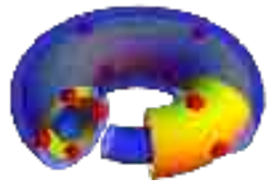
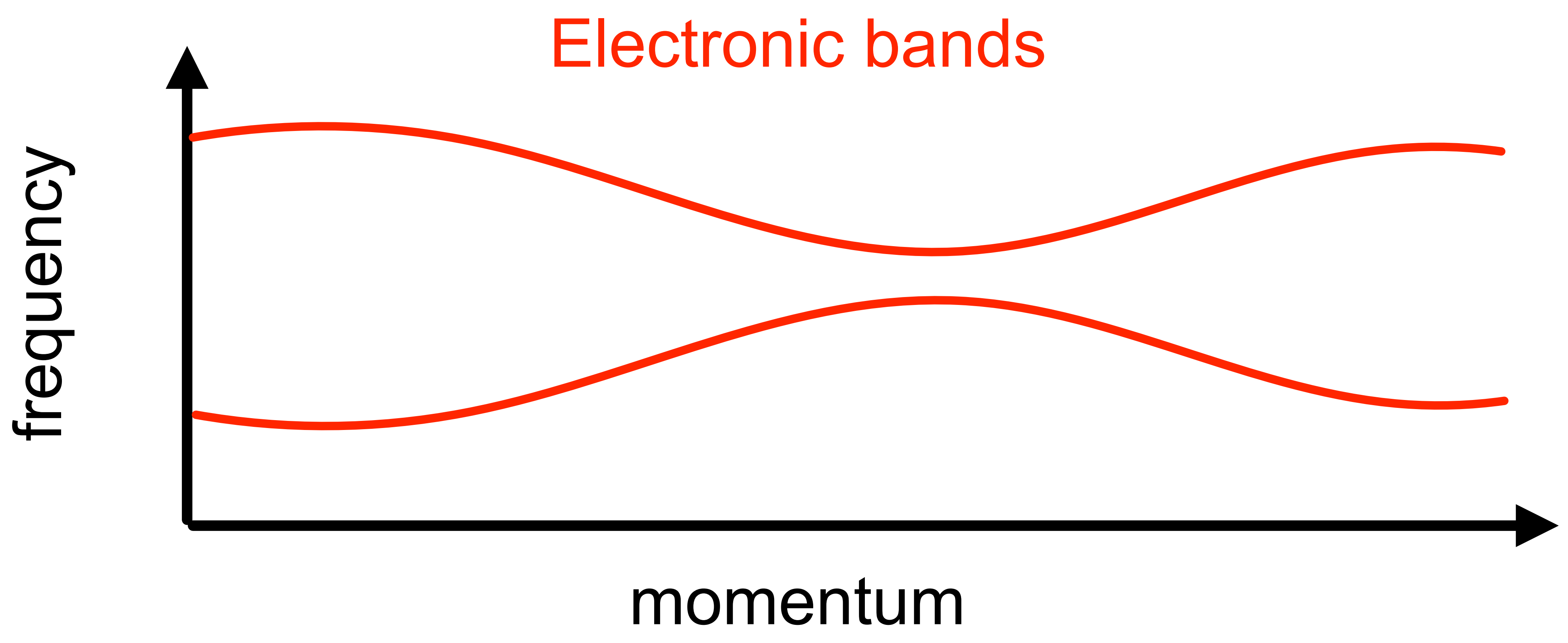
Topological Mechanical Metamaterials

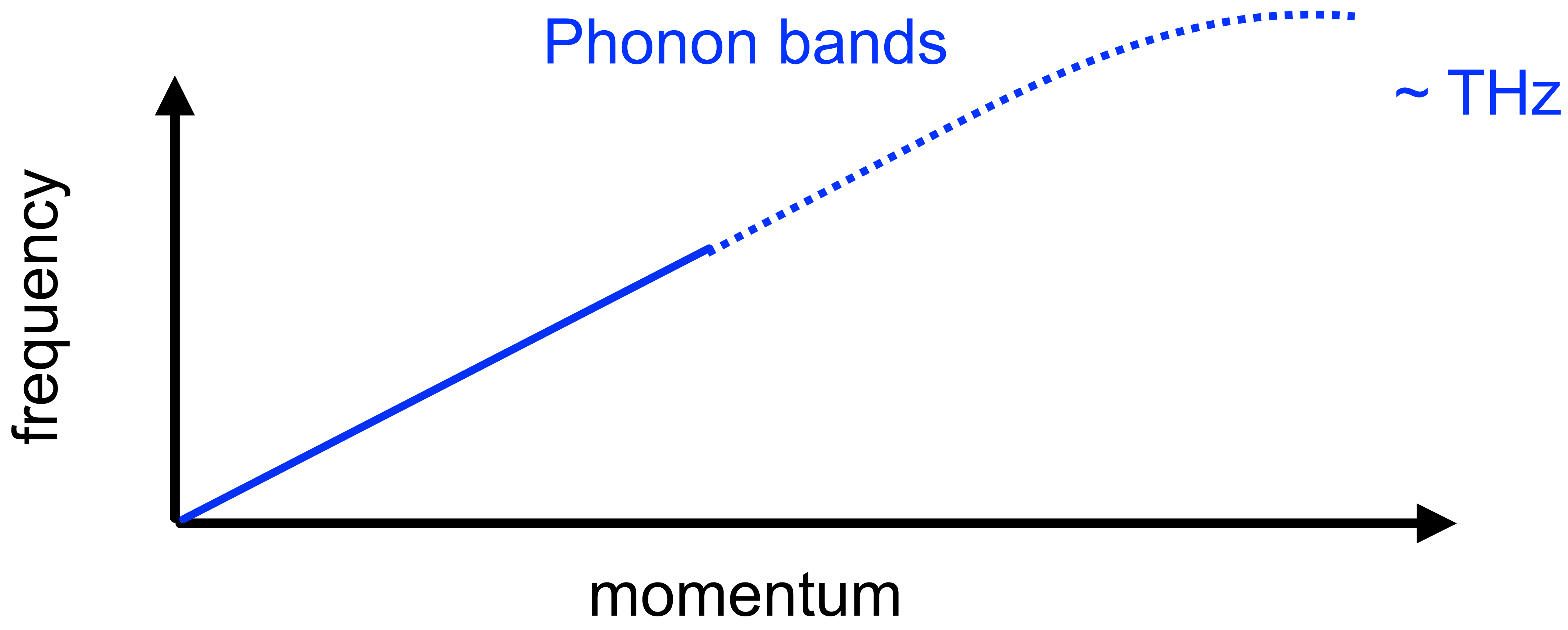
Sebastian Huber

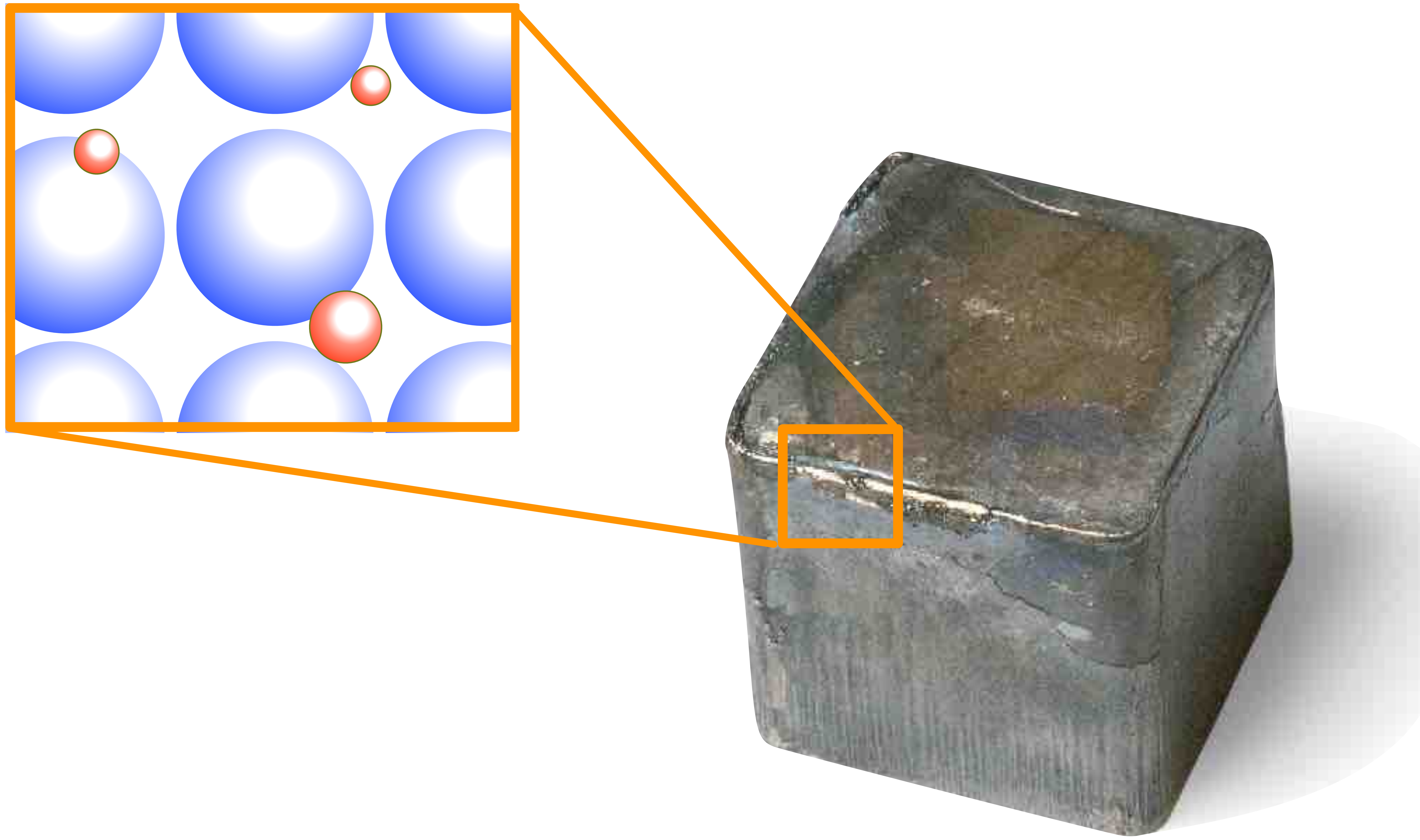
Nature Materials (2018)
Nature (2018)
Nature Physics (2016)



