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SYMPOSIUM

Financial markets with endogenous transaction costs

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Abstract The paper proposes an alternative general equilibrium formulation of financial asset economies with transaction costs. Transaction costs emerge endogenously at equilibrium and reflect agents' decisions of intermediating financial activities at the expense of providing labor services.

Keywords Competitive equilibrium · Incomplete markets · Endogenous transaction costs

JEL Classification D23 · D41 · D52

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1 Introduction

Transaction costs such as, brokerage commissions, market impact costs and transaction taxes are an inherent feature of modern financial markets.[1] Researchers have questioned the impact of transaction costs on financial markets focusing particularly on the way these costs affect the trading behavior of market participants as well as on the role they have in determining equilibrium prices and trading volume. Roughly speaking, theoretical models that study the implications of transaction costs in financial markets have their roots in two broad branches of economic literature: dynamic asset pricing theory and general equilibrium theory.[2]

This paper follows the second branch of literature and proposes a general equilibrium formulation of financial asset economies with transaction costs. We believe that a convincing model with transaction costs should provide an economic rationale for the emergence of these costs. To address this issue we provide a model in which transaction costs emerge endogenously at equilibrium, reflecting agents' decisions of intermediating financial activities at the expense of providing labor services.

1.1 Relation to the literature

Transaction costs were originally introduced in the standard general equilibrium model of Arrow and Debreu (1954) in order to explain the sequential opening of commodity markets.[3] The traditional approach to transaction costs assumes that real resources are used in the process of transaction.[4] Early studies visualize market as a profit maximizing transactor who uses real resources to transform sold goods into bought goods according to an exogenously specified technology. Hahn (1971, 1973) refer to such a technology as the transaction technology for the economy. To pay for those resources the transactor will benefit from the difference in the selling and buying price of goods. In equilibrium this difference reflects the transaction cost.

Kurz (1974a,b) push the theory further by allowing traders to have their own exchange technology. In this environment consumers are assumed to satisfy an additional (to their budget constraint) technological constraint. Each agent is characterized

[1] The market impact cost occurs because the transaction itself may change the market price of the asset. The difference between the transaction price and what the market price would have been in the absence of the transaction is termed the market impact of the transaction. The market impact is a price-per-share amount. Multiplying the market impact by the number of shares traded gives the market impact cost of the transaction.

[2] Contributions on asset pricing under transaction costs include partial equilibrium models of optimal portfolio design (see Magill and Constantinides 1976; Constantinides 1986; Duffie and Sun 1990) as well as general equilibrium models of endogenous price formation (see Vayanos 1998; Lo et al. 2004). Transaction costs have also been proposed (see Heaton and Lucas 1986; Aiyagari and Gertler 1991; Aiyagari 1993) as an explanation to various asset pricing puzzles that emerge in empirical literature (e.g., the equity premium puzzle, Mehra and Prescott 1985).

[3] In the classical multi-period Arrow-Debreu economy, agents' trading opportunities are not affected by opening markets at futures dates.

[4] For instance, search costs, transportation costs, storage costs and costs of measurement may involve losses of real inputs and labor.

by a technology that describes all the transactions he is able to undertake given that he sacrifices some resources. The transaction cost (which is agent specific) involves the value of total resources that are used to undertake the transactions in each good market. Further developments encompass models with uncertainty. Shefrin (1981) shows that under uncertainty transaction costs can account for the possibility of inactive future markets. Repullo (1987) shows that the set of equilibria in a Radner (1972) economy with incomplete markets coincides with the set of equilibria of a Kurz (1974a) economy where agents have a special kind of transaction technology.[5]

In the aforementioned models, transaction costs involve real resources that are burnt in the process of exchanging real commodities. Subsequent models studied environments where transaction costs, in the form of real resources, are imposed on trading financial contracts. Laitenberger (1996) proposes a two-period model with real financial assets and transaction costs that are incurred at the second period. The level of transaction costs is exogenously specified in this model, involving a reduced real claim from the viewpoint of a buyer and an increased real obligation from the viewpoint of a seller. Arrow and Hahn (1999) study a two-period financial market economy with real transactions costs incurred at the first period. Following Hahn (1971) they assume a common asset transaction technology, but they restrict further by requiring the technology to be of a fixed coefficient. Financial markets are complete and asset returns are dominated in units of account. In this setting, Arrow and Hahn (1999) show that the presence of transaction costs turns out equilibria to be always constrained Pareto-inefficient.[6] Jin and Milne (1999) (see also Milne and Neave 2003) propose a model where financial activities are intermediated by some brokers/intermediaries that trade assets between buyers and sellers using a costly technology. The intermediaries act competitively taking buying and selling prices as given. They also are allowed to hold portfolios and trade on their own account. At equilibrium their revenues are eventually redistributed back to consumers.

An alternative to real transaction costs approach argues for commission fees derived from the monopolistic power of a privately owned brokerage house. The commissions are subsequently redistributed to consumers according to their equity shares. In this setting the transaction cost is interpreted as an intermediation cost since it only implies a transfer of income across individuals (no real resources are used in the process of transaction).

Préchac (1996) proposes a model in which the commission fees are assumed to be proportional to the value of trade. Such an assumption makes it difficult to deal with assets having negative and positive payoffs since in that case their prices can be close to zero.[7] Markeprand (2008) allows for arbitrary asset returns and for a general intermediation cost function that takes into account the volume of trade in addition to the value of trade.

[5] Specifically, the transaction technology assigns zero costs if at some date-event the markets are open, and infinite otherwise.

[6] Herings and Schmedders (2006) provide an algorithm for the computation of equilibria in an particular version of this model with one good (wealth).

[7] In Préchac (1996) returns are non-negative.

Both Préchac (1996) and Markeprand (2008) deal with real asset markets. In both models intermediation costs serve to bound endogenously portfolios, ruling out situations where equilibria fail to exist (see Hart 1975). Duffie and Shafer (1985, 1986) showed that Hart's example is an exception and that the existence of a Radner equilibrium is assured generically. However, subsequent contributions showed that the generic existence argument is no more valid in cases where asset dividends are not linear with respect to prices (see Ku and Polemarchakis 1990) or preferences are not strictly convex (see Busch and Govindan 2004).

Préchac (1996) and Markeprand (2008) do not provide any rationale for the level of intermediation costs. The bid-ask spread in their models is specified outside the functioning of the economy. Moreover, there is no market for equity shares and their initial distribution is exogenous. The papers of Pesendorfer (1995) and Bisin (1998) try to address these shortcomings. Pesendorfer (1995) considers a setting where perfectly competitive intermediaries choose optimally to intermediate a class of derivative securities on a set of basic securities whose payoffs are dominated in units of account. The trading of those derivatives takes place in two different markets, an institutional and a retail market, and involves different marketing costs. At equilibrium the price of those derivatives will be different in each market, giving rise to endogenous price differentials. Bisin (1998) considers an alternative model where intermediaries have a monopoly power in a fix number of security markets. Intermediaries act as a monopolist choosing optimally the securities' payoffs and the bid-ask spread they charge.[8] In this setting, strategic interaction (Bertrand competition) among firms leads to endogenous bid-ask spreads. In both Pesendorfer (1995) and Bisin (1998) intermediation involves fixed costs as well as costs that are proportional to trading volume. Equilibrium prices and asset demands are rationally anticipated by intermediaries when evaluating their profits.

Our purpose in this paper is to propose an alternative general equilibrium formulation of financial asset economies with transaction costs. We argue that it is unreasonable to neglect any form of real transaction costs in financial markets as it is the case in the models proposed by Pesendorfer (1995), Préchac (1996), Bisin (1998) and Markeprand (2008). In that respect, our formulation is related to the one proposed by Laitenberger (1996) and Arrow and Hahn (1999) but it differentiates from them in two crucial aspects.

First, it is difficult to accept that trading in financial markets involves general real resources that are associated with search, transportation, storage and measurement costs. In modern financial markets most trading is centralized and there are no issues of measuring and verifying the quantities exchanged. It is true that some physical inputs are burnt in the transaction process, but empirical evidence reports that the financial sector is more labor intensive. Indeed, comparing the labor cost figures for financial intermediation with those for the whole economy it is clear that the former are in many cases higher than the latter (see Carley 2002). In addition, the financial sector in developed economies exhibits higher wage differentials with respect to other industry-specific sectors (see Genre et al. 2005). It is therefore reasonable to think that

8 Payoffs are assumed to be nominal.

an important part of the costs generated in modern financial markets are associated with the provision of labor services required to intermediate the financial activities. Taking this perspective, we propose a model in which intermediation is remunerated by a "price", essentially the value of labor services at the margin, the wage.

Second, we propose a closed model in which transaction costs are determined endogenously and reflect agents' decisions of intermediating financial activities. In that respect our formulation is related to the one of Pesendorfer (1995), Bisin (1998) and Jin and Milne (1999). Pesendorfer (1995) provides an explanation based on differentiating the marketing cost of financial innovations in different markets. Bisin (1998) offers an explanation based on Bertrand competition among monopolists. We follow another route and explain how transaction costs emerge endogenously in a competitive market of brokers. Our choice is partly justified in the light of mixed empirical assessments on the competitiveness of financial sector in specific economies.[9] Our formulation of transaction costs has an advantage with respect to the one proposed by Jin and Milne (1999). In an environment where the outcome of production activity is uncertain and financial markets are incomplete, treating intermediaries as privately owned firms is difficult to justify since there may be disagreement among the shareholders on the objective assigned to the firm.[10]

1.2 The setting and results

There is a fixed number of real assets available for trade. Each agent is characterized by a consumption and investment set as well as by an abstract set representing his available labor or effort. Agents' actions involve three activities. The exchange of commodity goods, the purchase or sale of assets and the intermediation of financial orders.[11]

The demand for commodities is determined by the prices of goods. Investment decisions are driven by security prices and the commission fee per unit of transactions. The choice of supplying labor to intermediate transactions is determined by the price of labor (wage). The provision of labor services implies a revenue that agents can dispose in the consumption of goods but simultaneously it incurs a loss in utility due to the reduction in leisure. Consumers can intermediate in any security market and they can act simultaneously as investors and intermediaries.

[9] Shaffer (1989, 1993) present results that strongly reject collusive conduct and support perfect competition in U.S. and Canada while Shaffer (2001) reports that European banks appear to suffer from a measurable but limited degree of banking market power. Evidence from Battelino (2008) suggests that the Australian financial sector is competitive. Bandt and Davis (2000) find that the U.S. banking sector is highly competitive whereas in some European countries like Germany and France the banking system is characterized by monopolistic competition.

