = 0$. For every n, p^n lie in a $\|.\|_\infty$-compact set K. Without any loss of generality we can suppose that for every n, $\|p^n\| \leqslant 1$ and $p^n \geqslant \eta 1_K$. From (5.2), for every n,

$$\eta \|x^n_\infty\| \leqslant \sum_{j\in J}(\vartheta^n - \vartheta^n_j)| \langle p^n, y^n_j\rangle |.$$

Since for each $j \in J$, $\lim\limits_{n\to\infty} \langle p^n, y^n_i\rangle = \langle p^*, y^*_j\rangle$, it follows, using Fact 5.1, that $\lim\limits_{n\to\infty} \|x^n_\infty\| = 0$.

Now we prove that the sequence $(x^n)_n$ is integrably bounded. Let $A' \in \mathcal{A}$ be a measurable subset of $A \setminus A_0$ with $\mu(A \setminus A') = 0$ and such that all *almost everywhere* assumptions and properties are satisfied for every $a \in A'$. Let $a \in A'$, from (5.1), for every n,

$$\langle p^n, x^n(a)\rangle = \langle p^n, e^n(a)\rangle + \sum_{j\in J} \theta^n_j(a) \langle p^n, y^n_j\rangle .$$

Since for every $j \in J$, $\lim_{n\to\infty} \langle p^n, y^n_j\rangle = \langle p^*, y^*_j\rangle$, there exists $M > 0$ such that

$$\eta \|x^n(a)\| \leqslant \|e^n(a)\| + M \sum_{j\in J} \theta^n_j(a).$$

Following Fact (5.1), for every n,

$$\|x^n(a)\| \leqslant \frac{(1 + MJ)(1 + g(a))}{\eta}.$$

□

Applying Theorem B.3 and passing to a subsequence if necessary, there exists a Gelfand integrable mapping $x^* : A \to M(T)$, such that

$$\int_A x^*(a)d\mu(a) = w^*\text{-}\lim_{n\to\infty} \int_A x^n(a)d\mu(a),$$

and such that for a.e. $a \in A^{na}$, $x^*(a) \in w^*\text{-}\overline{\text{co}}\,[w^*\text{-ls}\{x^n(a)\}]$ and for every $a \in A^{pa}$, $x^*(a) \in w^*\text{-ls}\{x^n(a)\}$ where A^{na} is the non-atomic part of $(A, \mathcal{A}, \mu)$ and A^{pa} is the purely atomic part of $(A, \mathcal{A}, \mu)$.

5.3 The element (x^*, y^*, p^*) is a quasi-equilibrium of $\mathcal{E}$

The condition (b) of the definition of a quasi-equilibrium has already been proved in Claim 5.2 . Since $\lim_{n\to\infty} \int_A x^n(a)d\mu(a) = \omega + \sum_{j\in J} y^*_j$, to get the condition (c) of the definition of a quasi-equilibrium for the economy $\mathcal{E}$, it is sufficient to prove that $x^* \in S^1(X)$. We recall that

$$A_0 = \bigcup_{n\in\mathbb{N}} A \setminus (\cup_{i\in I^n} A^n_i) .$$

Let A' be a subset of $A \setminus A_0$ with $\mu(A \setminus A') = 0$ and such that all *almost everywhere* assumptions and properties are satisfied for every $a \in A'$. We propose to prove that for every $a \in A'$, $x^*(a) \in X(a)$. Let $a \in A'$, by construction, we have that for every n, $x^n(a) \in X^n(a)$, and thus, for every n, $d(x^n(a), X^n(a)) = 0$. Let $m \in d\text{-ls}\,\{x^n(a)\}$, applying Fact 5.1, $d(m, X(a)) = 0$. Since $X(a)$ is d-closed, it means that $m \in X(a)$. Thus $d\text{-ls}\,\{x^n(a)\} \subset X(a)$, and under Assumption C, it follows that $x^*(a) \in X(a)$.

Now we prove that (x^*, y^*, p^*) satisfies the condition (a') of the definition of a quasi-equilibrium of $\mathcal{E}$. Let $a \in A'$. First, with (5.1), Claim 5.2 and Fact 5.1, we easily check that

$$\langle p^*, x^*(a)\rangle = \langle p^*, e(a)\rangle + \sum_{j \in J} \theta_j(a) \left\langle p^*, y_j^* \right\rangle .$$

Second, we will prove that

$$\forall x' \in P_a(x^*(a)), \quad \langle p^*, x'\rangle \geqslant \langle p^*, x^*(a)\rangle .$$

Let $x' \in P_a(x^*(a))$. Since $X(a) = d\text{-cl}\,\{f_k(a)\colon k \in \mathbb{N}\}$, we can suppose (extracting a subsequence if necessary) that $(f_k(a))_k$ is d-convergent to x'. But $P_a(x^*(a))$ is d-open in $X(a)$, thus there exists $k_0 \in \mathbb{N}$, such that for every $k \geqslant k_0$, $f_k(a) \in P_a(x^*(a))$. To prove that $\langle p^*, x'\rangle \geqslant \langle p^*, x^*(a)\rangle$, it is sufficient to prove that for every k large enough, $\langle p^*, f_k(a)\rangle \geqslant \langle p^*, x^*(a)\rangle$. Now, let $k \geqslant k_0$.