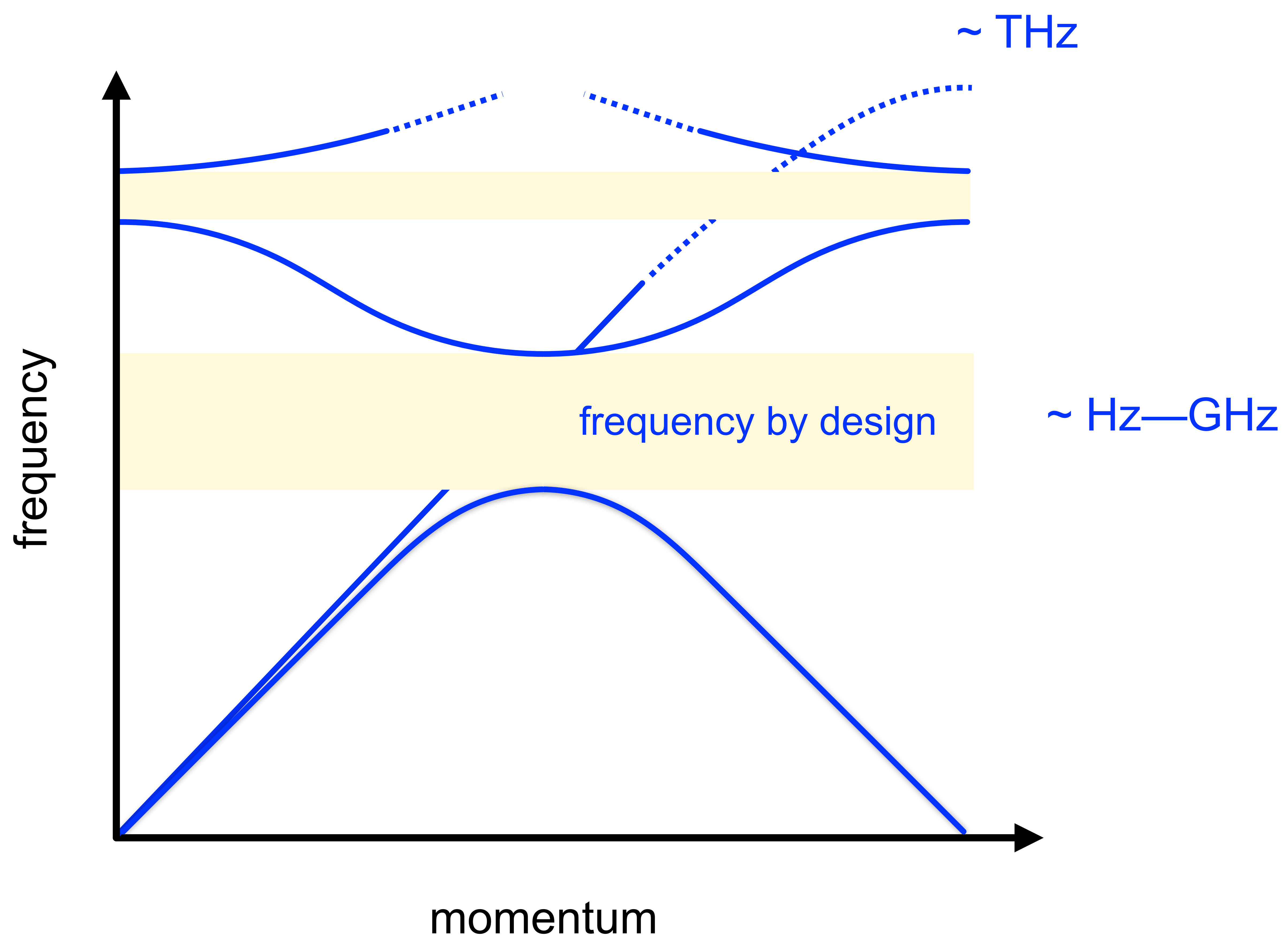
A mechanical metamaterial: tailoring the phonons to our liking





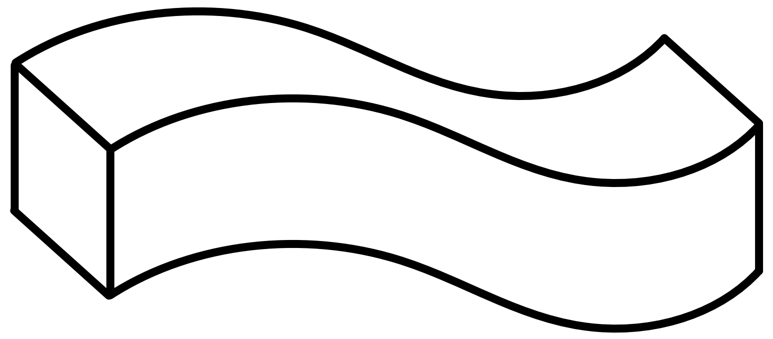


A mechanical metamaterial: tailoring the phonons to our liking



Flavours of topological mechanics

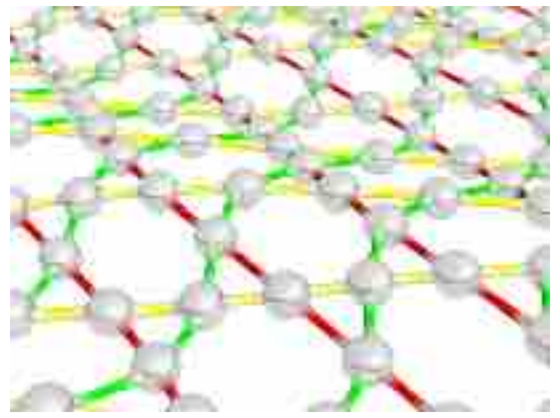
Elastic waves



$$\rho\partial_t^2 u_m(\mathbf{x}) = \frac{\partial \tau_{mk}}{\partial x_k}$$

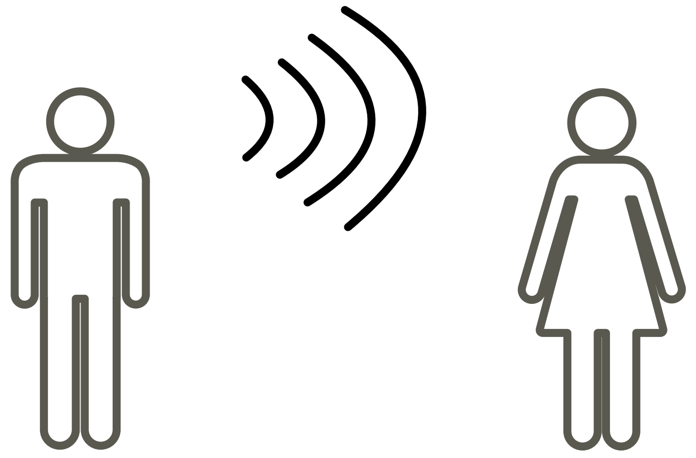
$$\tau_{mk}(\mathbf{x}) = c_{kmij}\left[\partial_{x_i} u_j(\mathbf{x}) + \partial_{x_j} u_i(\mathbf{x})\right]$$

Mass-spring models



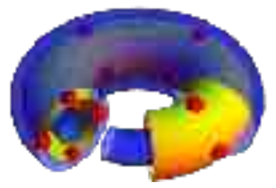
$$m\partial_t^2 u_i^\alpha = -D_{ij}^{\alpha\beta} u_j^\beta$$

Acoustic waves



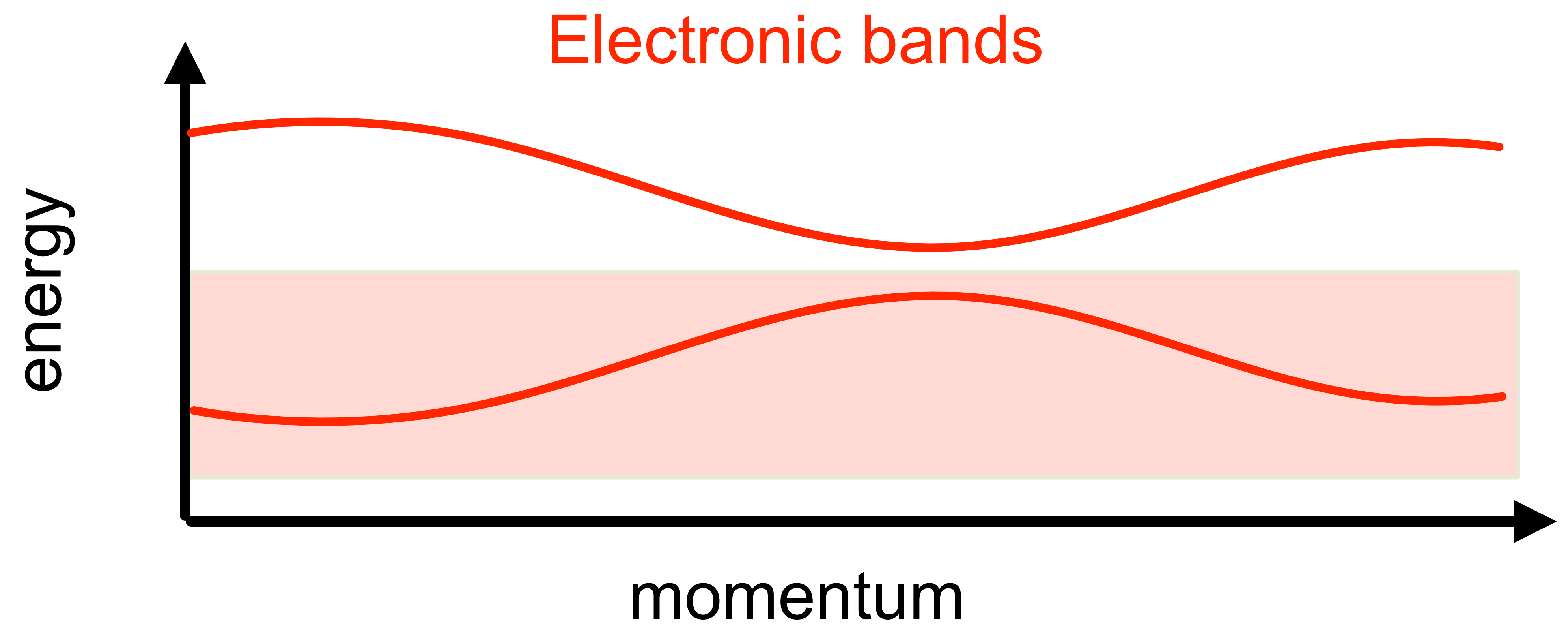
$$\partial_t^2 \rho(\mathbf{x}) = c^2 \nabla^2 \rho(\mathbf{x})$$