[10] Another drawback of Jin and Milne (1999) is the fact that assets pay in unspecified units of account (i.e., the asset structure is nominal). This is problematic due to the "spurious" real effects of changing units of account (see Magill and Quinzii 1996 for a thorough discussion).

[11] Our formulation is close to Kurz (1974a,b) in the sense that transactions are directly employed by consumers.

We assume that there are no barriers to entry on intermediating financial activities. The entry costs are low enough such that any agent can be a potential broker.[12] Therefore, instead of considering that there is a finite set of brokerage houses with monopoly power, we assume that brokers act competitively in the sense that there is a competitive market for their labor services. In this setting, the price of labor coincides with the commission fee at equilibrium. This in turn implies that the commission fee is endogenously determined, reflecting agents' productivity of intermediating transactions and their intratemporal preferences for labor and leisure.

The paper is structured as follows. In Sect. 2 we present the model, introduce notation and assumptions and define agents' objectives. In Sect. 3 we present the concept of competitive equilibrium, highlight its properties and prove its existence. Section 4 relates our work with the literature on exogenous intermediation costs and in particular with the models of Préchac (1996) and Markeprand (2008). We argue that both models are incomplete since they do not provide any rational for the level of commission fees. An attempt to make the choice of commission fees endogenous in Préchac (1996) raises serious difficulties that stem from the fact that profits depend on the equilibrium outcome which in turn depends on the level of the chosen commission fees. The problem becomes more serious in the setting proposed by Markeprand (2008). The specification of his cost function induces a budget restriction at $t = 0$ that is no longer homogeneous of degree zero with respect to prices. This may raise questions about the normalization of prices that already appear in the literature of financial markets with nominal assets. Appendix A is technical in nature and is devoted to prove an intermediate result (i.e., existence of equilibria with bounded action sets) that is used in the proof of our main existence theorem. We propose in Appendix B a list of conditions on the primitives of the economy (utility functions and production technologies) that imply the assumptions imposed in Sect. 3 on the indirect payoff functions.

2 The model

We consider a pure exchange economy that extends over two periods $t \in \{0, 1\}$. There is exogenous uncertainty about consumers' characteristics at $t = 1$ represented by a finite set S of states. Markets open sequentially. The economy consists of a finite set I of agents, indexed by $i \in I$. Each consumer i has unlimited abilities to form expectations and thus can perfectly forecast endogenous macroeconomic variables. At every period t there is a finite set L_t of commodities available for trade. Let $X_t^i \subset X_t \equiv \mathbb{R}_+^{L_t}$ denote agent i's set of commodity bundles available for consumption and $P_t \equiv \mathbb{R}_+^{L_t}$ denote the set of commodity prices at period t.

12 Modeling explicitly entry costs leads to non-convex action sets. This is because the consumers' maximization problem involves a choice variable that takes discrete values: one value represents the decision to pay the entry cost and be a broker while a second value represents the decision not to be a broker. In such a setting the salary should be such that the entry costs of those who decided to be brokers have to be compensated by the utility of the goods they can consume using the revenue from the commission fees. One could deal with non-convexities by introducing a setting with a continuum of agents. While such a formulation is interesting, tackling the existence problem becomes technically more complex. We postpone such an attempt in a future paper.

At the first period $t = 0$ consumers can trade commodities and a finite set J of financial assets. The financial structure is assumed to be exogenous. The payoff of asset j in state s, denominated in units of account, is given by $V_j(p_1(s), s)$ where $p_1(s)$ is the commodity price prevailing in that state. In order to avoid issues related to monetary policy, we restrict our attention to *real assets*.

Assumption 2.1 The payoffs of each asset j are real in the sense that for each state s, the function $V_j(\cdot, s) : P_1 \to \mathbb{R}$ is homogenous of degree 1, i.e.,

$$\forall \lambda \in \mathbb{R}_+, \quad \forall p_1(s) \in P_1, \quad V_j(\lambda p_1(s), s) = \lambda V_j(p_1(s), s).$$

Moreover, we assume that the function $V_j(\cdot, s) : P_1 \to \mathbb{R}$ is continuous.

Remark 2.1 In contrast with Préchac (1996) we do not impose that asset payoffs are non-negative. As in Markeprand (2008) we relax the assumption of linear dependent payoff-functions to allow for general assets like (real) options and futures.[13] However, we rule out nominal assets.

At $t = 0$, each agent chooses to purchase an amount $\theta_j \geq 0$ and to sell an amount $\varphi_j \geq 0$ of asset j. Let $\Theta^i \subset \Theta \equiv \mathbb{R}^J_+$ denote the space of available purchases and $\Phi^i \subset \Phi \equiv \mathbb{R}^J_+$ the space of available sales. We let $Q \equiv \mathbb{R}^J$ be the space of asset prices at $t = 0$.

Assumption 2.2 For each i, the sets Θ^i and Φ^i are closed and convex. Moreover, every agent i is allowed to sell or purchase a small amount of each asset, i.e.,

$$\exists v \in \mathbb{R}^J_{++}, \quad [0, v] \subset \Theta^i \cap \Phi^i.$$

We assume that markets for consumption are frictionless: there are no transaction costs. However, purchasing or selling assets requires the intermediation of financial brokers. Every agent i is a potential broker. He is characterized by an abstract set Y^i representing potential labor or effort and a correspondence $F^i : Y^i \to \mathbb{R}^J_+$ representing his *production technology*: if agent i chooses a level of effort y, then he can intermediate a volume z_j of financial transactions for each asset j where the vector $z = (z_j)_{j \in J}$ belongs to $F^i(y)$. We assume that agent i's ability to intermediate transactions is asset-dependent but independent of the type of transactions: buying or selling.

Each agent i has a utility/disutility function $U^i_0 : X^i_0 \times Y^i \to \mathbb{R}$ for consumption and labor at $t = 0$ and is endowed with a bundle $e^i_0 \in X^i_0$ of consumption goods. For each possible realization of exogenous uncertainty s at $t = 1$, agent i has a utility function $U^i_1(s, \cdot) : X^i_1 \to \mathbb{R}$ for consumption at $t = 1$ and is endowed with a bundle $e^i_1(s) \in X^i_1$ of consumption goods. At $t = 0$ agent i discounts future consumption with a discount factor $\beta^i > 0$ and has beliefs about exogenous uncertainty represented by a probability $\nu^i \in \text{Prob}(S)$.

[13] Consider that there is an asset j which dividend in state s is the market value of a bundle $A_j(s) \in \mathbb{R}^{L_1}_+$. Our framework allows for a call option on this asset with strike the market value of a fixed bundle $\xi \in \mathbb{R}^{L_1}_+$. The dividend of this call option in state s is given by $[p_1(s) \cdot A_j(s) - p_1(s) \cdot \xi]^+$ where $p_1(s)$ is the vector of commodity market prices in state s.

Assumption 2.3 Every agent i considers that every state of nature s is probable, i.e., $\nu^i(s) > 0$ for each $s \in S$.

Purchasing or selling assets induces a commission fee that is paid to those agents who act as intermediaries (brokers). The decision of participating in the intermediation process involves the devotion of working time. For a fixed volume of financial transactions, different agents may need a different level of labor to intermediate this volume. We assume that there are no barriers to entry in intermediating financial activities. The entry costs are low enough such that any agent can be a potential broker. At equilibrium the wage (the price of labor) received by brokers for the provision of their labor services will coincide with the commission fee paid by investors. Therefore, in our framework the commission fee is endogenously determined, reflecting agents' productivity of intermediating transactions and their intratemporal preferences for labor and leisure. We denote by $\kappa_j \in \mathbb{R}_+$ the salary paid to each broker for intermediating a unit of asset j. Given the previous discussion, κ_j is also the transaction cost paid by investors per unit of asset j's purchase or sale.

It is assumed that the transacting costs are proportional to the volume of transactions a consumer/broker can intermediate. We allow for the possibility of an agent to be simultaneously an investor (purchasing or selling an asset j) and a broker (intermediating exchanges of asset j).

When agent i chooses at $t = 0$ to consume the bundle x_0^i and to supply the labor/effort y^i, he gets a utility $U_0^i(x_0^i, y^i)$ and he is able to intermediate a vector $z^i \in \mathrm{F}^i(y^i)$ of asset transactions. We propose to represent agent i's action by the couple (x_0^i, z^i) of consumption/intermediation instead of the couple (x_0^i, y^i) of consumption/effort. We denote by Z^i the set of transactions that agent i can intermediate, i.e.,

$$Z^i \equiv F^i(Y^i) \subset Z \equiv \mathbb{R}_+^J.$$

Assumption 2.4 The production set Z^i is a closed convex subset of $\mathbb{R}_+^J$. Moreover, inaction is possible, i.e., $0 \in Z^i$ and every agent is able to make an effort sufficient to produce a non-negative amount of transactions for each asset, i.e.,

$$\exists v \in \mathbb{R}_{++}^J, \quad [0, v] \subset Z^i.$$

We subsequently define the payoff that agent i obtains when he chooses an action (x_0^i, z^i) in the feasible set $X_0^i \times Z^i$. The natural definition of the payoff $\Pi_0^i(x_0^i, z^i)$ is the following:

$$\Pi_0^i(x_0^i, z^i) \equiv \sup\{U_0^i(x_0^i, y) : \ y \in Y^i \text{ and } z^i \in F^i(y)\}.$$

In what follows we abstract from specific assumptions on the production technology (Y^i, F^i) that give rise to a production set Z^i that satisfies the requirements in Assumption 2.4. Similarly, we do not refer explicitly on the assumptions imposed on the utility function U_0^i that imply our requirements on agent i's period-zero payoff function $\Pi_0^i : X_0^i \times Z^i \to [-\infty, +\infty)$ (see Assumption 2.5(b) below). We refer to

Appendix B for a list of possible assumptions on the primitives (i.e., production technology (Y^i, F^i) and the utility function U_0^i) that are consistent with Assumption 2.4 and Assumption 2.5(b).

Assumption 2.5 For each agent i,

(a) initial endowments in commodities are strictly positive, i.e.,

$$e_0^i \in \operatorname{int} X_0^i \subset \mathbb{R}_{++}^{L_0} \quad \text{and} \quad \forall s \in S, \quad e_1^i(s) \in \operatorname{int} X_1^i \subset \mathbb{R}_{++}^{L_1};$$

(b) the payoff function $\Pi_0^i : X_0^i \times Z^i \to [-\infty, \infty)$ is continuous, concave, strictly increasing in consumption x_0 and strictly decreasing in production z;[14]
(c) the utility function $U_1^i(\cdot, s)$ is continuous, concave and strictly increasing.