Claim 5.6. There exist an increasing mapping $\varphi : \mathbb{N} \to \mathbb{N}$ and such that for every n,

$$f_k^{\varphi(n)}(a) \in P_a^{\varphi(n)} \left(x^{\varphi(n)}(a) \right) .$$

Proof. Suppose that for every increasing mapping $\varphi : \mathbb{N} \to \mathbb{N}$, there exists an increasing mapping $\phi : \mathbb{N} \to \mathbb{N}$, such that for every n,

$$d\left(x^{\varphi \circ \phi(n)}(a), R_k^{\varphi \circ \phi(n)}(a) \right) = 0.$$

Applying Fact 5.1, it follows that for every $\ell \in w^*\text{-ls}\,\{x^n(a)\colon n \in \mathbb{N}\}$, we have $d(\ell, R_k(a)) = 0$. Then following Assumption C,

$$d\text{-}\overline{\text{co}}\,[d\text{-ls}\,\{x^n(a)\colon n \in \mathbb{N}\}] \subset R_k(a),$$

if a belongs to the non-atomic part of $(A, \mathcal{A}, \mu)$, and

$$d\text{-ls}\,\{x^n(a)\colon n \in \mathbb{N}\} \subset R_k(a)$$

elsewhere. It follows that $x^*(a) \in R_k(a)$, i.e., $f_k(a) \notin P_a(x^*(a))$. Contradiction. □

With Claim 6 , (5.1) and (5.3), for every n,

$$\left\langle p^{\varphi(n)}, f_k^{\varphi(n)}(a) \right\rangle \geqslant \left\langle p^{\varphi(n)}, e^{\varphi(n)}(a) \right\rangle + \sum_{j \in J} \theta_j^{\varphi(n)}(a) \left\langle p^{\varphi(n)}, y_j^{\varphi(n)} \right\rangle .$$

Passing to the limit, we get that

$$\langle p^*, f_k(a)\rangle \geqslant \langle p^*, e(a)\rangle + \sum_{j \in J} \theta_j(a) \left\langle p^*, y_j^* \right\rangle = \langle p^*, x^*(a)\rangle .$$

A Finitely many agents

We provide in this section, an equilibrium existence result for economies with finitely many consumers. The following Lemma A.1 is mostly inspired from the existence result in Zame [48]. We say that an economy $\mathcal{E}$ has finitely many agents if A is a finite set, $\mathcal{A} = 2^A$ is the set of all subsets of A and μ is the counting measure on A.

Assumption ($\mathbf{B}_f$) *The set of realizable consumption plans $\mathcal{A}_X(\mathcal{E})$ is compact in $\prod_{a \in A} X_a$ for the product w^*-topology, where*

$$\mathcal{A}_X(\mathcal{E}) := \left\{ x \in \prod_{a \in A} X_a \colon \sum_{a \in A} x_a \in \{\omega\} + Y_\Sigma \right\}.$$

Lemma A.1. *Let $\mathcal{E}$ be an economy with finitely many consumers satisfying Assumptions* **C**, **M**, **P**, **S**, $\mathbf{B}_f$ *and* **UP**. *Let $v \in \Gamma \cap M(T)_+$ be a properness vector and let V be a bw^*-open convex and symmetric subset of $M(T)$ such that $\{v\} + V \subset \Gamma$. Then there exists a quasi-equilibrium (x^*, y^*, p^*), with $\langle p^*, v\rangle = 1$. Moreover there exist individual prices $p^a \in V^\circ$, $a \in A$, such that $p^* = \sup\{p^a \colon a \in A\}$.*

Remark A.2. The arguments of Theorem 1 in Zame [48] can be adjusted to the set of assumptions of Lemma A.1. However, the content of Lemma A.1 carries over to more abstract settings, and some assumptions may be weakened. For precisions, we refer to Martins-da-Rocha [27].

