Solution 10. *Let us denote by μ the probability law under consideration and μ_n the law such that $\mu = \mu_n^{*n}$.*

(1) $\mu_n = \mathcal{N}(\frac{m}{n}, \frac{\sigma^2}{n})$;
(2) $\mu_n = \mathcal{P}(\lambda/n)$;
(3) *pick μ_n as the law of a compound Poisson process with intensity λ/n and jump distribution ν, evaluated at time 1;*
(4) *pick $\mu_n = \Gamma(\frac{a}{n}, b)$ (when $k \in \mathbb{N}$ recall that $\Gamma(k, b)$ is the law of the sum of k i.i.d. $\mathcal{E}(b)$ random variables);*
(5) $\mu_n = \delta_{a/n}$.

Solution 11. *Let X be a $\mathrm{Ber}(p)$ random variable. Suppose that $X = Y_1 + Y_2$ with Y_1 and Y_2 independent and identically distributed. Prove that necessarily $\mathrm{P}(Y_1 = 1/2) = \sqrt{p}$ and $\mathrm{P}(Y_1 = 0) = \sqrt{1-p}$. This is impossible when $p \in (0,1)$. See Example 9 in [11] for a full solution.*

Solution 12. (1) $\Psi(u) = ium - \frac{u^2\sigma^2}{2}$;
(2) $\Psi(u) = \lambda(e^{iu} - 1)$;
(3) $\Psi(u) = \lambda \int (e^{iuz} - 1)\nu(\mathrm{d}z)$;
(4) $\Psi(u) = -a\log(1 - i\frac{u}{b})$. *First compute it when $a \in \mathbb{N}$ using the interpretation of $\Gamma(a, b)$ as the sum of independent exponential variables, then extend the formula to $a \in \mathbb{Q} \cap (0, +\infty)$ using infinite divisibility. To extend it to $a > 0$, verify that for all $u \in \mathbb{R}$ the mapping $a > 0 \mapsto \int_0^\infty e^{iut} t^{a-1} e^{-t} \mathrm{d}t$ is continuous (by dominated convergence).*
(5) $\Psi(u) = iua$.

Solution 13. *For any fixed $u \in \mathbb{R}^d$, the function $z \mapsto e^{i\langle u,z\rangle} - 1 - i\langle u, z\rangle 1_{\{|z|\le 1\}}$ is bounded (in modulus) by some constant times $1 \wedge |z|^2$. Since ν is a Lévy measure, the integral is well-defined.*

Solution 14. *First of all, the assumption that ν is finite allows us to split in two the integral in (3.2) and proves that c is well-defined. Moreover,*

$$\begin{aligned} \phi_Y(u) &= \exp\Big(-\frac{1}{2}\langle u, Au\rangle\Big), \\ \phi_{\widetilde{Y}}(u) &= \exp\Big(\int (e^{i\langle u,z\rangle} - 1)\nu(\mathrm{d}z)\Big) \qquad \textit{(see Exercise 4)}. \end{aligned} \tag{A.98}$$

We conclude by using the independence of Y and $\widetilde{Y}$.