SDH Nature Phys. (2016)

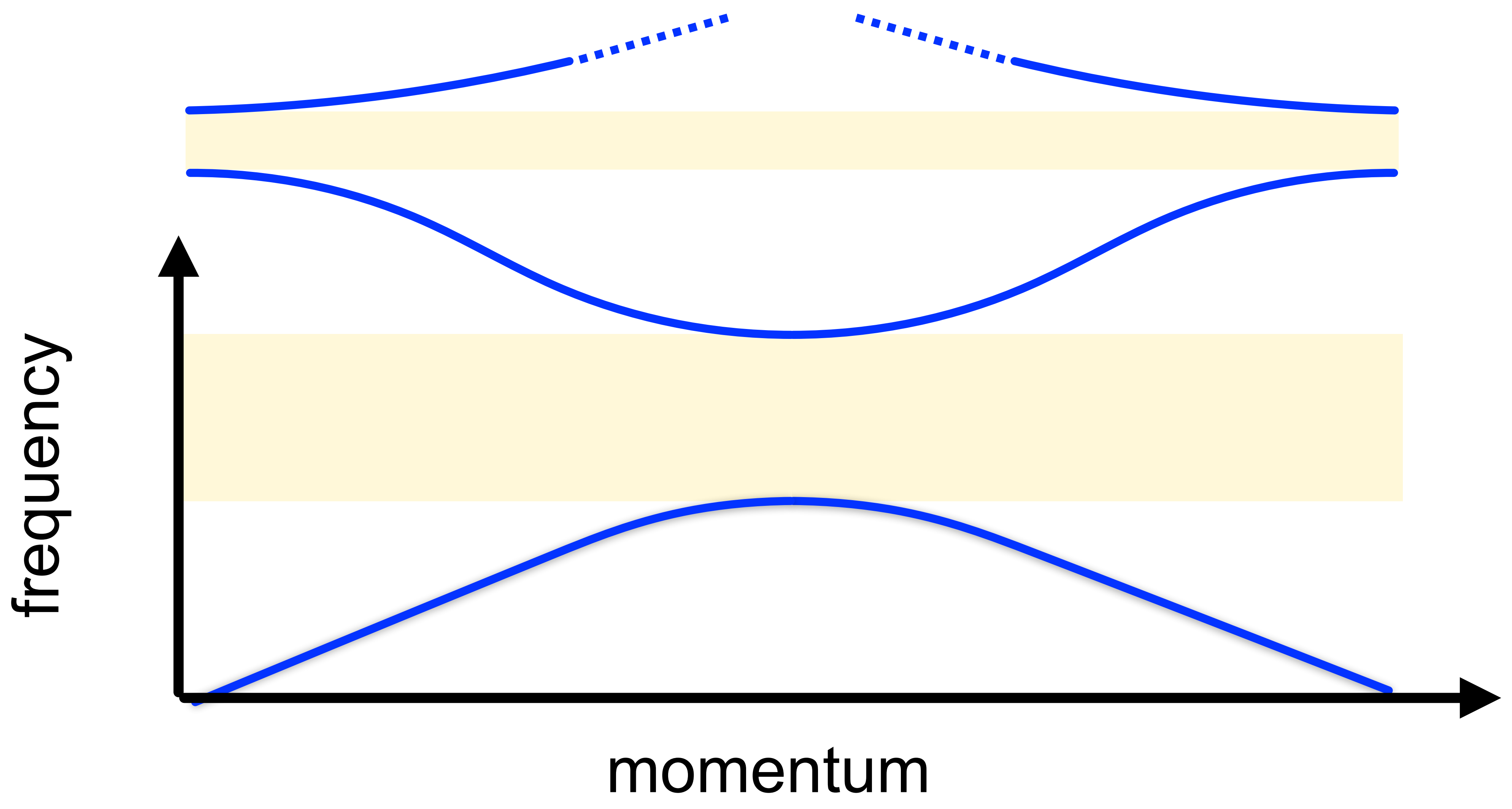


From electrons to phonons

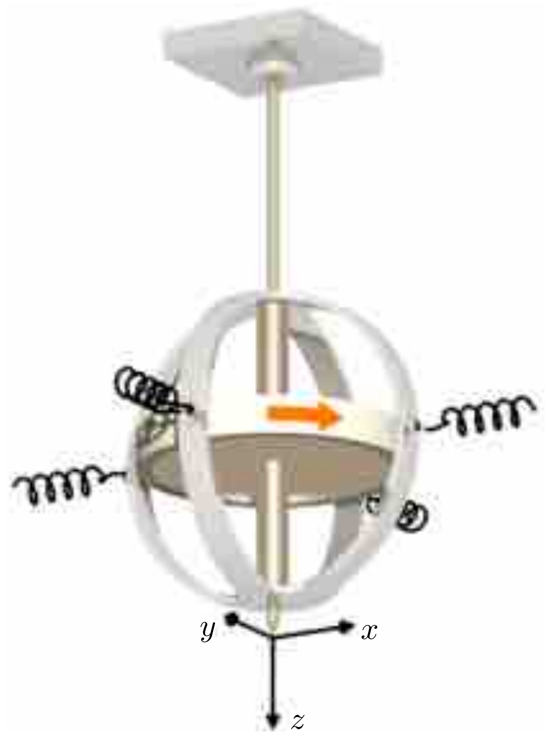
$$i\hbar\partial_t\psi = H\psi \quad \Rightarrow \quad \psi_{\rm gs}(\theta_x,\theta_y)$$



$$\partial_t^2 x = -Dx \quad \Rightarrow$$



A more concrete mapping



A conservative discrete system

$$\ddot{x}_i(t) = -D_{ij}\, x_j(t) + \Gamma_{ij}\, \dot{x}_j(t) \qquad D^{\mathrm{T}} = D, \quad D > 0, \quad \Gamma^{\mathrm{T}} = -\Gamma$$

First order time derivative

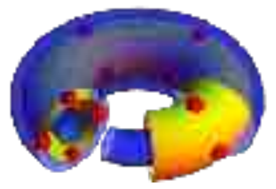
$$\frac{d}{dt}\begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} = \begin{bmatrix} 0 & \mathbb{1} \\ -D & \Gamma \end{bmatrix}\begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} \quad \Rightarrow \quad \boldsymbol{\psi} = \begin{bmatrix} Q^{\mathrm{T}} & 0 \\ 0 & i\mathbb{1} \end{bmatrix}\begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix}$$