Consider a consumption plan $x^i = (x_0^i, \boldsymbol{x}_1^i)$ where $x_0^i \in X_0^i$ and $\boldsymbol{x}_1^i = (x_s^i)_{s\in S} \in [X_1^i]^S$ and a production vector $z^i \in Z^i$. The expected discounted payoff of the action $a^i = (a_0^i, \boldsymbol{x}_1^i)$ where $a_0^i = (x_0^i, \theta^i, \varphi^i, z^i)$ is defined by

$$\Pi^i(a^i) \equiv \Pi_0^i(x_0^i, z^i) + \beta^i \sum_{s\in S} \nu^i(s) U_1^i(s, x^i(s)).$$

Remark 2.2 Observe that agent i's actions may be restricted to strict subsets $X_0^i \subset X_0$, $Z^i \subset Z$ and $X_1^i \subset X_1$. These restrictions may be problematic if agents are satiated on some feasible consumption plans. To avoid this problem, we make the following non-satiation assumption.

Assumption 2.6 The restrictions on each agent's consumption sets do not prevent them to purchase the aggregate endowment, i.e.,

$$\forall i \in I, \quad e_0 \equiv \sum_{k\in I} e_0^k \in \operatorname{int} X_0^i \quad \text{and} \quad e_1(s) \equiv \sum_{k\in I} e_1^k(s) \in \operatorname{int} X_1^i, \quad \forall s \in S.$$

We denote by A_0^i the set of actions at $t = 0$, i.e., $A_0^i \equiv X_0^i \times \Theta^i \times \Phi^i \times Z^i$ and we denote by A_1^i the set of actions for $t = 1$, i.e., $A_1^i \equiv [X_1^i]^S$. The set $A^i \equiv A_0^i \times A_1^i$ represents the set of intertemporal actions available for agent i.

3 Competitive equilibrium

We assume that markets are competitive and agents are price-takers. At $t = 0$, given

[14] In the sense that if x_0 and $\widehat{x}_0$ are two consumption bundles in X_0^i satisfying $\widehat{x}_0 > x_0$ then $\Pi_0^i(\widehat{x}_0, z) > \Pi_0^i(x_0, z)$ for any $z \in Z^i$. Similarly, if z and $\widehat{z}$ are two production plans in Z^i satisfying $\widehat{z} > z$ then $\Pi_0^i(x_0, \widehat{z}) < \Pi_0^i(x_0, z)$ for any consumption bundle $x_0 \in X_0^i$.

- a family (p_0, q, κ) of commodity prices $p_0 \in P_0$, asset prices $q \in Q$ and transaction costs/salaries $\kappa \in \mathbb{R}^J_+$;
- a family of (perfectly anticipated) future prices $\boldsymbol{p}_1 = (p_1(s))_{s\in S}$

each agent i chooses an action $a^i = (a^i_0, \boldsymbol{x}^i_1)$ where $a^i_0 = (x^i_0, \theta^i, \varphi^i, z^i) \in A^i_0$ is the vector of actions at $t = 0$ and $\boldsymbol{x}^i_1 = (x^i_s)_{s\in S} \in A^i_1$ is the vector of consumption bundles, such that

$$p_0 \cdot x^i_0 + (q + \kappa) \cdot \theta^i \le p_0 \cdot e^i_0 + (q - \kappa) \cdot \varphi^i + \kappa \cdot z^i \tag{1}$$

and for each $s \in S$,

$$p_1(s) \cdot x^i_1(s) + V(p_1(s), s) \cdot \varphi^i \le p_1(s) \cdot e^i_1(s) + V(p_1(s), s) \cdot \theta^i. \tag{2}$$

The set of all actions satisfying the budget restrictions (1) and (2) is called the budget set and denoted by $B^i(p, q, \kappa)$. We recall that the (discounted and expected) payoff of an action $a^i = (a^i_0, \boldsymbol{x}^i_1)$ is defined by

$$\Pi^i(a^i) \equiv \Pi^i_0(x^i_0, z^i) + \beta^i \sum_{s\in S} \nu^i(s) U^i_1(s, x^i(s)).$$

Definition 3.1 A competitive equilibrium $\{(p, q, \kappa), (a^i)_{i\in I}\}$ of an economy $\mathcal{E} \equiv \left\{X^i, \Theta^i, \Phi^i, Z^i\right\}_{i\in I}$ is a family composed of prices (p, q, κ) and an allocation $(a^i)_{i\in I}$ of intertemporal actions

$$a^i = (a^i_0, a^i_1) \quad \text{with} \quad a^i_0 = (x^i_0, \theta^i, \varphi^i, z^i) \quad \text{and} \quad a^i_1 = (x^i_1(s))_{s\in S}$$

such that

(a) actions are optimal, i.e.,

$$\forall i \in I, \quad a^i \in \operatorname{argmax}\{\Pi^i(a) : a \in B^i(p, q, \kappa)\} \tag{3}$$

(b) commodity markets clear, i.e.,

$$\sum_{i\in I} x^i_0 = \sum_{i\in I} e^i_0 \quad \text{and} \quad \sum_{i\in I} x^i_1(s) = \sum_{i\in I} e^i_1(s), \quad \forall s \in S \tag{4}$$

(c) asset markets clear, i.e.,

$$\forall j \in J, \quad \sum_{i\in I} \theta^i_j = \sum_{i\in I} \varphi^i_j \tag{5}$$

(d) transactions are feasible, i.e.,

$$\sum_{i\in I} \varphi^i + \theta^i = \sum_{i\in I} z^i. \tag{6}$$

3.1 Equilibrium properties

Before presenting general conditions that ensure existence of a competitive equilibrium we propose to underly some of its properties.

Definition 3.2 The asset structure is said *non-degenerate and positive* if for every possible vector of strictly positive prices $(p_1(s))_{s\in S}$[15] and for every asset j, payoffs are non-negative, i.e., $V_j(p_1(s), s) \geq 0$ for each $s \in S$ and non-degenerate, i.e., there exists $s_j \in S$ such that $V_j(p_1(s_j), s_j) > 0$.

Remark 3.1 If each asset j promises to deliver the units of account corresponding to the market value of a bundle $A_j(s)$ contingent to state s, then the asset structure is non-degenerate and positive if $A_j(s)$ belongs to $\mathbb{R}^{L_1}_{+}$ and for at least one state s_j the promise $A_j(s_j)$ is not zero.

Consider a competitive equilibrium $\{(p, q, \kappa), (a^i)_{i\in I}\}$. A direct consequence of Assumptions 2.5 and 2.6 is that commodity prices are strictly positive, i.e., $p_0 \in \mathbb{R}^{L_0}_{++}$ and $p_1(s) \in \mathbb{R}^{L_1}_{++}$ for every state s. Assume that the asset structure is non-degenerate and positive. If there are no restrictions on portfolios, i.e., $\Theta^i = \Theta$ and $\Phi^i = \Phi$ for each agent i, then the vector of asset prices must be strictly positive, i.e., $q \in \mathbb{R}^{J}_{++}$. In that case we can reinterpret our model by considering that transaction costs and salaries are proportional to transaction *values* instead of volumes. Indeed, let $c = (c_j)_{j\in J}$ be defined by

$$\forall j \in J, \quad c_j \equiv \frac{\kappa_j}{q_j}.$$

The number c_j represents the transaction cost paid by investors per unit of account invested or borrowed on asset j. The budget set at $t = 0$ can be rewritten in the following way:

$$p_0 \cdot x_0 + \sum_{j\in J}(1 + c_j)q_j\theta_j \leq p_0 \cdot e^i_0 + \sum_{j\in J}(1 - c_j)q_j\varphi_j + \sum_{j\in J} c_j q_j z^i_j. \tag{7}$$

This budget set is similar to the one proposed by Préchac (1996). Two main differences are in order. In Préchac (1996) the cost c_j is an exogenous parameter and the "reward" of each agent i is a fixed share on the profits of a monopolistic brokerage house. We highlight further these differences in Sect. 4.

Observe that if $c_j > 1$ in the budget restriction (7) then there is a problem.[16] An interesting property of our equilibrium is that, provided that there is trade in asset j, the cost c_j is endogenously determined at a level strictly lower than 1. Indeed, assume

15 In the sense that $p_1(s) \in \mathbb{R}^{L_1}_{++}$ for each state $s \in S$.

16 In Préchac (1996) it is not explicitly assumed that the exogenous cost parameter should satisfy $c_j \leq 1$.

that there is trade in the market of asset j, i.e.,

$$\sum_{i \in I} \theta_j^i + \varphi_j^i > 0.$$

The transaction cost κ_j must necessarily be strictly lower than the asset price q_j. By way of contradiction, assume that it is not the case, i.e., $q_j \leq \kappa_j$. There exists at least one agent k that is selling an amount $\varphi_j^k > 0$ of that asset. By doing so, he is paying at $t = 0$ the amount $\kappa_j - q_j$ in exchange of the obligation to deliver non-negative amounts in every state and a strictly positive amount in state s_j.[17] Agent k would be better-off canceling his sales and replacing them by strictly positive consumption at least in state s_j. This contradicts the optimality of action a^k. The above reasoning illustrates the importance of allowing the transaction costs to be determined endogenously and not to be fixed exogenously as in Préchac (1996) and Markeprand (2008). We come back on this issue in Sect. 4. In particular, we argue that the lack of a rationale for specifying the level of transaction costs in the models of Préchac (1996) and Markeprand (2008) raises a number of serious questions.

Definition 3.3 An economy $\mathcal{E} = \{X^i, \Theta^i, \Phi^i, Z^i\}_{i \in I}$ is said *particular* if there is only one good in both periods and for each agent i,

- there are no restrictions, i.e.,

$$X_0^i = X_0, \quad X_1^i = X_1, \quad \Theta^i = \Theta, \quad \Phi^i = \Phi \quad \text{and} \quad Z^i = Z;$$

- the payoff function Π_0^i is decomposed as follows:

$$\forall (x_0, z) \in X_0 \times Z, \quad \Pi_0^i(x_0, z) = U_0^i(x_0) - E^i(z);$$

- the function U_0^i is continuous, strictly increasing and strictly concave on $[0, \infty)$, differentiable on $(0, \infty)$, and satisfies Inada's condition at $x_0 = 0$, i.e.,

$$\lim_{h \to 0^+} \frac{U_0^i(h) - U_0^i(0)}{h} = \infty;$$

- for each asset j, there exists a function $z_j \mapsto E_j^i(z_j)$ differentiable, strictly increasing and strictly convex on $[0, \infty)$ such that

$$\forall z \in Z, \quad E^i(z) = \sum_{j \in J} E_j^i(z_j).$$

The term $E_j^i(z_j)$ represents the loss in terms of utility due to the effort required to intermediate a volume z_j of asset j.