B Mathematical auxiliary results

B.1 Asymptotic cones

Following Section 2, we recall that if X is a subset of $M(T)$, then we let $A_{w^*}(X)$ be the set of elements $x \in L$ such that $x = w^*\text{-}\lim_{n\to\infty} \lambda_n x_n$ where $(\lambda_n)_n$ is a real sequence decreasing to 0 and $(x_n)_n$ is a sequence in X.

Proposition B.1. *Let X, Y two subsets of $M(T)$ and G be a bounded subset of $M(T)$. Suppose that $X \subset G + M(T)_+$ and that $A_{w^*}(X) \cap A_{w^*}(Y) = \{0\}$, then $X \cap Y$ is $\|.\|$-bounded.*

Proof. Suppose in the contrary, that $X \cap Y$ is not $\|.\|$-bounded. We can thus extract a sequence $(x^n)_n$ in $X \cap Y$, such that for every n, $\|x^n\| \geqslant n$. Let, for every n, $v^n := x^n / \|x^n\|$. By the Banach-Alaoglu Theorem, we can suppose, without any loss of generality, that the sequence $(v^n)_n$ is w^*-convergent to $v \in M(T)$. Since for every n, there exists $g^n \in G$ such that $v^n - g^n / \|x^n\| \geqslant 0$, then

$$\langle 1_K, v^n - g^n / \|x^n\|\rangle = \|v^n - g^n / \|x^n\|\| \geqslant |\|v^n\| - \|g^n\| / \|x^n\||.$$

Passing to the limit, we get that $\langle 1_K, v\rangle \geqslant 1$ and then $v \neq 0$. But $v \in A_{w^*}(X) \cap A_{w^*}(Y)$. Contradiction.

B.2 Measurability and integration

We consider $(A, \mathcal{A}, \mu)$ a finite complete measure space and (D, d) a complete separable metric space. Following Aumann [8], graph measurable correspondences have measurable selections.

Proposition B.2. *Consider F a graph measurable correspondence from A into D with non-empty values. Then there exists a sequence $(z_n)_n$ of measurable selections of F, such that for every $a \in A$, $(z_n(a))_n$ is dense in $F(a)$.*

We provide hereafter a classical version of Fatou's Lemma for Gelfand integrable mappings.

Theorem B.3. *Let $(f^n)_n$ a sequence of Gelfand integrable mappings from A into $M(T)$. If $(f^n)_n$ is integrably bounded, then there exists an increasing mapping $\phi : \mathbb{N} \to \mathbb{N}$ and a Gelfand integrable mapping f^* from A to $M(T)$, such that*

$$w^*\text{-}\lim_{n\to\infty} \int_A f^{\phi(n)}(a)d\mu(a) \ = \int_A f^*(a)d\mu(a),$$

$$\textit{for a.e. } a \in A^{na}, \quad f^*(a) \in w^*\text{-}\overline{co}\left[w^*\text{-}ls\left\{f^{\phi(n)}(a)\right\}\right]$$

and

$$\textit{for every } a \in A^{pa}, \quad f^*(a) \in w^*\text{-}ls\left\{f^{\phi(n)}(a)\right\},$$

where A^{na} is the non-atomic part of $(A, \mathcal{A}, \mu)$ and A^{pa} is the purely atomic part of $(A, \mathcal{A}, \mu)$.

Proof. Let, for each n, $v^n := \int_A f^n$. Since the sequence $(f^n)_n$ is integrably bounded, the sequence $(v^n)_n$ is bounded and there exists a subsequence w^*-converging to some $v^* \in M(T)$. Applying Lemma 6.6 in Podczeck [34] and following the proof of Corollary 4.4 in Balder and Hess [10], the result follows.

For more precisions about measurability and integration of correspondences, we refer to papers [44] and [45] of Yannelis.

B.3 Compactness and lattice operations

Proposition B.4. *Let $V \subset M(T)$ be a bw^*-neighborhood V of zero. The following set $K(V) \subset C(T)$ is relatively $\|.\|_\infty$-compact,*

$$K(V) = \left\{ \bigvee_{i=1}^{n} p_i : n \geqslant 1 \quad \textit{and} \quad \forall i \in \{1, \cdots, n\}, \quad p_i \in V^\circ \right\}.$$

Proof. Following Holmes [22], without any loss of generality, we can assume that there exists B a $\|.\|_\infty$-compact convex and circled[12] subset of $C(T)$ such that $V^\circ \subset B$. Since the set B is equicontinuous and norm-bounded, it follows that $K(V)$ is also equicontinuous and norm-bounded. The end of the proof follows from Ascoli's Theorem.

[12] A set A in a vector space X is circled if for each $x \in A$ the line segment joining x and $-x$ lies in A.

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