$$D = Q^{\mathrm{T}} Q$$

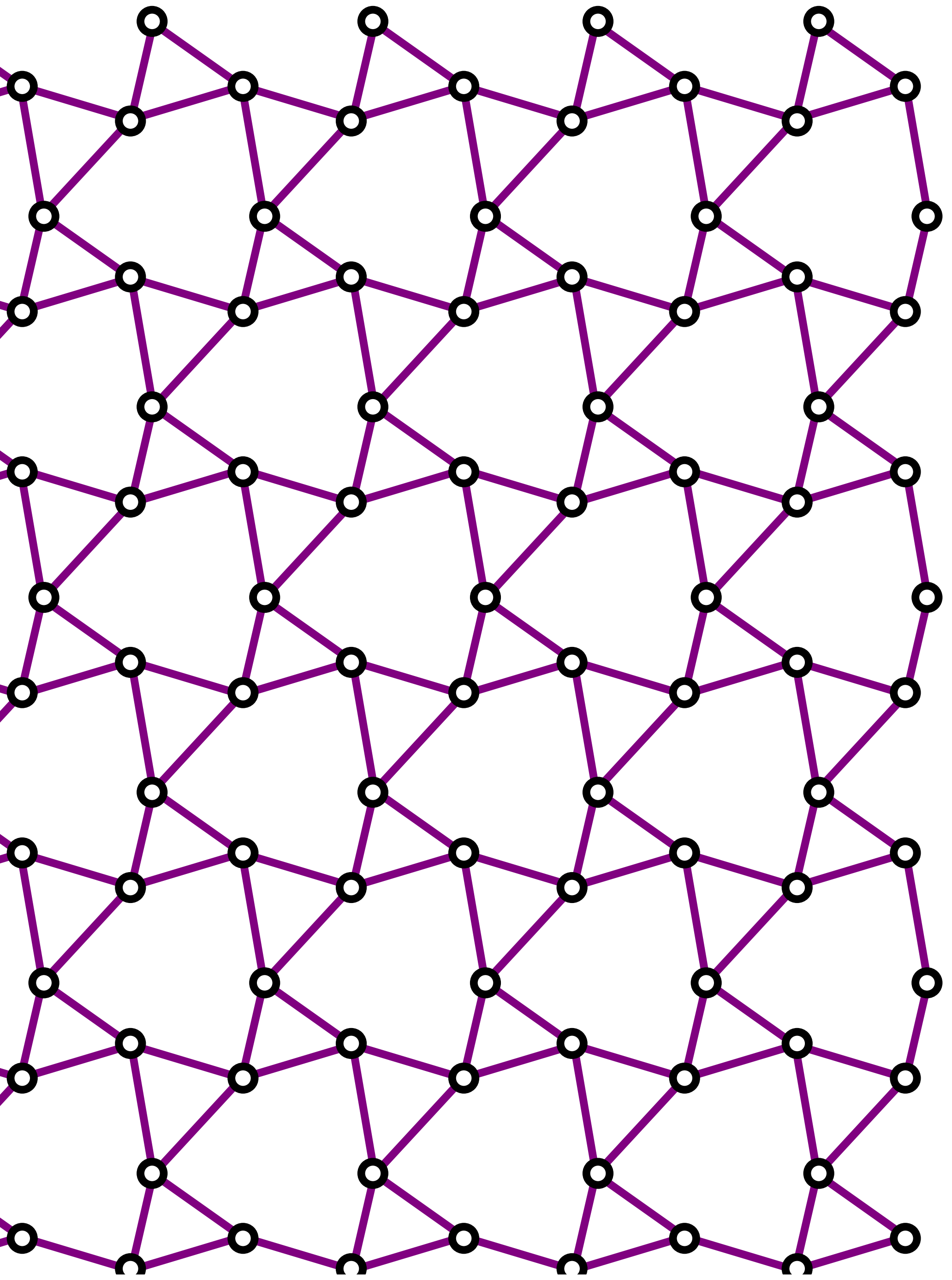
Formally: a BdG-type problem

$$i\frac{d}{dt}\boldsymbol{\psi} = \underbrace{\begin{bmatrix} 0 & Q^{\mathrm{T}} \\ Q & i\Gamma \end{bmatrix}}_{H}\boldsymbol{\psi} \quad \Rightarrow \quad U_{\mathcal{C}} = \begin{bmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{bmatrix} K \qquad H U_{\mathcal{C}} + U_{\mathcal{C}} H = 0$$

Kane & Lubensky, Nature Phys (2013)
Süsstrunk and SDH, PNAS 2016

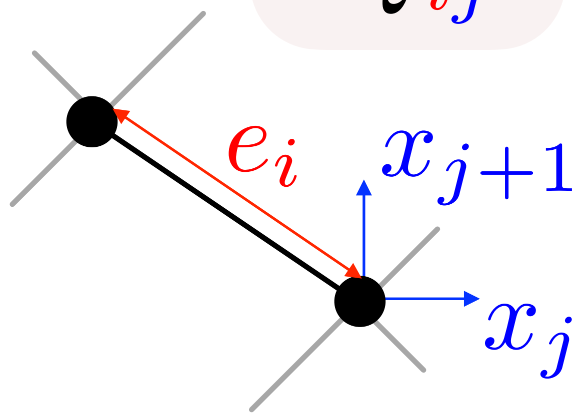


Half the dynamical matrix



Isostatic lattices

Q_{ij}



$$D = Q^{\mathrm{T}} Q$$

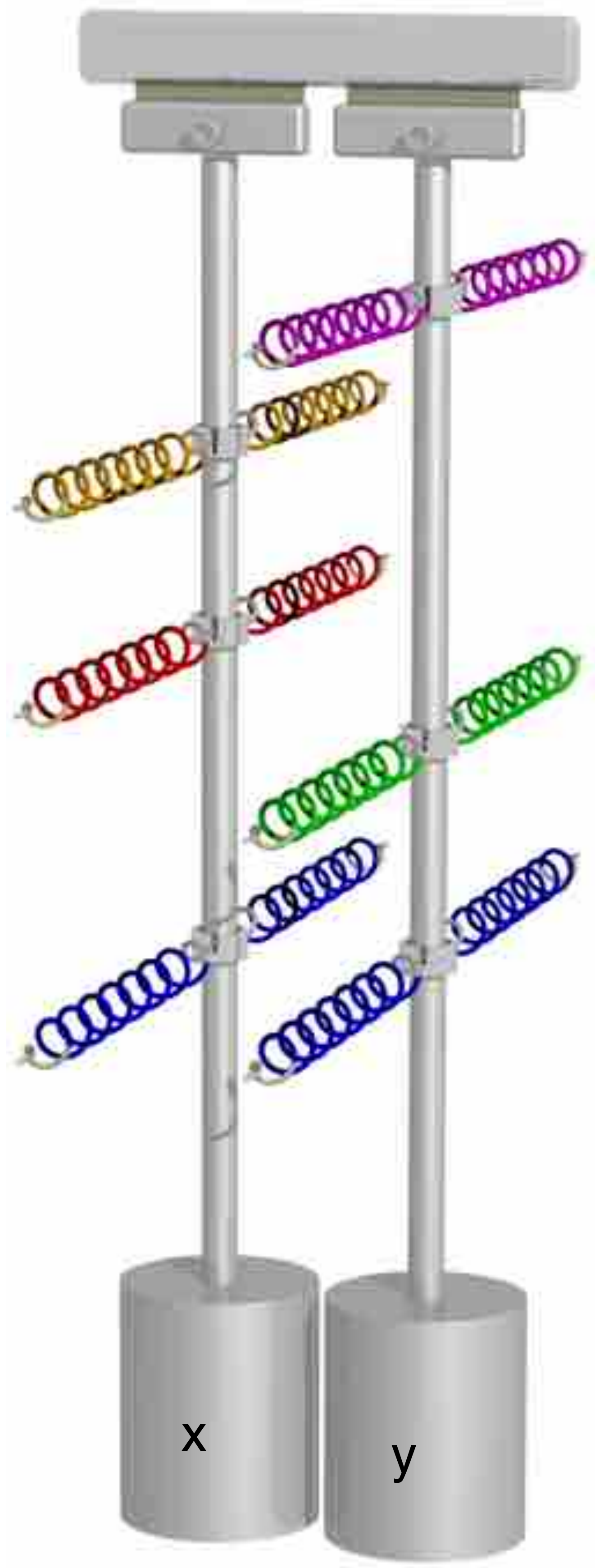
$$Q^{\mathrm{T}} \neq Q$$

Generic lattices

$$D > 0 \;\Rightarrow$$

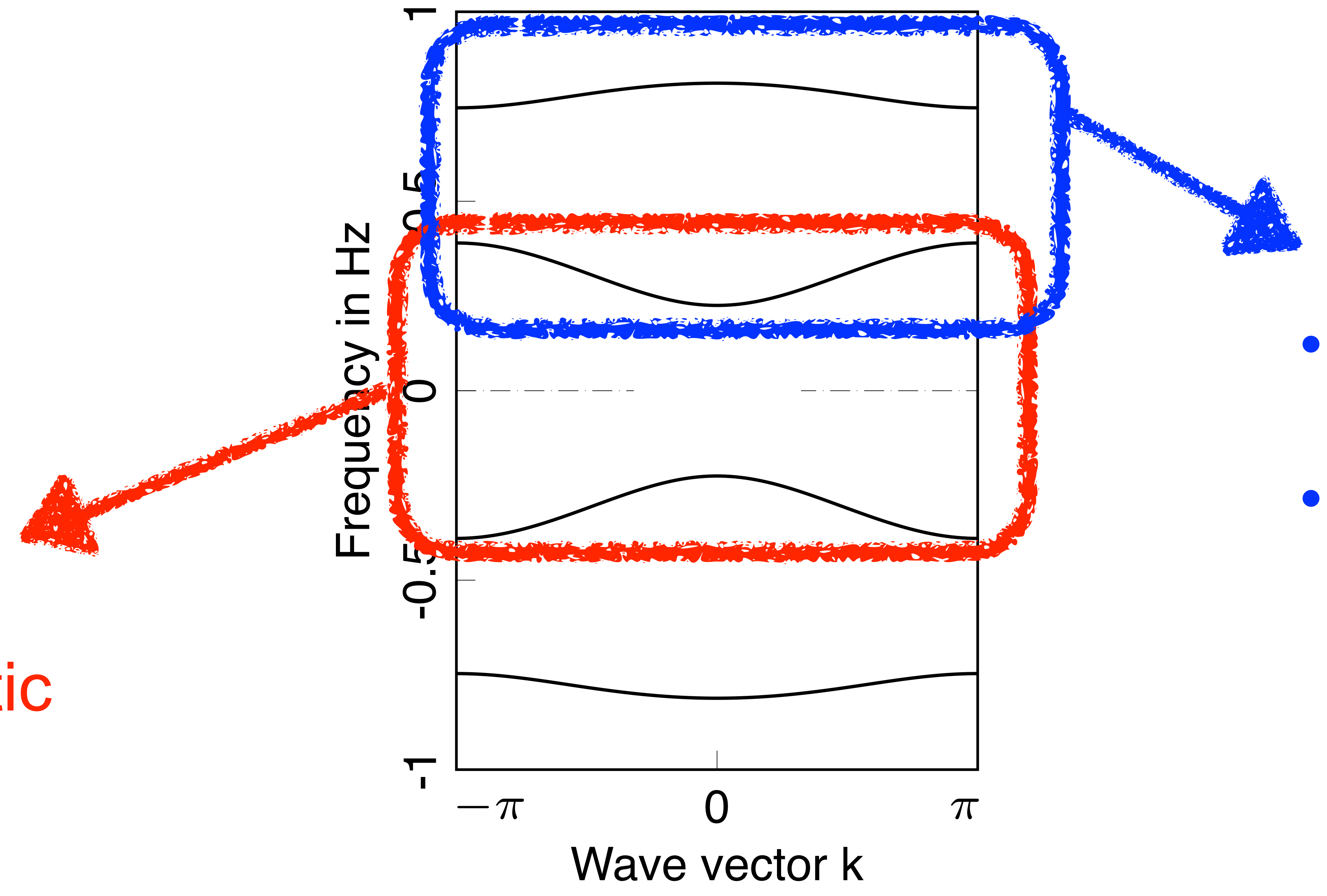
$$Q = \sqrt{D}$$

$$Q^{\mathrm{T}} \equiv Q$$



Kane & Lubensky, Nature Phys (2013)
Süsstrunk and SDH, PNAS 2016

Generic dispersion: flavors of TI's



- Isostatic lattices
- Zero-frequency: static properties
  - Floppy modes
  - States of self-stress