[17] Remember that in state s_j we have $V_j(p_1(s_j), s_j) > 0$.

Assume that the economy is particular. Let $(\pi, \boldsymbol{a})$ be a competitive equilibrium with $\pi = (p, q, \kappa)$, $\boldsymbol{a} = (a^i)_{i \in I}$ and $a^i = (a_0^i, a_1^i)$ where $a_0^i = (x_0^i, \theta^i, \varphi^i, z^i)$. Since the function $x_0 \mapsto \Pi_0^i(x_0, z^i)$ is strictly increasing, we can assume without any loss of generality that $p_0 = 1$. Fix an asset j for which there exists trade. Then, there exists at least one agent i such that $z_j^i > 0$. From the Inada's condition we have $x_0^i > 0$. It follows from the first order conditions that[18]

$$\kappa_j = \frac{\nabla E_j^i(z_j^i)}{\nabla U_0^i(x_0^i)}.$$

At equilibrium, the salary for the intermediation of one unit of asset j equals the marginal rate of substitution between the disutility of effort and the utility of consumption.

3.2 Alternative modeling

One may consider the following alternative modeling:[19] if agent i supplies the labor y^i he can intermediate the volume of asset j corresponding to $g_j^i(y^i)$ units of account, i.e., if q_j is the price of asset j, agent i can intermediate z_j^i units of assets where

$$z_j^i \equiv \frac{1}{q_j} g_j^i(y^i).$$

This modeling seems to make sense only if the price q_j is strictly positive. This is problematic since we do not want to restrict the asset structure to be non-degenerate and positive (see Definition 3.2).

Assume that the asset structure is non-degenerate and positive. Observe that for this alternative modeling the budget set restriction (1) should be replaced by

$$p_0 \cdot x_0^i + (q + \kappa) \cdot \theta^i \leq p_0 \cdot e_0^i + (q - \kappa) \cdot \varphi^i + \sum_{j \in J} \frac{\kappa_j}{q_j} g^i(y^i) \tag{8}$$

and the market clearing condition (6) for transaction costs should be replaced by

$$\forall j \in J, \quad q_j \sum_{i \in I} \varphi_j^i + \theta_j^i = \sum_{i \in I} g_j^i(y^i).$$

It is not clear to us what is the interpretation of a unit of account. Here we cannot consider that it is a specific currency that serves as a medium of exchange since we do not model any government institution and we do not address the political and economic structure that determines the value of money. Since the production function

[18] If $f : [0, \infty) \to \mathbb{R}$ is a differentiable function, the differential of f is denoted by ∇f.

[19] We thank an anonymous referee for raising this point.

g^i is independent of the normalization of the prices (p_0, q, κ) determined endogenously at $t = 0$, the budget constraint (8) is not homogeneous of degree zero. This is problematic as it generates "spurious" real effects of changing units of account.[20]

3.3 Endogenous bounds on portfolio transactions

When there are no transaction costs, existence of equilibrium is not ensured. The non-existence arises from the discontinuity of demand for assets when commodity prices converge to prices for which the payoff matrix drops in rank (see Hart 1975). One way of preventing this discontinuity is to put some bounds on the portfolio transactions. We provide a framework where transaction costs imply the existence of an endogenous bound on physically feasible and individually rational portfolios. In what follows we discuss two cases that imply such a bound. First, we consider the case where the bound is a consequence of limited intermediation possibilities. This case is close to the model proposed by Laitenberger (1996) where the bound comes from the scarcity of commodities. However, the level of transaction costs in Laitenberger (1996) is specified exogenously. The second case has no counterpart in the literature and replaces limited intermediation by making the bound to be a consequence of a strong disutility for effort.

3.3.1 Limited intermediation

An allocation is said *physically feasible* if the market clearing conditions (4)–(6) are satisfied. The set of physically feasible actions is denoted by $\mathrm{F}((X^i, \Theta^i, \Phi^i, Z^i)_{i \in I})$. It is trivial to exhibit an exogenous upper-bound on the consumption allocations that are physically feasible since

$$\mathrm{F}((X^i, \Theta^i, \Phi^i, Z^i)_{i \in I}) \subset \prod_{i \in I} \left\{ [0, e_0] \times \Theta^i \times \Phi^i \times Z^i \times \prod_{s \in S} [0, e_1(s)] \right\}.$$

In particular, Assumption 2.6 implies that each agent is non-satiated at each feasible consumption plan.

Definition 3.4 An economy $\mathcal{E} = \{X^i, \Theta^i, \Phi^i, Z^i\}_{i \in I}$ is said to have *limited intermediation* if the production set Z^i of each agent i is bounded.

If the production possibilities of each agent are bounded then it is possible to exhibit an exogenous bound on portfolio allocations that are physically feasible.

Proposition 3.1 *If an economy has limited intermediation then the set of physically feasible allocations is bounded.*

[20] Similar problems arise in Markeprand (2008), see Sect. 4.2.

Proof of Proposition 3.1 Consider an economy $\mathcal{E} = \{X^i, \Theta^i, \Phi^i, Z^i\}_{i \in I}$ with limited intermediation. It follows that

$$\forall i \in I, \quad \exists \overline{z}^i \in \mathbb{R}^J_+, \quad Z^i \subset [0, \overline{z}^i]. \tag{9}$$

As a consequence we obtain that $\mathrm{F}((X^i, \Theta^i, \Phi^i, Z^i)_{i \in I})$ is a subset of the following bounded set

$$\prod_{i \in I} \left\{ [0, e_0] \times [0, \overline{z}] \times [0, \overline{z}] \times [0, \overline{z}^i] \times \prod_{s \in S} [0, e_1(s)] \right\},$$

where $\overline{z} \equiv \sum_{i \in I} \overline{z}^i$ is the maximum level of transactions that can be implemented by the labor market. □

There is another interesting situation where an exogenous bound on portfolios can be exhibited.

3.3.2 Strong disutility for effort

We say that an allocation $\boldsymbol{a} = (a^i)_{i \in I}$ is individually rational if

$$\forall i \in I, \quad \Pi^i(a^i) \geq \Pi^i(e^i_0, 0, e^i_1).$$

If agent i does not participate in any market, neither consumption nor labor market, he gets the payoff $\Pi^i(e^i_0, 0, e^i_1)$. The set of individually rational and physically feasible allocations is denoted by

$$\text{Ir-F}((X^i, \Theta^i, \Phi^i, Z^i)_{i \in I}).$$

We propose to replace the assumption that the volume of intermediation of each agent is limited by the assumption that agents need a huge effort to intermediate large amounts of financial activities.

Definition 3.5 Consider an economy $\mathcal{E} = \{X^i, \Theta^i, \Phi^i, Z^i\}_{i \in I}$. An agent i is said to exhibit *strong disutility for effort* if there is no physically feasible consumption that can compensate the effort for intermediating an arbitrarily large volume of transactions, i.e.,[21]

$$\liminf_{\|z\| \to \infty} \Pi^i(e_0, z, e_1) < \Pi^i(e^i_0, 0, e^i_1). \tag{10}$$

Proposition 3.2 *If every agent in an economy exhibits strong disutility for effort then the set of individually rational and physically feasible allocations is bounded.*

[21] If K is a finite set then $\|\cdot\|$ represents the norm defined by $\|y\| \equiv \sum_{k \in K} |y_k|$ for each vector $y = (y_k)_{k \in K}$ in $\mathbb{R}^K$.

Proof of Proposition 3.2 Assume by contradiction that the set of individually rational and physically feasible allocations is not bounded. Then there exists an unbounded sequence of allocation $(\boldsymbol{a}_n)_n$ satisfying[22]

$$\forall n \in \mathbb{N}, \quad \boldsymbol{a}_n \in \text{Ir-F}((X^i, \Theta^i, \Phi^i, Z^i)_{i\in I}).$$

Each consumption allocation $\boldsymbol{x}_n = (x_n^i)_{i\in I}$ is physically feasible and in particular we have

$$\forall i \in I, \quad 0 \leq x_n^i \leq e, \tag{11}$$

where $e = (e_0, e_1)$ is the aggregate intertemporal endowment. By market clearing of the labor markets, we have

$$\forall n \in \mathbb{N}, \quad \sum_{i\in I} z_n^i = \sum_{i\in I} \theta_n^i + \varphi_n^i.$$

Since the sequence $(\boldsymbol{a}_n)_n$ is unbounded, we must have that the sequence of production allocations $(\boldsymbol{z}_n)_{n\in\mathbb{N}}$ is unbounded. Passing to a subsequence if necessary, we can assume that there exists an agent $i \in I$ such that

$$\lim_{n\to\infty} \|z_n^i\| = \infty.$$

Since for each n, the allocation $\boldsymbol{a}_n$ is individually rational, we must have

$$\forall n \in \mathbb{N}, \quad \Pi^i(x_{0,n}^i, z_n^i, x_{1,n}^i) \geq \Pi^i(e_0^i, 0, e_1^i).$$

Since the function $(x_0, x_1) \mapsto \Pi^i(x_0, z, x_1)$ is strictly increasing for any z, it follows from (11) that

$$\forall n \in \mathbb{N}, \quad \Pi^i(e_0, z_n^i, e_1) \geq \Pi^i(e_0^i, 0, e_1^i).$$

This leads to the following contradiction

$$\liminf_{n\to\infty} \Pi^i(e_0, z_n^i, e_1) \geq \Pi^i(e_0^i, 0, e_1^i).$$

□

3.4 Existence result

If it is possible to exhibit a bound on individually rational and physically feasible allocations, then existence of a competitive equilibrium follows from standard fixed-point

[22] Recall that $\boldsymbol{a}_n = (a_n^i)_{i\in I}$ and $a_n^i = (a_{0,n}^i, a_{1,n}^i)$ where $a_{0,n}^i = (x_{0,n}^i, \theta_n^i, \varphi_n^i, z_n^i)$ and $a_{1,n}^i = (x_{1,n}^i(s))_{s\in S}$. The intertemporal consumption plan $(x_{0,n}^i, (x_{1,n}^i(s))_{s\in S})$ is denoted by x_n^i. The production allocation $(z_n^i)_{i\in I}$ is denoted by $\boldsymbol{z}_n$.

arguments. The only novelty of the existence proof is to show that it is possible to clear via competitive prices the markets for commodities, assets and labor at period $t = 0$. In what follows we propose to show that condition (b) of Assumption 2.5 (i.e., the payoff function is concave and strictly decreasing in production) implies that agents have either limited intermediation or they exhibit strong disutility for effort.

