

PROPOSITIONS DE STAGES DE M2

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The research internship will be about **polymer models and/or local times**. Below is a short explanation of these two terms.

(i) *Random polymer models* belong to the field of Probability Theory and Statistical Mechanics but their motivations often come from Biology and Physics. The goal is to investigate the typical conformations (shapes) of a polymer (macromolecule) and possible phase transitions (drastic changes due to a modification of parameters such as temperature). One common approach is to model the polymer by the path of a simple random walk $S = (S_n)_{1 \leq n \leq N}$ in \mathbb{Z}^d under the polymer (probability) measure $P_{N,\beta}$ defined by:

$$(0.1) \quad \frac{dP_{N,\beta}}{dP}(S) = \frac{\exp(-\beta H_N(S))}{Z_{N,\beta}}$$

where P is the law of simple random walk, β is the inverse temperature and H (for Hamiltonian) is a function which gives the energy of each path and the expression of which depends on the phenomenon under study. Sometimes the Hamiltonian depends on an extra randomness ω which accounts for inhomogeneities of the solvent or of the polymer constituents (heteropolymer). Here are two examples of such models which still present many open questions :

(1) Charged polymer [1]: the energy of a path is given in terms of its self-intersections

$$(0.2) \quad H_N(S, \omega) = \sum_{1 \leq i, j \leq N} \omega_i \omega_j \mathbf{1}\{S_i = S_j\},$$

where $\omega = (\omega_i)_{i \geq 1}$ stands for the charges along the polymer and is drawn as a sequence of i.i.d. real random variables independent from S . The polymer may attract or repel itself depending on the charge distribution, leading to a phase transition between a collapsed and an extended phase.

(2) Directed polymer among many repulsive interfaces [2]: in this one-dimensional model, the energy of a path is given in terms of the number of contacts with the interface

$$(0.3) \quad H_N(S, \tau) = \sum_{1 \leq i \leq N} \mathbf{1}\{S_i \in \tau\},$$

where $\tau = \{\tau_n\}_{n \geq 0}$ is a random set of \mathbb{Z} (the positions of the interfaces). Here, the polymer tends to localize in a region free from interfaces.

(ii) *Local times* count how many times a random process visits a particular element of its state space. For instance,

$$(0.4) \quad \ell_n(x) = \text{card}\{1 \leq i \leq n : S_i = x\}, \quad x \in \mathbb{Z}^d,$$

for a random walk which is discrete in time and space. These quantities appear naturally in several polymer models, hence the relevance of their study (representations of their law, probability of large deviations [3], etc).

The internship is intended for a second-year master student in mathematics with a good background in Probability Theory (Markov chains, martingales, random walks).

REFERENCES

- [1] F. Caravenna, F. den Hollander, N. Pétrelis and J. Poisat, Annealed scaling for a charged polymer, *Mathematical Physics, Analysis and Geometry* 19 (2016), Article 2, 1–87.
- [2] J. Poisat and F. Simenhaus, A limit theorem for the survival probability of a simple random walk among power-law renewal traps, <https://arxiv.org/abs/1809.08866>.
- [3] Donsker, M. D. and Varadhan, S. R. S., Asymptotic evaluation of certain Markov process expectations for large time. I-IV, *JOURNAL = Comm. Pure Appl. Math.*, VOLUME = 28, 29, 36,