$$i\frac{d}{dt}\psi = \begin{bmatrix} 0 & Q^{\mathrm{T}} \\ Q & i\Gamma \end{bmatrix}\psi$$

$$D = Q^{\mathrm{T}}Q$$

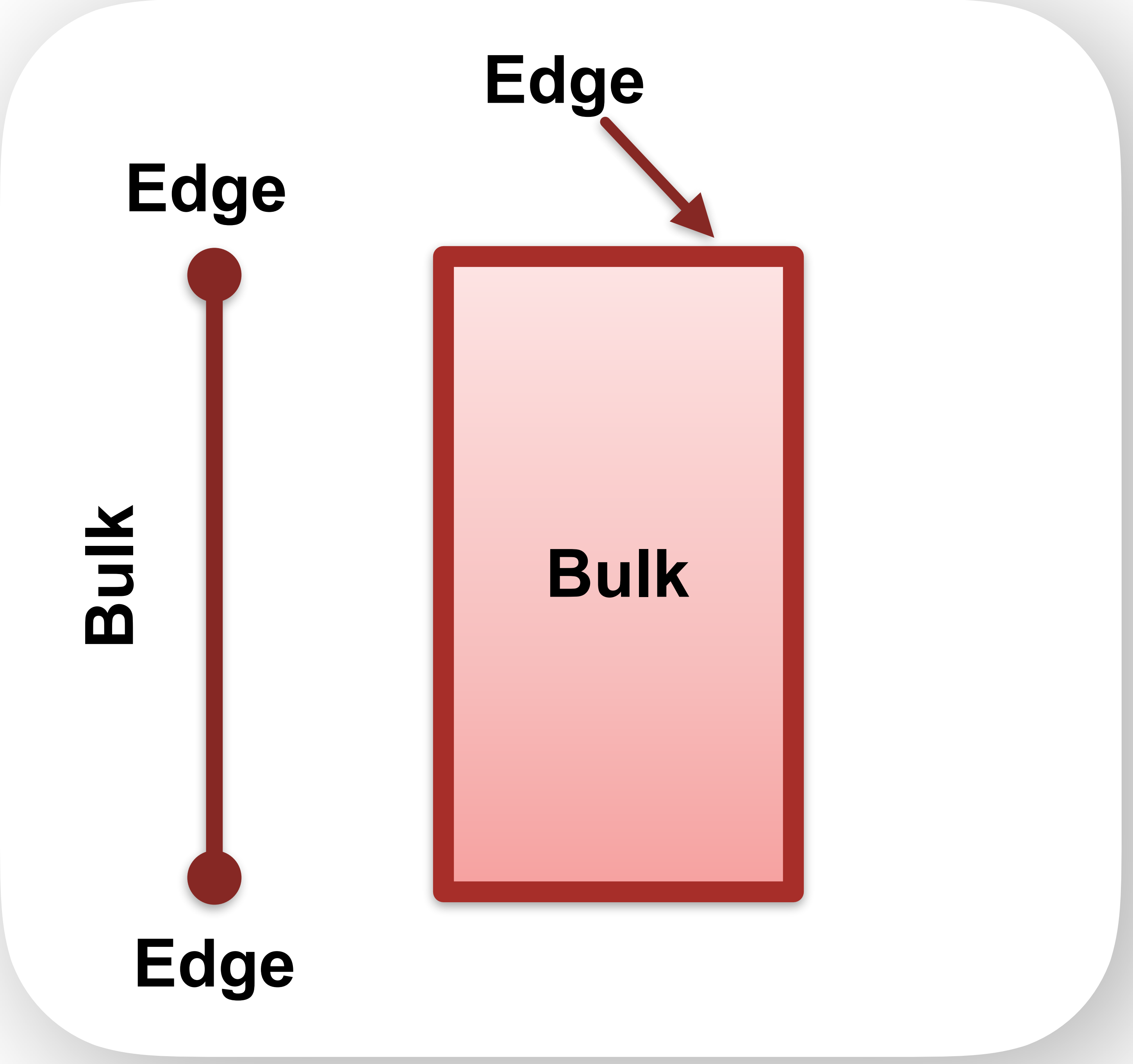
- Generic lattices
- High-frequency properties
  - wave guides
  - vibration isolation
  - heat-diodes

Kane & Lubensky, Nature Phys (2013)
Lubensky et al., Rep. Prog. Phys. (2015)