Proposition 3.3 *Consider an economy satisfying the aforementioned assumptions. Every agent i has either limited intermediation in the sense that Z^i is bounded, or exhibits strong disutility for effort. In particular, the set of individually rational and physically feasible allocations is bounded.*

Proof of Proposition 3.3 Assume that there exists an agent i whose production set Z^i is not bounded. Let $(z_n)_{n\in\mathbb{N}}$ be a sequence in Z^i such that

$$\lim_{n\to\infty} \|z_n\| = \infty.$$

Passing to a subsequence if necessary, one can assume that there exists at least one asset j such that the sequence $(z_n(j))_{n\in\mathbb{N}}$ is strictly increasing and unbounded (in particular converges to ∞). We let ξ_n be the vector in $\mathbb{R}^J_+$ defined by $\xi_n \equiv z_n(j)\mathbf{1}_{\{j\}}$.[23] Since the function $z \mapsto \Pi^i(e_0, z, e_1)$ is decreasing, we have

$$\forall n \in \mathbb{N}, \quad \Pi^i(e_0, z_n, e_1) \le \Pi^i(e_0, \xi_n, e_1).$$

Since the function $h \mapsto \Pi^i(e_0, h\mathbf{1}_{\{j\}}, e_1)$ is concave and strictly decreasing, we must have[24]

$$\lim_{n\to\infty} \Pi^i(e_0, \xi_n, e_1) = -\infty$$

implying that agent i exhibits strong disutility for effort. □

As a consequence, we can state our main result.

Theorem 3.1 *There exists a competitive equilibrium $((p, q, \kappa), \boldsymbol{a})$ with non-negative transaction costs, i.e., $\kappa \in \mathbb{R}^J_+$.*

Our proof of Theorem 3.1 is based on a limiting argument. We first state that existence is ensured when the sets of actions are bounded.[25]

Proposition 3.4 *Assume that each action set A^i is bounded. Then, there exists a competitive equilibrium $((p, q, \kappa), \boldsymbol{a})$ with non-negative transaction costs.*

[23] If K is a finite set and H is a subset of K then $\mathbf{1}_H$ denotes the vector $y = (y_k)_{k\in K}$ in $\mathbb{R}^K$ defined by $y_k = 1$ if $k \in H$ and $y_k = 0$ elsewhere.

[24] Let $f : [0, \infty) \to \mathbb{R}$ be a concave and strictly decreasing function. For every increasing sequence $(x_n)_{n\in\mathbb{N}}$ converging to ∞ one must have $\lim_n f(x_n) = -\infty$. Indeed, since the sequence converges to ∞, for n large enough we have $x_n \ge 1$. It implies by concavity that $f(x_n) \le f(0) + x_n[f(1) - f(0)]$. Since f is strictly decreasing we have $f(1) - f(0) < 0$ and we get the desired result.

[25] The proof of Proposition 3.4 is postponed to Appendix A.

We subsequently consider a sequence of suitably truncated economies and apply Proposition 3.4 to obtain a sequence of competitive equilibria for the corresponding truncated economies. The final argument amounts to show that there exists a truncated economy for which every competitive equilibrium is actually an equilibrium of the initial economy.

Proof of Theorem 3.1 Consider an economy $\mathcal{E} = \{X^i, \Theta^i, \Phi^i, Z^i\}_{i\in I}$ satisfying the assumptions of this paper and such that the set of individually rational and physically feasible allocations is bounded. We fix an integer $n \in \mathbb{N}$ and we propose to truncate the economy $\mathcal{E}$ in a suitable manner, such that the truncated economy $\mathcal{E}_n$ still satisfies the assumptions of the paper. Consider the economy

$$\mathcal{E}_n \equiv \left\{X_n^i, \Theta_n^i, \Phi_n^i, Z_n^i\right\}_{i\in I}$$

where

- consumption sets are defined by

$$X_{0,n}^i \equiv X_0^i \cap \left[0, e_0 + n\mathbf{1}_{L_0}\right] \quad \text{and} \quad X_{1,n}^i \equiv X_1^i \cap \left[0, \bar{e}_1 + n\mathbf{1}_{L_1}\right],$$

  where $\bar{e}_1(\ell) = \max\{e_1(s,\ell) \colon s \in S\}$ for each $\ell \in L_1$;
- portfolio sets are defined by

$$\Theta_n^i \equiv [0, n\mathbf{1}_J] \cap \Theta^i \quad \text{and} \quad \Phi_n^i \equiv [0, n\mathbf{1}_J] \cap \Phi^i;$$

- production sets are defined by

$$Z_n^i \equiv [0, n\mathbf{1}_J] \cap \Theta^i.$$

We can apply Proposition 3.4 to each economy $\mathcal{E}_n$ to get a sequence $(\pi_n, \boldsymbol{a}_n)_{n\in\mathbb{N}}$ of competitive equilibrium where $\pi_n = (p_n, q_n, \kappa_n)$ with $\kappa_n \in \mathbb{R}_+^J$. Since the set of individually rational and physically feasible allocations is bounded, there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$,

$$\text{Ir-F}((X^i, \Theta^i, \Phi^i, Z^i)_{i\in I}) \subset \prod_{i\in I} X_{0,n}^i \times \Theta_n^i \times \Phi_n^i \times Z_n^i \times [X_{1,n}^i]^S.$$

We let $\nu \equiv n_0 + 1$. It follows from standard arguments based on the concavity of each expected payoff function Π^i that $(\pi_\nu, \boldsymbol{a}_\nu)$ is actually a competitive equilibrium of the initial economy $\mathcal{E}$. □

4 Discussion

In this section we argue that specifying exogenously the level of transaction costs, as it is the case in the models proposed by Préchac (1996) and Markeprand (2008), it raises a number of serious questions. Préchac (1996) and Markeprand (2008) propose a model where no real resources are burnt in the process of transaction and labor costs are negligible. They assume that there is one firm in the market that they refer to as the brokerage house. Trade can only be implemented through this firm, i.e., this firm has the monopoly of intermediating financial activities. The brokerage house is assumed to be privately owned by consumers/investors. Each agent i is endowed with an equity share $\sigma^i \in (0, 1)$ that determines at equilibrium his share of profits. Using its monopoly power, the brokerage house fixes a commission fee on transactions that can be proportional to volume and/or value of assets traded. More precisely, if an agent chooses a financial strategy (θ, φ) and if the asset price is q, then the agent should pay to the brokerage house the amount $c(q, \theta, \varphi)$ of units of account given by

$$c(q, \theta, \varphi) \equiv \sum_{j \in J} c_j q_j (\theta_j + \varphi_j) + \kappa_j (\theta_j + \varphi_j), \tag{12}$$

where $c \in \mathbb{R}^J_+$ and $\kappa \in \mathbb{R}^J_+$. Préchac (1996) assumes that assets are non-degenerate and positive, and he considers the special case

$$c \in \mathbb{R}^j_{++} \quad \text{and} \quad \kappa = 0$$

while Markeprand (2008) allows for a more general asset structure but he needs to assume that[26]

$$\kappa \in \mathbb{R}^j_{++}.$$

Agents operate in a perfectly competitive environment taking not only prices as given but also the profit π of the brokerage house. We denote by $B^i_{\mathrm{M}}(p, q, \pi)$ the set of agent i's actions $a^i = (a^i_0, \boldsymbol{x}^i_1)$ with $a^i_0 = (x^i_0, \theta^i, \varphi^i)$ satisfying the following budget constraint at $t = 0$

$$p_0 \cdot x^i_0 + q \cdot \theta^i + c(q, \theta^i, \varphi^i) \le p_0 \cdot e^i_0 + q \cdot \varphi^i + \sigma^i \pi \tag{13}$$

and the budget constraint at $t = 1$ and each state s that we consider in our model, i.e.,

$$p_1(s) \cdot x^i_1(s) + V(p_1(s), s) \cdot \varphi^i \le p_1(s) \cdot e^i_1(s) + V(p_1(s), s) \cdot \theta^i. \tag{14}$$

Since labor costs are negligible in Préchac (1996) and Markeprand (2008), we denote by V^i the expected utility function of agent i defined by

$$V^i(a^i) \equiv U^i_0(x^i_0) + \beta^i \sum_{s \in S} \nu^i(s) U^i_1(s, x^i_1(s)).$$

[26] Actually Markeprand (2008) allows for a more general form of cost function. When the cost function takes the form defined by (12), then it is needed to assume that $\kappa_j > 0$ for each asset j since the price q_j may be zero at equilibrium.

We now consider the definition of equilibrium adapted to this model and introduced by Préchac (1996).

Definition 4.1 A competitive equilibrium of the economy with a monopolistic brokerage house is a family $\{(p, q, \pi), (a^i)_{i\in I}\}$ composed of prices (p, q), profit π and an allocation $(a^i)_{i\in I}$ of intertemporal actions such that

(a) actions are optimal, i.e.,

$$\forall i \in I, \quad a^i \in \operatorname{argmax}\{V^i(a) : a \in B^i_{\mathrm{M}}(p, q, \pi)\} \tag{15}$$

(b) commodity markets clear, i.e.,

$$\sum_{i\in I} x^i_0 = \sum_{i\in I} e^i_0 \quad \text{and} \quad \sum_{i\in I} x^i_1(s) = \sum_{i\in I} e^i_1(s), \quad \forall s \in S \tag{16}$$

(c) asset markets clear, i.e.,

$$\forall j \in J, \quad \sum_{i\in I} \theta^i_j = \sum_{i\in I} \varphi^i_j \tag{17}$$

(d) profits are correctly anticipated, i.e.,

$$\sum_{i\in I} c(q, \theta^i, \varphi^i) = \pi. \tag{18}$$

4.1 Endogenous intermediation costs with a monopolistic brokerage house

When assets are non-degenerate and positive, Préchac (1996) proved that an equilibrium with a monopolistic brokerage house always exists. A serious drawback of the model proposed by Préchac (1996) is the lack of a rationale for the determination of intermediation costs. Since the brokerage house has the monopoly power to choose the vector $c = (c_j)_{j\in J}$, an issue that naturally arises concerns the way the brokerage house chooses the vector c. An obvious answer is to say that c is determined such that the brokerage house maximizes its profit. However, such a modification introduces serious difficulties for tackling the existence problem. This is because, in such a setting, profits depend on the equilibrium outcome which in turn depends on the level of the chosen intermediation costs. Additional complications arise from the existence of multiple equilibria. To simplify things, consider that the primitives of the economy are such that for each vector c, there is a unique equilibrium and therefore the profit function

$$c \mapsto \pi(c) = \sum_{i\in I} \sum_{j\in J} c_j q_j(c)(\theta^i_j(c) + \varphi^i_j(c))$$

is well-defined. Following the idea proposed (although in a different framework) by Bisin (1998), a simple and natural way to make c endogenous is to solve the following maximization problem

$$\operatorname{argmax}\{\pi(c)\ :\ c \in (0,1)^J\}.$$

It is not clear whether this maximization problem has always a solution. Moreover, for economies exhibiting the non-existence phenomena la Hart (1975), it follows from Proposition 2 in Markeprand (2008) that

$$\lim_{c \to 0} \sum_{i \in I} \|\theta^i(c)\| + \|\varphi^i(c)\| = \infty.$$

This observation implies that it is far from clear that the profit function $c \mapsto \pi(c)$ is bounded from above in a neighborhood of 0. Therefore, one cannot conclude whether the non-existence phenomenum à la Hart (1975) is ruled out when the intermediation cost becomes endogenous.

