Morohogénèse cardiaque

Cardiac morphogenesis

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Abstract

The master thesis present a modelisation for the septation of the atrio−ventricular tube. We will consider that the cell proliferation is due only to shear stress exerted by the blood flow inside the tube. We obtain, under some assumptions, a two dimensional free boundary evolution model of the type Hele Shaw, where the velocity of the boundary depend nonlinearly on the normal derivative of the velocity of the blood.

First, we present the mathematical model then we give some numerical results.

At the beginning of the cardiac morphogenesis, the heart is a tube. During its development, this tube bends and its cross-section is no longer circular. We have to note that during the formation of the heart, many kind of septation occurs.

The purpose of this master thesis is to present a mathematical model suitable for describe the septation of the atrio−ventricular tube.
We consider that the cell proliferation is due to shear stress exerted by the blood flow. We note that many experimental studies prove that shear stress play an important role on the development of the embryonic heart [1,2].
We suppose that the cardiac tube is cylindric with elliptic section . Let us denote $\mathbf{e}_z$ the unit vector along the centerline direction, and the velocity field $\mathbf{u} = u \cdot \mathbf{e}_z$ is invariant by translation along $\mathbf{e}_z$. Blood flow is mathematically modeled using Stokes equation for an incompressible flow. We consider that the pressure of the blood is constant in each section, supose that we have $\lambda \in \mathbb{R}$ is related to the pressure by $\lambda = -\nabla p$.

Mathematical formulation

We denote by $\Omega(t)$ the domain at time t, with boundary $\Gamma(t)$, and by $n(\gamma, t)$ the outside normal vector to $\Gamma(t)$ at $\gamma \in \Gamma(t)$. The motion of Γ is determined by a normal velocity field $V(\gamma, t)$, so that considering

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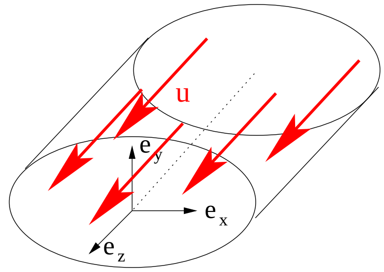

$$\begin{cases} \dfrac{\partial \gamma}{\partial t}(\gamma_0, t) &= V(\gamma(\gamma_0, t), t) n(\gamma(\gamma_0, t), t), \\ \gamma(\gamma_0, 0) &= \gamma_0 \quad \gamma_0 \in \Gamma(0). \end{cases} \tag{1}$$

The set Γ can be written as

$$\begin{cases} \Gamma(t) &= \{\gamma(\gamma_0, t), \gamma_0 \in \Gamma(0)\}, \\ &= \{\gamma_0 + \int\limits_0^t V(\gamma(\gamma_0, t), t) n(\gamma(\gamma_0, t), t) dt, \gamma_0 \in \Gamma(0)\}. \end{cases} \tag{2}$$

The velocity of the boundary $V(\gamma, t)$ depend on the shear stress exerted by the blood flow. the shear stress can be writen $\dfrac{\partial u}{\partial n}$ at γ, where the velocity u solves the system

$$\begin{cases} -\Delta u &= \lambda \text{ on } \Omega(t), \\ u &= 0 \text{ on } \partial\Omega(t). \end{cases} \tag{3}$$

The normal velocity V can be written

$$V(\gamma(\gamma_0, t), t) = \frac{1}{\tau} \int\limits_0^t e^{-\frac{t-s}{\tau}} \psi\left(\frac{\partial u(\gamma(\gamma_0, s), s)}{\partial n}\right) ds,$$

τ is the typical duration of activity induced by shear stress and ψ is a function defined as below

$$\psi(X) = \frac{k}{2}\left(tanh\left(\frac{X - \delta_0}{\varepsilon}\right) - 1\right),$$

where $\varepsilon > 0$ is the stiffness, and k is the maximal velocity.

1. Numerical result

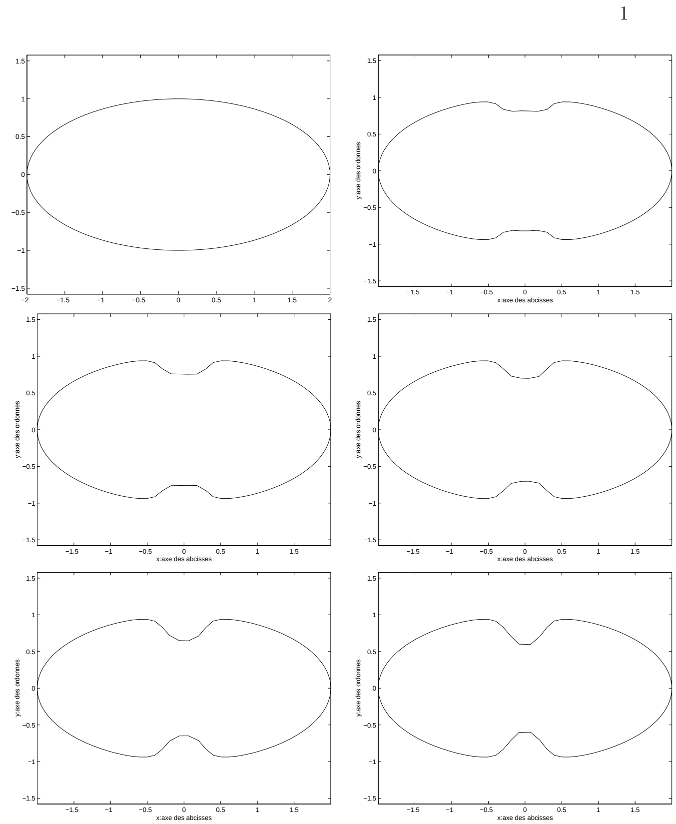

Figure 1. Evolution of the section of the atrio-ventricular tube

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[2] C.G.DeGroff, B.L.Thornburg, J.O Pentecost, K.L.Thornburg, M.Gharib, D.J.Shan, A.Baptista, *Flow in the early embryonic human heart: a numerical study*, Pediatric Cardiology. 24 (2003) 375 - 380 .