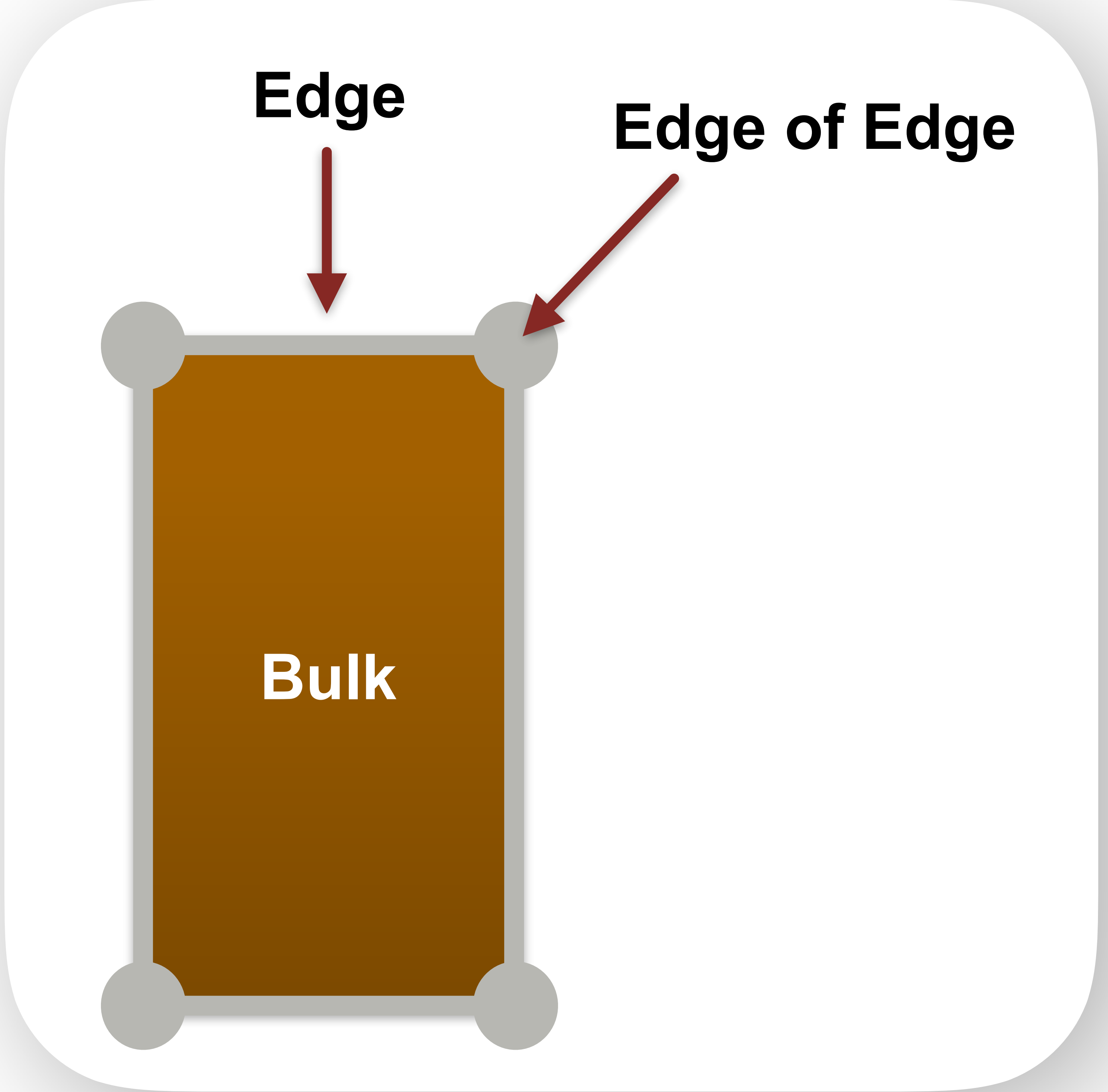
Süsstrunk and SDH, PNAS 2016

A selection of topological effects

Separate channels in real space



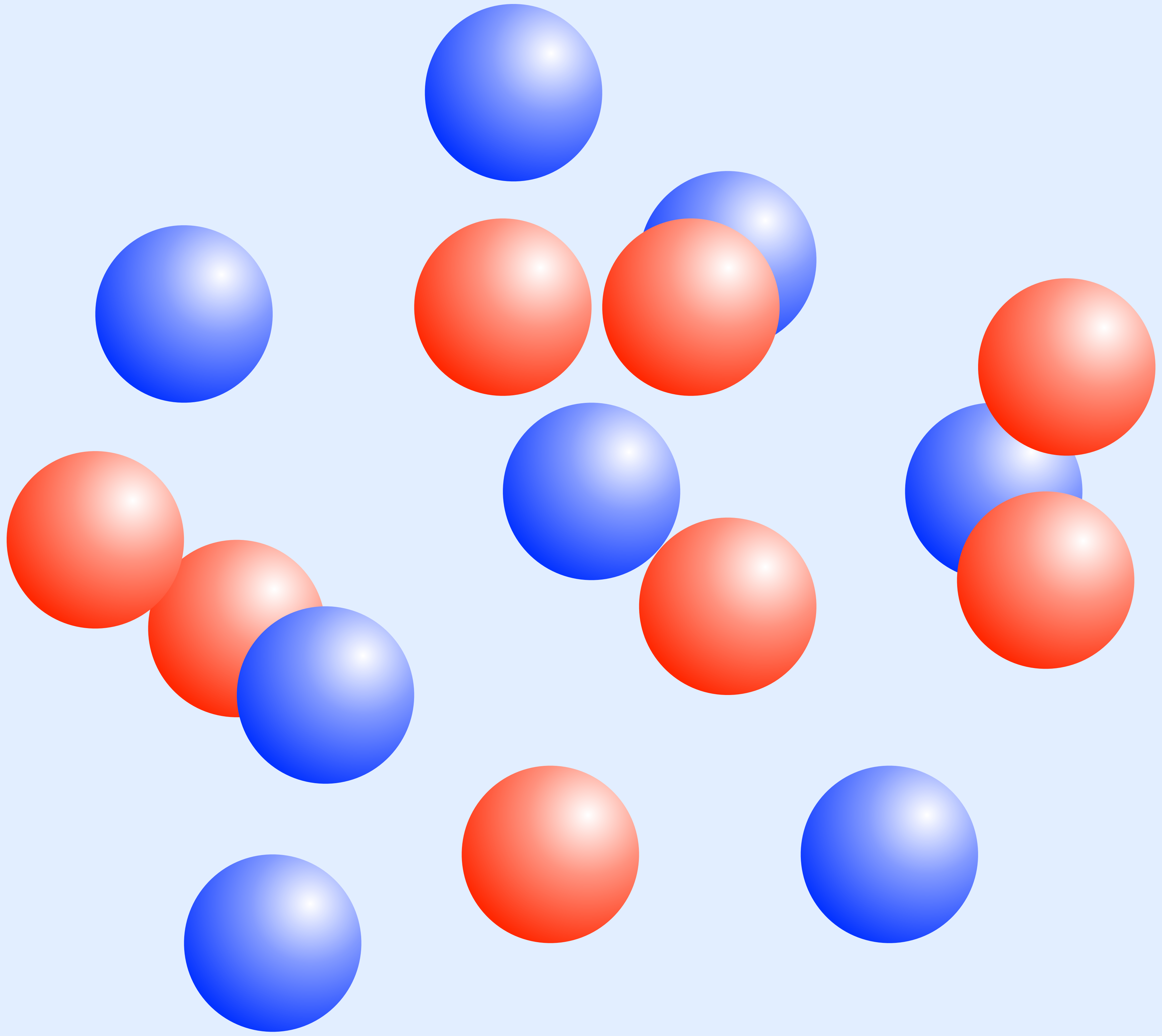
Separate them even "further"

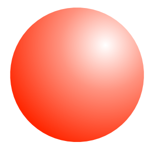


Multipole expansion

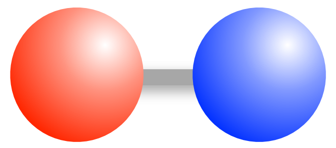
$$\varphi(\mathbf{r}) = \frac{1}{4\pi}\int \mathbf{d}\mathbf{r}' \, \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

$$\varphi(\mathbf{r}) = \frac{1}{4\pi}\frac{Q}{r} + \frac{1}{4\pi}P_i\frac{r_i}{r^3} + \frac{1}{4\pi}Q_{ij}\frac{3r_ir_j - r^2\delta_{ij}}{2r^5} + \cdots$$

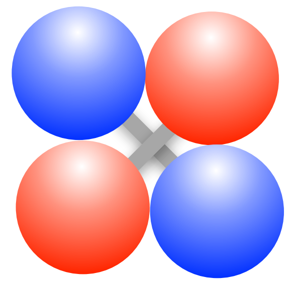




$$Q = \int \mathbf{d}\mathbf{r}\,\rho(\mathbf{r})$$ Monopole



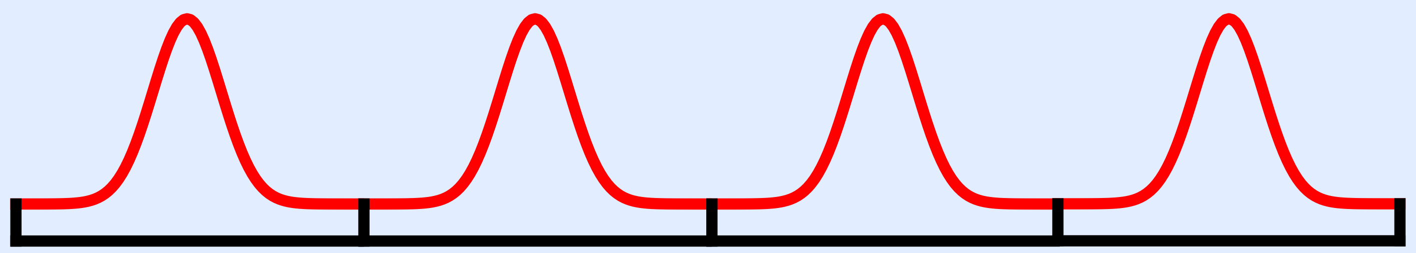
$$P_i = \int \mathbf{d}\mathbf{r}\, r_i\rho(\mathbf{r})$$ Dipole



$$Q_{ij} = \int \mathbf{d}\mathbf{r}\, r_ir_j\rho(\mathbf{r})$$ Quadrupole

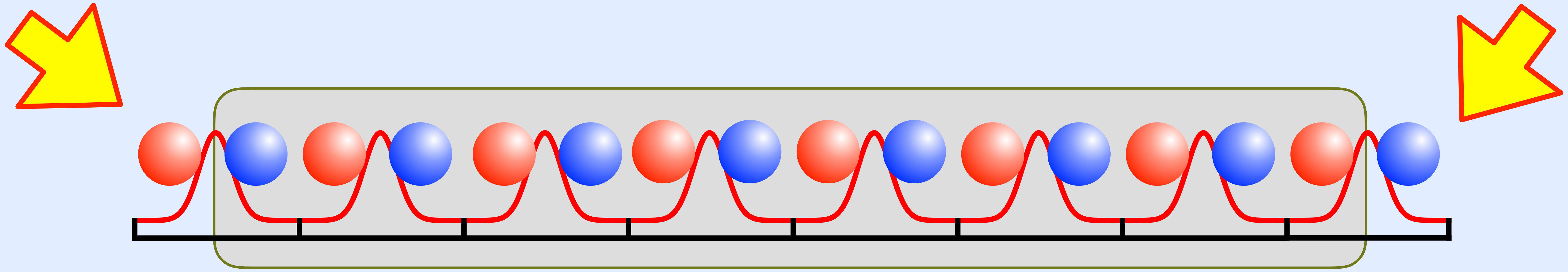
Dipole density

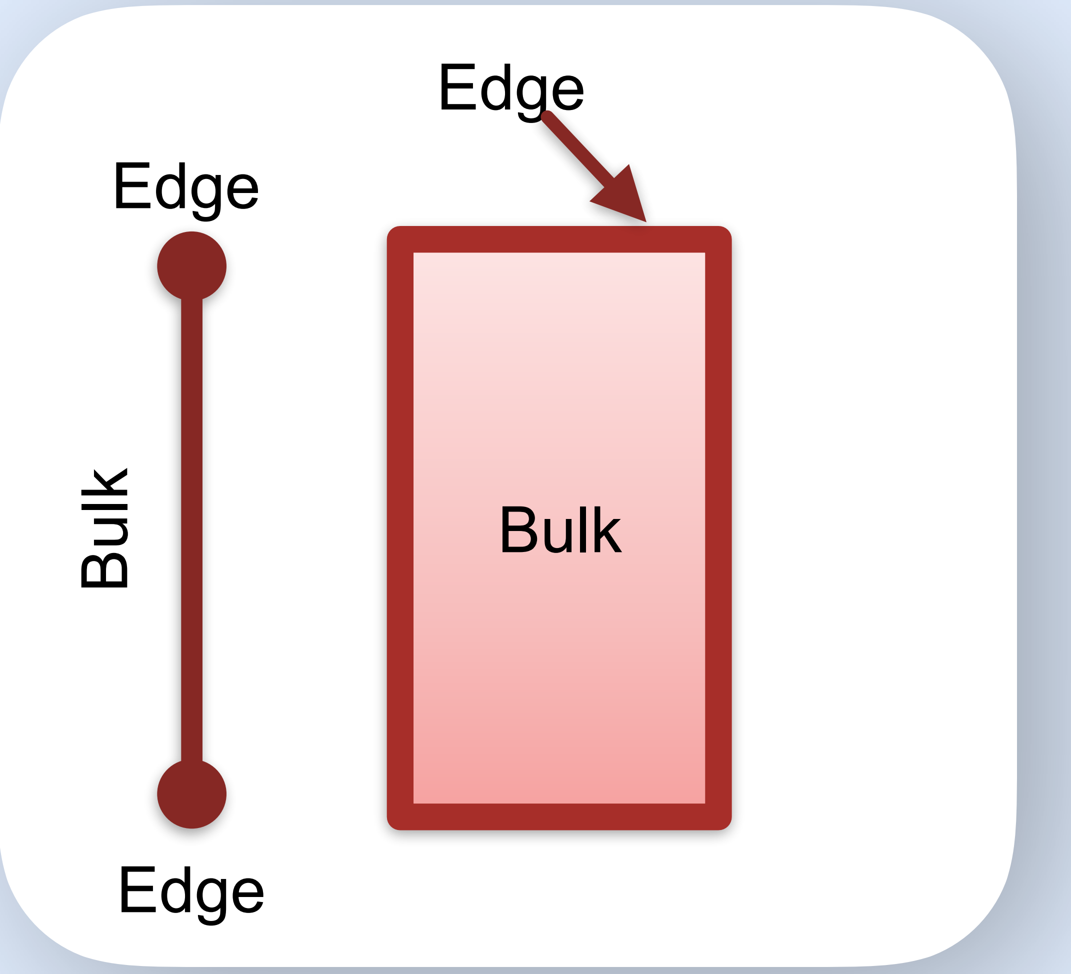
- $$P = \int dr\, r\rho(r) \quad \Rightarrow \quad P = \langle\Psi|r|\Psi\rangle$$