4.2 Non-homogeneous budget restriction at $t = 0$

In Préchac (1996) intermediation costs are proportional to the value of the transactions. In other words, costs are denominated in units of assets: in order to trade one unit of asset j, each agent should give to the brokerage house c_j units of the same asset as a fee. This kind of intermediation costs ensures existence when assets are non-degenerate and positive. Markeprand (2008) showed that in order to deal with a more general asset structure including options, the cost function should satisfy extra-properties since asset prices may be 0 at equilibrium. Markeprand (2008) claims that if the cost function

$$c(q,\theta,\varphi) = \sum_{j \in J} c_j q_j(\theta_j + \varphi_j) + \kappa_j(\theta_j + \varphi_j)$$

is such that $\kappa_j > 0$ for each asset j, then existence is guaranteed even if the assets are not non-degenerate and positive. The only property that the payoff function of each asset should satisfy is continuity with respect to commodity prices. This kind of cost function proposed by Markeprand (2008) introduces an important difference with respect to the model proposed by Préchac (1996). It implies that the budget restriction at $t = 0$ is no more homogeneous of degree zero with respect to prices. If we multiply the prices (p_0, q) by 2, intermediation becomes less costly, while if we divide the prices by 2 intermediation becomes more costly. This may lead to a serious problem of interpretation and raises questions that already appear in the literature of financial markets with nominal assets. Since the *level of prices* matters, who determines this level? Is it endogenously determined through market forces? If not, is there any institution or agent that is fixing the equilibrium level of prices? When does the brokerage house choose the commission fee κ? Before or after observing the price level?

We would like to stress that endogeneity of transaction costs as modeled in Sect. 2 re-establishes homogeneity of budget constraints at $t = 0$ and pin down these complicated issues. This is also true in the endogenous models proposed by Pesendorfer (1995) and Bisin (1998).[27]

Markeprand (2008) does not discuss the implications of his cost function on the homogeneity of period $t = 0$ budget restriction. More problematic is the proof of the existence result found in his paper. The inherited nominal feature due to the specification of his cost function seems to play no role. Indeed, it is claimed in Markeprand (2008, see Sect. 3, p. 152) that, independently of the *nominal* level of the vector of commission fees κ, there exists a competitive equilibrium

$$\{(p, q, \pi), (a^i)_{i \in I}\}$$

where prices at $t = 0$ satisfy the following conditions[28]

$$p_0 \in \mathbb{R}_+^{L_0}, \quad \|p_0\| = 1 \quad \text{and} \quad \|q\| \leq 1. \tag{19}$$

We propose to show that his claim is not correct. Indeed, for a competitive equilibrium to exist, the level of prices at $t = 0$ matters and should depend on the nominal commission fee κ.

We consider the simplest case: one good per date, no uncertainty at $t = 1$ and one asset delivering one unit of the unique good at $t = 1$. The cost function is defined as follows:

$$c(q, \theta, \varphi) \equiv \kappa(\theta + \varphi).$$

To make the analysis simpler, we adopt the notation in Markeprand (2008) and denote by $z = \theta - \varphi$ the net trade in the asset market.[29] Given a vector of commodity prices $p = (p_0, p_1)$, an asset price q and the profit π, the budget set of agent i, denoted by $B^i_{\mathrm{M}}(p, q, \pi)$ is the set of all actions $(x_0, z, x_1) \in \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+$ such that at $t = 0$

$$p_0 x_0 + qz + \kappa|z| \leq p_0 e^i_0 + \sigma^i \pi \tag{20}$$

and at $t = 1$

$$p_1 x_1 \leq p_1(e^i_1 + z). \tag{21}$$

Each agent i is assumed to have the same utility function given by

$$U^i(x_0, x_1) \equiv \sqrt{x_0} + \sqrt{x_1}.$$

[27] However, in Pesendorfer (1995) and Bisin (1998) the budget constraints at $t = 1$ are not homogenous of degree zero with respect to consumption prices.

[28] We recall that if K is a finite set, we let $\|z\|$ be the norm of a vector $z = (z_k)_{k \in K}$ in $\mathbb{R}^K$ defined by $\|z\| \equiv \sum_{k \in K} |z_k|$.

[29] An agent will optimally choose (θ, φ) such that $\theta\varphi = 0$. In other words, either we have $\theta = 0$ or $\varphi = 0$. This implies that the action (θ, φ) of an agent on the asset market can be replaced by $z = \theta - \varphi$.

What differentiates agents are initial endowments. We will assume that there are two agents $I = \{i_0, i_1\}$. Agent i_0 has a larger endowment at $t = 0$ while agent i_1 has a larger endowment at $t = 1$. More precisely, we will assume that there exists $M > 1$ such that

$$e_0^{i_0} = Me_1^{i_0} \quad \text{and} \quad e_1^{i_1} = Me_0^{i_1}.$$

This economy satisfies all the assumptions of Theorem 1 in Markeprand (2008). Therefore, independently of κ, there exists a competitive equilibrium

$$\{(p, q, \pi), (x_0^i, z^i, x_1^i)_{i \in I}\}$$

satisfying

$$p_0 = 1, \quad p_1 = 1 \quad \text{and} \quad |q| \leq 1.$$

We show that there is a problem if $\kappa \geq 1$.

Proposition 4.1 *If $\kappa \geq 1$ then we can choose initial endowments such that there does not exist a competitive equilibrium with prices satisfying* (19).

Proof of Proposition 4.1 Assume that $\kappa \geq 1$ and choose M large enough such that $M > (\kappa + 1)^2$. Since $|q| \leq 1$ we must have $q - \kappa \leq 0$. This implies that if agent i chooses to short-sell one unit of the asset, he has to deliver $\kappa - q$ units of the good at $t = 0$ and 1 unit at $t = 1$. He is clearly better off not short-selling. Since the asset market clears, there is no transaction in the market, i.e.,

$$z^{i_0} = z^{i_1} = 0 \quad \text{and} \quad \pi = 0.$$

However, agent i_0 prefers to transfer wealth from period $t = 0$ to period $t = 1$. Indeed, let $\widetilde{a}^{i_0}(\varepsilon)$ be the alternative action

$$\widetilde{a}^{i_0}(\varepsilon) = (\widetilde{x}_0^{i_0}(\varepsilon), \widetilde{z}^{i_0}(\varepsilon), \widetilde{x}_1^{i_0}(\varepsilon))$$

defined by

$$\widetilde{z}^{i_0}(\varepsilon) = \varepsilon, \quad \widetilde{x}_0^{i_0}(\varepsilon) = e_0^{i_0} - (q + \kappa)\varepsilon \quad \text{and} \quad \widetilde{x}_1^{i_0}(\varepsilon) = e_1^{i_0} + \varepsilon.$$

The action $\widetilde{a}^{i_0}(\varepsilon)$ belongs to the budget set $B_{\mathrm{M}}^i(p, q, \pi)$ and by the chain rule we have

$$\lim_{\varepsilon \to 0^+} \frac{U^i(\widetilde{x}^{i_0}(\varepsilon)) - U^i(x^{i_0})}{\varepsilon} = -\frac{q + \kappa}{2\sqrt{e_0^{i_0}}} + \frac{1}{2\sqrt{e_1^{i_0}}}.$$

Observe that

$$q + \kappa \leq 1 + \kappa < \sqrt{M} = \sqrt{e_0^{i_0}/e_1^{i_0}}$$

implying that for ε small enough, we have the following contradiction

$$U^i(\widetilde{x}^{i_0}(\varepsilon)) > U^i(x^{i_0}).$$

$\square$

Actually, the normalization (19) proposed by Markeprand (2008) is problematic even if there are no transaction costs.[30]

Proposition 4.2 *Assume that $\kappa = 0$ and $e_0 > e_1$. Then there does not exist a competitive equilibrium with prices satisfying* (19).

Proof of Proposition 4.2 Assume that $\kappa = 0$. We have complete markets and actually the optimal action $a^i = (x_0^i, z^i, x_1^i)$ of agent i is also a solution to the maximization of $U^i(x_0, x_1)$ under the constraint

$$x_0 + qx_1 \leq e_0^i + qe_1^i.$$

First order condition implies $q = \sqrt{x_0^i}/\sqrt{x_1^i}$. Since markets clear, one must have $e_0 = q^2 e_1$. If $e^0 > e^1$ we get the contradiction: $q > 1$. $\square$

Let us replace the normalization (19) by the classical one[31]

$$(p_0, q, p_1) \in \mathbb{R}_+^3, \quad p_0 + q = \chi_0 \quad \text{and} \quad p_1 = \chi_1 \tag{22}$$

where $\chi_0 > 0$ and $\chi_1 > 0$. One may wonder if the arguments in Markeprand (2008) can be corrected when considering this new normalization. The answer is yes, but if κ is larger than χ_0, the only possible equilibrium is no-trade. We do not provide the general proof of our claim. For the simplicity of the presentation, we prefer to illustrate this result using the specific economy we have been considering.

Proposition 4.3 *Assume that $\kappa \geq \chi_0$. Then no-trade is the only possible competitive equilibrium with prices satisfying* (22).

Proof of Proposition 4.3 Assume that there exists a competitive equilibrium

$$\{(p, q, \pi), (x_0^i, z^i, x_1^i)_{i \in I}\}$$

with

$$(p_0, q, p_1) \in \mathbb{R}_+^3, \quad p_0 + q = \chi_0 \quad \text{and} \quad p_1 = \chi_1.$$

30 In Markeprand (2008), it is shown that the existence is guaranteed only if $\kappa > 0$. However, in our specific example, assets are numéraire. This implies that we can find an exogenous bound on actions that is not binding at equilibrium. If the arguments of Lemmas 1 and 3 in Markeprand (2008) were correct, existence of an equilibrium satisfying the normalization (19) should be ensured even if there are no transaction costs.