- 
  $$|W_R(r)\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi} dk\, e^{ik(R-r)}|\phi_k\rangle$$

- $$p = \int dr\, \langle W_0(r)|r|W_0(r)\rangle = \frac{1}{(2\pi)^2}\iint_{-\pi}^{\pi}\int_{-\pi}^{\pi} dr dk dq\, e^{ir(k-q)}\langle\phi_k|i\partial_q|\phi_q\rangle$$
  $$= \frac{i}{2\pi}\int_{-\pi}^{\pi} dk\, \langle\phi_k|\partial_k|\phi_k\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi} dk\, \mathcal{A}_x(k)$$

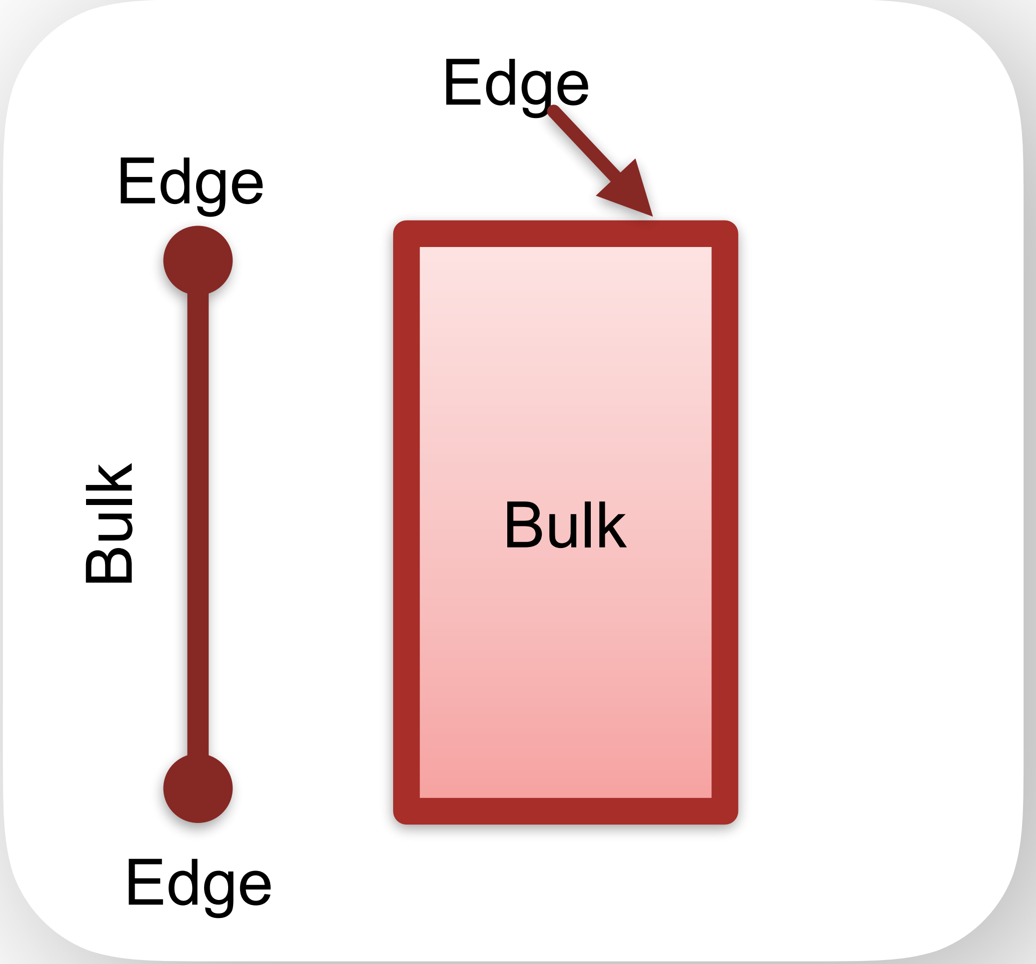
Dipole based topological systems





$$\rho = \partial_i p_i$$

$$\sigma = n_i p_i$$



A conventional topological system

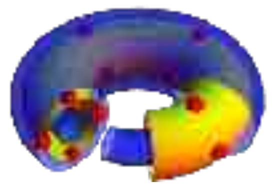
Two copies of a Chern insulator

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}_\Phi & 0 \\ 0 & \mathcal{H}_{-\Phi} \end{pmatrix} = \begin{pmatrix} \mathcal{H}_\Phi & 0 \\ 0 & \mathcal{H}^*_\Phi \end{pmatrix}$$

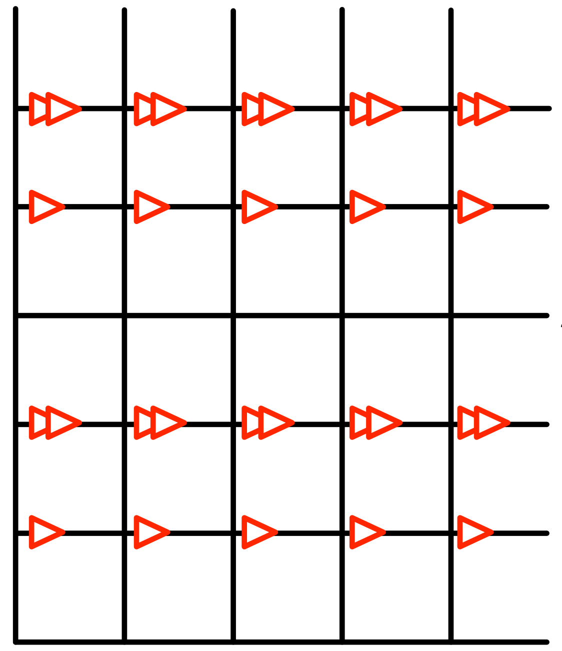
$$\begin{pmatrix} x_{r,s} \\ y_{r,s} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \begin{pmatrix} \psi^+_{r,s} \\ \psi^-_{r,s} \end{pmatrix}$$

$$U^\dagger \mathcal{H} U = \mathcal{D} = \begin{pmatrix} \mathrm{Re}\mathcal{H}_\Phi & \mathrm{Im}\mathcal{H}_\Phi \\ \mathrm{Im}\mathcal{H}_\Phi & \mathrm{Re}\mathcal{H}_\Phi \end{pmatrix}$$

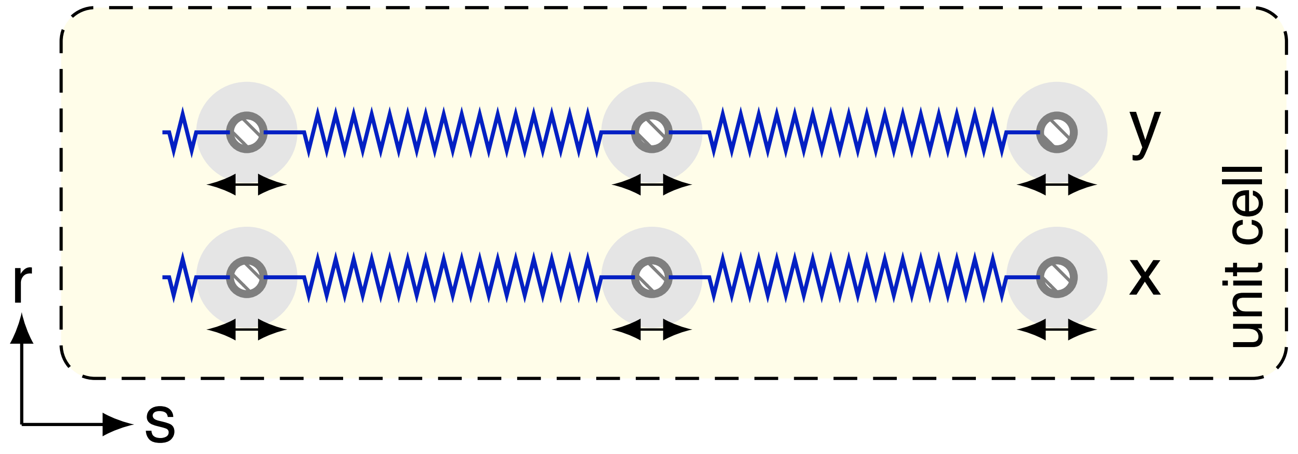
Süsstrunk & SDH, PNAS (2016)