31 In our specific example, the asset structure is non-degenerate and positive. If assets may have negative payoff as in Markeprand (2008), the normalization has to be adapted to the following one:

$$(p_0, q, p_1) \in \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+, \quad p_0 + |q| = \chi_0 \quad \text{and} \quad p_1 = \chi_1.$$

Since $q - \kappa \leq \chi_0 - \kappa \leq 0$, no agent will short-sell the asset. By market clearing, we must have no trade. Therefore, no-trade is the only possible equilibrium. Actually, it is an equilibrium. We only have to choose q close enough to χ_0 such that

$$\frac{q+\kappa}{\chi_0 - q} > \sqrt{M}.$$

Indeed, we claim that

$$\{(p, q, \pi), (e_0^i, 0, e_1^i)_{i\in I}\} \quad \text{with} \quad p_0 = \chi_0 - q \quad \text{and} \quad p_1 = \chi_1$$

is a competitive equilibrium. We only have to prove that for each agent i, no-trade is the optimal action. Fix an agent i and assume by way of contradiction that there exists a budget feasible action $a^i = (x_0^i, z^i, x_1^i)$ such that $U^i(x^i) > U^i(e^i)$. Since we have $q - \kappa \leq 0$ the agent i will not short-sell the asset. As a consequence, we must have $z^i > 0$. Since a^i is budget feasible, we have

$$x_0^i \leq e_0^i - \frac{q+\kappa}{\chi_0 - q} z^i \quad \text{and} \quad x_1^i \leq e_1^i + z^i.$$

We let $\widetilde{a}^i$ be the action $(\widetilde{x}_0^i, z^i, \widetilde{x}_1^i)$ defined by

$$\widetilde{x}_0^i = e_0^i - \frac{q+\kappa}{\chi_0 - q} z^i \quad \text{and} \quad \widetilde{x}_1^i = e_1^i + z^i.$$

This action is budget feasible and satisfies $U^i(\widetilde{x}^i) - U^i(e^i) > 0$. By concavity, this implies

$$\lim_{\varepsilon\to 0^+} \frac{1}{\varepsilon}\left[\left(\sqrt{e_0^i - \frac{q+\kappa}{\chi_0 - q}\varepsilon} - \sqrt{e_0^i}\right) + \left(\sqrt{e_1^i + \varepsilon} - \sqrt{e_1^i}\right)\right] > 0.$$

Therefore, chain rule implies the following contradiction

$$\frac{q+\kappa}{\chi_0 - q} < \sqrt{\frac{e_0^i}{e_1^i}} \leq \max\{1/\sqrt{M}, \sqrt{M}\} \leq \sqrt{M}.$$

□

Proposition 4.3 illustrates that the level of prices χ_0 is a relevant parameter. If it is not large enough (i.e., larger that the commission fee κ) only *no-trade* equilibrium exists. Obviously, neither the brokerage house nor the agents have an incentive to preclude trade. The model proposed by Markeprand (2008) shares with the model developed by Préchac (1996) the same drawback: the commission fee is exogenous and the objective of the brokerage house is not explicitly modeled. But in Markeprand (2008) a rationale for an endogenous level of prices is also missing. Observe that our model suffers from none of the two drawbacks.

Appendix A: Proof of Proposition 3.4

Consider an economy $\mathcal{E} = \{X^i, \Theta^i, \Phi^i, Z^i\}_{i\in I}$ satisfying the list of assumptions of this paper and such that the sets X^i, Θ^i, Φ^i and Z^i are compact. We let Price_0 be the auctioneer's action set at $t = 0$ where[32]

$$\text{Price}_0 \equiv \{\pi_0 = (p_0, q, \kappa) \in \mathbb{R}^{L_0} \times \mathbb{R}^J \times \mathbb{R}^J_+ \;:\; \|\pi_0\| \equiv \|p_0\| + \|q\| + \|\kappa\| \leq 1\}.$$

We would like to stress that almost all arguments of the proof are standard. The only difficult step is to clear simultaneously at $t = 0$ the consumption, the financial and the labor market. The choice made above of the price space Price_0 is crucial. Other choices of price normalization may lead to problems like those illustrated in Propositions 4.1–4.3.

Since the vector $(0, 0, 0)$ belongs to the price set Price_0, we follow Bergstrom (1976) and consider the following *slack* function

$$\forall \pi_0 = (p_0, q, \kappa) \in \text{Price}_0, \quad \gamma(\pi_0) = 1 - \|\pi_0\|.$$

We let Price_1 be the auctioneer's action set at $t = 1$ where

$$\text{Price}_1 \equiv \{p \in \mathbb{R}^{L_1}_+ \;:\; \|p\| = 1\}.$$

We let Price be the set of auctioneer's intertemporal actions defined by

$$\text{Price} \equiv \text{Price}_0 \times [\text{Price}_1]^S.$$

We slightly modify each agent's budget set as follows: for each agent i and price family $\pi = (\pi_0, \pi_1)$ with $\pi_0 = (p_0, q, \kappa)$ and $\pi_1 = (p_1(s))_{s\in S}$, we let $B^i_\gamma(\pi)$ be the set of all actions $a = (a_0, a_1)$ with

$$a_0 = (x_0, \theta, \varphi, z) \in A^i_0 = X^i \times \Theta^i \times \Phi^i \times Z^i$$

and $a_1 = (x_1(s))_{s\in S}$ with $x_1(s) \in X^i_1$ such that the following budget constraints are satisfied:

$$p_0 \cdot x_0 + (q + \kappa) \cdot \theta \leq p_0 \cdot e^i_0 + (q - \kappa) \cdot \varphi + \kappa \cdot z + \gamma(\pi_0) \tag{23}$$

and for each state s,

$$p_1(s) \cdot x_1(s) + V(p_1(s), s) \cdot \varphi \leq p_1(s) \cdot e^i_1(s) + V(p_1(s), s) \cdot \theta. \tag{24}$$

The function γ has suitably been chosen in order to obtain the following continuity result.

[32] We recall that if K is a finite set then $\|\cdot\|$ represents the norm defined by $\|y\| \equiv \sum_{k\in K} |y(k)|$ for each $y = (y(k))_{k\in K}$ in $\mathbb{R}^K$.

Lemma A.1 *The correspondence B^i_γ is continuous on the set* Price *for each agent i.*

We omit the (standard) details of the proof (see Martins-da-Rocha and Vailakis 2008 for a rigorous proof). Following our normalization the vector $(0, 0, \mathbf{1}_{\{j\}})$ belongs to the price set Price_0.[33] Lower semi-continuity of the correspondence B^i_γ at any family $\pi = (\pi_0, \pi_1)$ with $\pi_0 = (0, 0, \mathbf{1}_{\{j\}})$ follows from the fact that agent i has the productive capacity to intermediate at least some units of asset j (Assumption 2.4).

Using the continuity and convexity assumptions made on payoff functions together with the continuity of the modified budget correspondence, it is straightforward to apply Berge's Maximum Theorem (see Berge 1963, pp. 115–116 or Aliprantis and Border 1999, Theorem 16.31) and obtain the upper semi-continuity of the modified demand correspondence as defined hereafter.

Lemma A.2 *For each agent i, the correspondence d^i_γ is continuous on the set* Price*, where*

$$\forall \pi \in \text{Price}, \quad d^i_\gamma(\pi) \equiv \text{argmax}\{\Pi^i(a) \ : \ a \in B^i_\gamma(\pi)\}.$$

Moreover, for every $\pi \in$ Price*, the set $d^i_\gamma(\pi)$ is non-empty, convex and compact.*

We let σ_0 be the correspondence from $\prod_{i\in I} A^i_0$ to Price_0 representing the auctioneer's demand at $t = 0$ and defined by

$$\sigma_0(\boldsymbol{a}_0) \equiv \text{argmax}\left\{\sum_{i\in I} p_0 \cdot [x^i_0 - e^i_0] + q \cdot [\theta^i - \varphi^i] + \kappa \cdot [\theta^i + \varphi^i - z^i] : \pi_0 \in \text{Price}_0\right\}$$

for all $\boldsymbol{a}_0 = (a^i_0)_{i\in I}$. For each state s, we let σ_s be the correspondence from $\prod_{i\in I} X^i_1$ to Price_1 representing the auctioneer's demand at $t = 1$ contingent to state s and defined by

$$\sigma_s(\boldsymbol{x}_1(s)) \equiv \text{argmax}\left\{\sum_{i\in I} p_1(s) \cdot [x^i_1(s) - e^i_1(s)] \ : \ \pi_1(s) \in \text{Price}_1\right\}$$

for all $\boldsymbol{x}_1(s) = (x^i_1(s))_{i\in I}$.

We omit the standard arguments to prove that these correspondences are upper semi-continuous.

Lemma A.3 *The correspondence σ_0 is upper semi-continuous on $\prod_{i\in I} A^i_0$ with non-empty, compact, convex values, and for each state s the correspondence σ_s is upper semi-continuous on $\prod_{i\in I} X^i_1$ with non-empty, compact and convex values.*

[33] We recall that if K is a finite set and H is a subset of K then $\mathbf{1}_H$ denotes the vector $y = (y(k))_{k\in K}$ in $\mathbb{R}^K$ defined by $y(k) = 1$ if $k \in H$ and $y(k) = 0$ elsewhere.

Let K be the compact, convex and non-empty set defined by

$$K \equiv \text{Price} \times \prod_{i \in I} A^i.$$

We let χ be the correspondence from K to K defined by

$$\forall (\pi, \boldsymbol{a}), \quad \chi(\pi, \boldsymbol{a}) \equiv \left[\sigma_0(\boldsymbol{a}_0) \times \prod_{s \in S} \sigma_s(\boldsymbol{x}_1(s)) \right] \times \prod_{i \in I} d^i_\gamma(\pi).$$

It follows from Lemma A.2 and A.3 that the correspondence χ is upper semi-continuous with compact, convex and non-empty values. Applying Kakutani's Fixed-Point Theorem (see Kakutani 1941 or Aliprantis and Border 1999, Corollary 16.51), we obtain the existence of a fixed-point $(\pi, \boldsymbol{a})$ of the correspondence χ, i.e.,

$$(\pi, \boldsymbol{a}) \in \chi(\pi, \boldsymbol{a}). \tag{25}$$

We split the rest of the proof in several steps.

Lemma A.4 *We have $\gamma(\pi_0) = 0$, i.e., the modified demand and the demand coincide.*

Proof of Lemma A.4 Assume by way of contradiction that $\gamma(\pi_0) > 0$, i.e.,

$$0 < \varepsilon \equiv 1 - (\|p_0\| + \|q\| + \|\kappa\|).$$

Following standard arguments,[34] we can prove that commodity markets clear at $t = 0$, i.e.,

$$\sum_{i \in I} [x^i_0 - e^i_0] = 0,$$

asset markets clear, i.e.,

$$\sum_{i \in I} \theta^i = \sum_{i \in I} \varphi^i$$

and labor markets clear with free-disposal, i.e.,

$$\sum_{i \in I} \theta^i + \varphi^i - z^i \leq 0.$$

Since commodity markets clear at $t = 0$, we must have

$$\forall i \in I, \quad x^i_0 \leq e_0.$$

[34] See Martins-da-Rocha and Vailakis (2008) for a rigorous proof.

Following Assumption 2.6, each agent i is non-satiated at (x_0^i, z^i) in terms of the payoff Π_0^i. This implies that the budget restriction of the first period $t = 0$ must be binding, i.e.,

$$p_0 \cdot [x_0^i - e_0^i] + q \cdot [\theta^i - \varphi^i] + \kappa \cdot [\theta^i + \varphi^i - z^i] = \gamma(\pi_0).$$

Summing over i and using the fact that commodity markets clear, asset markets clear and labor markets clear with free disposal, we must have

$$0 < \#I\gamma(\pi_0) = \kappa \cdot \sum_{i \in I} [\theta^i + \varphi^i - z^i] \leq 0,$$

which leads to a contradiction. □

Using the fact that the slack $\gamma(\pi_0)$ is zero, we can prove that commodity, asset and labor markets clear at $t = 0$.

Lemma A.5 *Commodity, asset and labor markets clear at $t = 0$, i.e.,*

$$\sum_{i \in I} x_0^i = \sum_{i \in I} e_0^i, \quad \sum_{i \in I} \theta^i = \sum_{i \in I} \varphi^i \quad \text{and} \quad \sum_{i \in I} \theta^i + \varphi^i = \sum_{i \in I} z^i.$$

Proof of Lemma A.5 It follows from (25) that

$$\widetilde{p}_0 \cdot A + \widetilde{q} \cdot B + \widetilde{\kappa} \cdot C \leq p_0 \cdot A + q \cdot B + \kappa \cdot C$$

for every $(\widetilde{p}_0, \widetilde{q}, \widetilde{\kappa})$ in Price_0 and where

$$A = \sum_{i \in I} [x_0^i - e_0^i], \quad B = \sum_{i \in I} [\theta^i - \varphi^i] \quad \text{and} \quad C = \sum_{i \in I} [\theta^i + \varphi^i - z^i].$$

Since the slack $\gamma(\pi_0)$ is zero, it follows from the first period budget constraint that

$$p_0 \cdot A + q \cdot B + \kappa \cdot C \leq 0$$

implying that

$$\forall (\widetilde{p}_0, \widetilde{q}, \widetilde{\kappa}) \in \mathrm{Price}_0, \quad \widetilde{p}_0 \cdot A + \widetilde{q} \cdot B + \widetilde{\kappa} \cdot C \leq 0.$$

Therefore, we must have that (A, B, C) belongs to the negative polar of Price_0, i.e.,

$$A = 0, \quad B = 0 \quad \text{and} \quad C \leq 0.$$

Since commodity markets clear at $t = 0$, the budget set restriction for the first period must be binding.[35] Therefore, we have

$$\kappa \cdot C = 0.$$

Remind that κ belongs to $\mathbb{R}^J_+$. Fix an asset j. If $\kappa_j > 0$ then we must have

$$\sum_{i \in I} \theta^i_j + \varphi^i_j - z^i_j = 0.$$

Assume next that $\kappa_j = 0$. Since the function $z \mapsto \Pi^i_0(x^i_0, z)$ is strictly decreasing, we must have $z^i_j = 0$ for every agent i. Since

$$0 \leq \sum_{i \in I} \theta^i_j + \varphi^i_j \leq \sum_{i \in I} z^i_j = 0$$

we get that the labor market for intermediation of asset j clears. □

Once we have proved that asset markets clear at the first period, it is straightforward and standard to prove that commodity markets clear at the second period. We refer to Martins-da-Rocha and Vailakis (2008) for details.

Lemma A.6 *For every possible state s at the second period $t = 1$, commodity markets clear, i.e.,*

$$\sum_{i \in I} x^i_1(s) = \sum_{i \in I} e^i_1(s).$$

In order to prove that $(\pi, \boldsymbol{a})$ is a competitive equilibrium we still have to prove that actions are optimal, i.e., $a^i \in d^i(\pi)$ for each agent i. Observe that (25) implies

$$\forall i \in I, \quad a^i \in d^i_\gamma(\pi). \tag{26}$$

The desired conclusion follows from Lemma A.4.

Appendix B: Assumptions on primitives

Recall that each agent (we omit the label i since there is no ambiguity) is endowed with

- a production technology (Y, F) where Y is the effort space and F is a correspondence from Y to $\mathbb{R}^J_+$;
- a utility function $U_0 : X_0 \times Y \to \mathbb{R}$ where X_0 is a non-empty subset of $\mathbb{R}^{L_0}_+$.

[35] See the argument used in the proof of Lemma A.4.

Denote by Z the space of possible productions defined as follows: $Z \equiv F(Y)$. Given these primitives, we can define the payoff function $\Pi_0 : X_0 \times Z \to [-\infty, \infty)$ as follows:

$$\Pi_0(x_0, z) \equiv \sup\{U_0(x_0, y) : y \in Y \quad \text{and} \quad z \in F(y)\}.$$

We propose to state explicitly the assumptions on production technology and preferences under which the assumptions imposed in Sect. 3 are satisfied.

Assumption 4.1 (a) The set X_0 is closed, convex and the function U_0 is continuous, concave, strictly increasing in consumption x_0 and strictly decreasing in effort y.

(b) For each asset j, there exists a non-empty convex subset Y_j of $[0, \infty)$ containing 0 such that Y is a closed, convex subset of $\prod_{j \in J} Y_j$ containing 0, satisfying a free-disposal property[36] and such that

$$Y \cap \mathbb{R}^J_{++} \neq \emptyset.$$

(c) For each asset j, there exists a continuous concave strictly increasing function $f_j : Y_j \to [0, \infty)$ with $f_j(0) = 0$ such that for any $y = (y_j)_{j \in J} \in Y$

$$F(y) = \prod_{j \in J} [0, f_j(y_j)].$$

For each asset j we denote by Z_j the set $f_j(Y_j)$. Under Assumption 4.1 the set Z_j is an interval of $[0, \infty)$ containing 0 and there exists a strictly increasing, continuous and convex function $h_j : Z_j \to Y_j$ satisfying

$$\forall z_j \in Z_j, \quad z_j = f_j \circ h_j(z_j) \quad \text{and} \quad \forall y_j \in Y_j, \quad y_j = h_j \circ f_j(y_j).$$

We denote by f the function from $\prod_{j \in J} Y_j$ to $\prod_{j \in J} Z_j$ defined by $f(y) \equiv (f_j(y_j))_{j \in J}$ and similarly we denote by h the function from $\prod_{j \in J} Z_j$ to $\prod_{j \in J} Y_j$ defined by $h(z) \equiv (h_j(z_j))_{j \in J}$.

Lemma B.1 *Assumption* 4.1 *implies Assumption* 2.4 *and Assumption* 2.5(b).

Proof of Lemma B.1 We first prove that Z satisfies Assumption 2.4. Since 0 belongs to Y and $f(0) = 0$ we get that 0 belongs to Z. Since f is concave and Y is convex we obtain that Z is convex. Since there exists $\widehat{y} \in \mathbb{R}^J_{++}$ that belongs to Y and each function f_j is strictly increasing, we get that $[0, \nu] \subset Z$ where $\nu = h(\widehat{y}) \in \mathbb{R}^J_{++}$. We still have to prove that Z is closed. Let $(z^n)_{n \in \mathbb{N}}$ be a sequence in Z converging to $z \in \mathbb{R}^J_+$. By definition of F for each n there exists $y^n \in Y$ such that $z^n \leq f(y^n)$. Moreover there exists ζ^n in $\prod_{j \in J} Y_j$ such that $h(z^n) = \zeta^n$. Observe that we have $\zeta^n \leq y^n$ and since Y satisfies a free-disposal property, we get that $\zeta^n \in Y$. The continuity of h implies that the sequence $(\zeta^n)_{n \in \mathbb{N}}$ converges to $h(z)$. Since Y is closed the vector $h(z)$ belongs to Y, implying that z belongs to Z.

[36] That is, if y belongs to Y then any vector y' satisfying $0 \leq y' \leq y$ also belongs to Y. For instance, one may have that Y is the set of all $(y_j)_{j \in J}$ in $\prod_{j \in J} Y_j$ satisfying $\sum_{j \in J} y_j \leq \overline{y}$ for a given $\overline{y} > 0$.

It follows from Assumption 4.1 that

$$\forall z=(z_j)_{j\in J}\in Z=F(Y),\quad \Pi_0(x_0,z)=U_0(x_0,h(z))\quad \text{where}\quad h(z)\equiv(h_j(z_j))_{j\in J}.$$

It is now straightforward to check that Π_0 satisfies Assumption 2.5(b). □

One may consider the following representation of the utility function U_0:

$$\forall(x_0,y)\in X_0\times Y,\quad U_0(x_0,y)\equiv V_0(x_0)-G(y)\quad \text{with}\quad G(y)=\sum_{j\in J}G_j(y_j),$$

where V_0 is continuous, strictly increasing and concave and for each j the function G_j is continuous, strictly increasing, convex and satisfies $G_j(0)=0$. Assuming that X_0 is closed, convex we get that Assumption 4.1(a) is satisfied. Under conditions (b) and (c) of Assumption 4.1 we can apply the arguments of the proof of Lemma B.1 to get that

$$\forall(x_0,z)\in X_0\times Z,\quad \Pi_0(x_0,z)=V_0(x_0)-E(z)\quad \text{with}\quad E(z)=\sum_{j\in J}E_j(z_j),$$

where the function E_j is defined by

$$E_j\equiv G_j\circ h_j.$$

Since h_j is continuous, strictly increasing and convex we get that Π_0 has a representation similar to the one introduced in Definition 3.3.

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