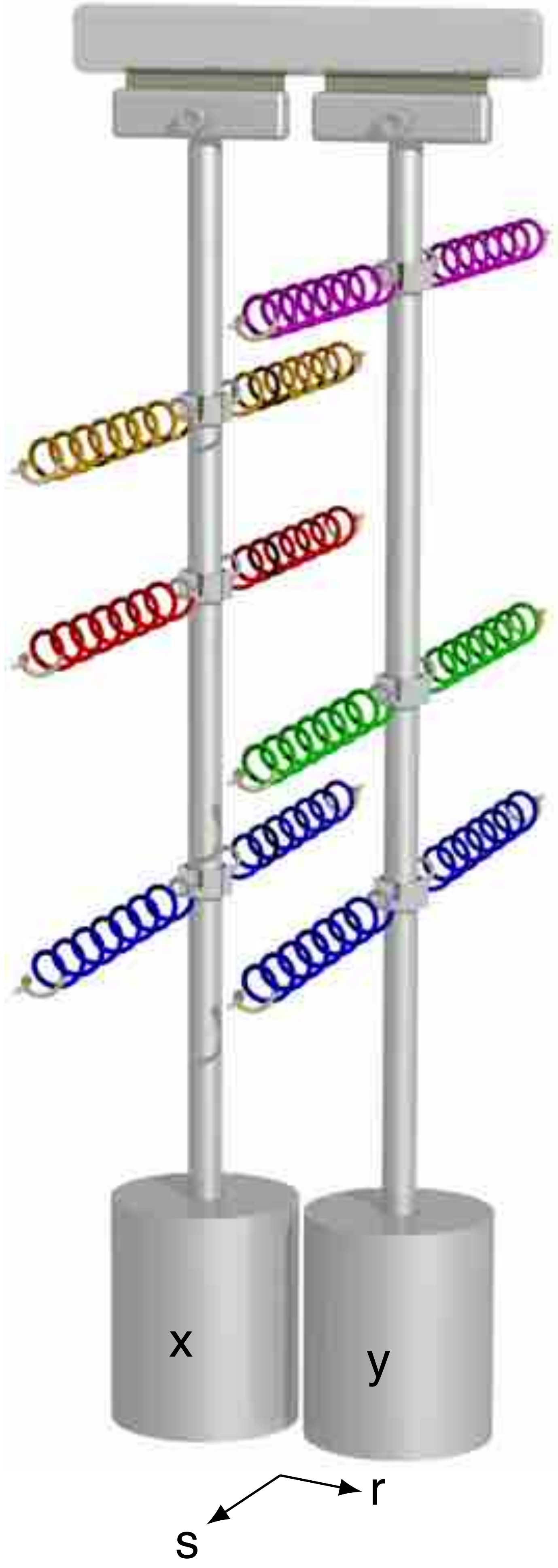


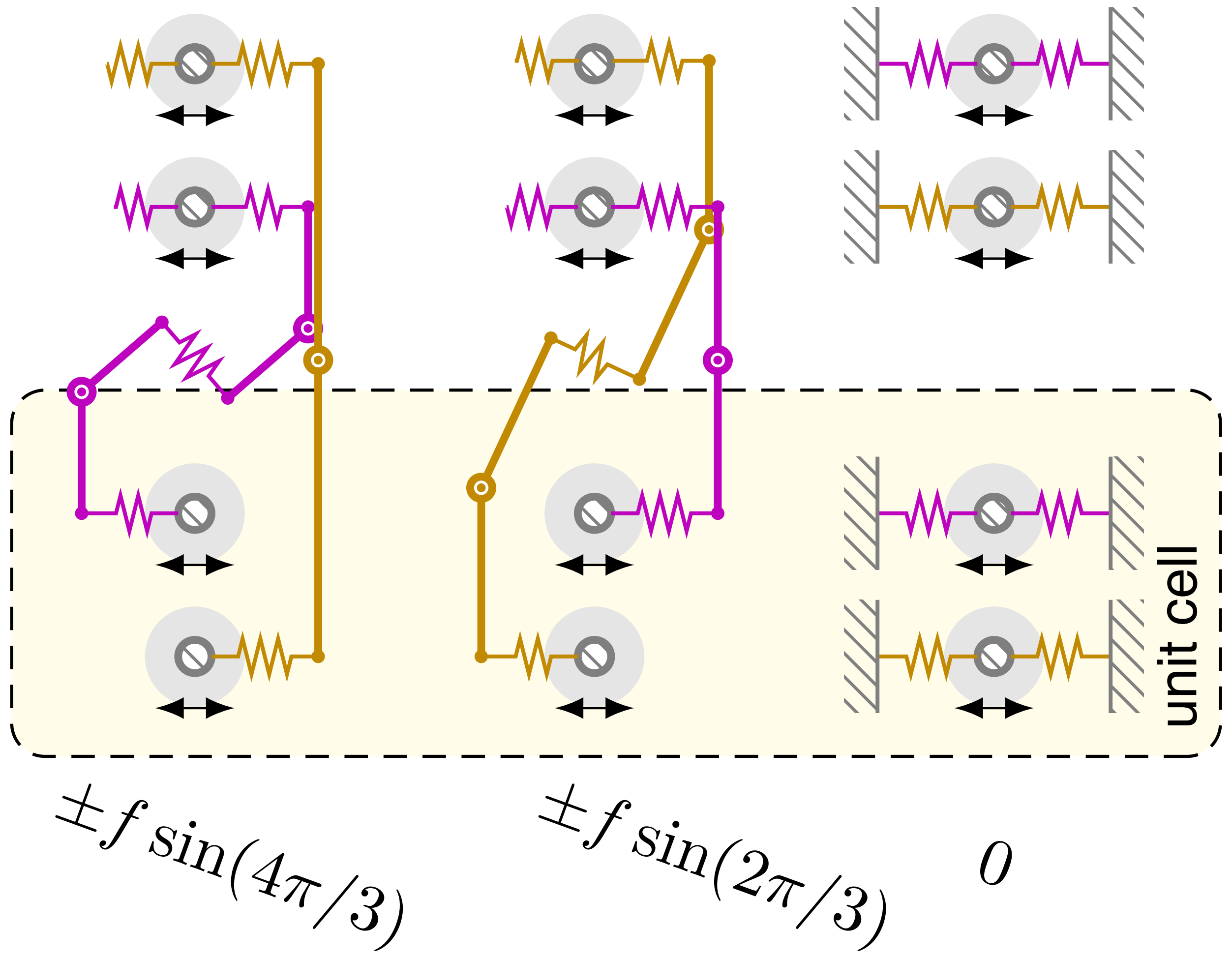
Our implementation

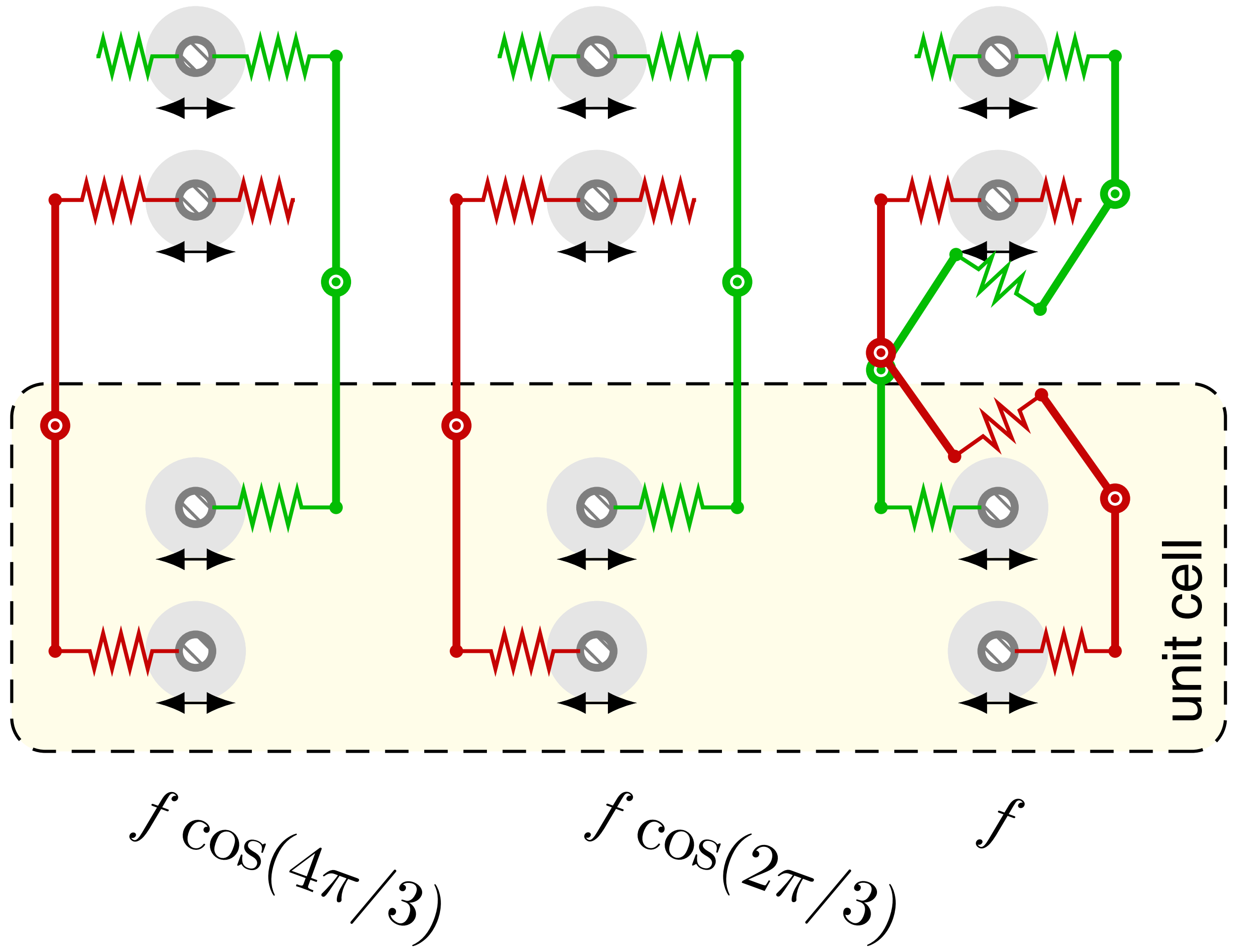


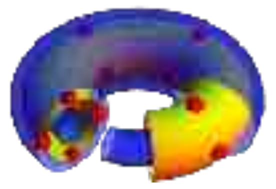
$$\mathcal{D} = \begin{array}{c} \begin{array}{cc} x & \quad y \end{array} \\ \begin{pmatrix} \mathrm{Re}\mathcal{H}_\Phi & \mathrm{Im}\mathcal{H}_\Phi \\ \mathrm{Im}\mathcal{H}_\Phi & \mathrm{Re}\mathcal{H}_\Phi \end{pmatrix} \end{array} \begin{array}{c} x \\ y \end{array}$$





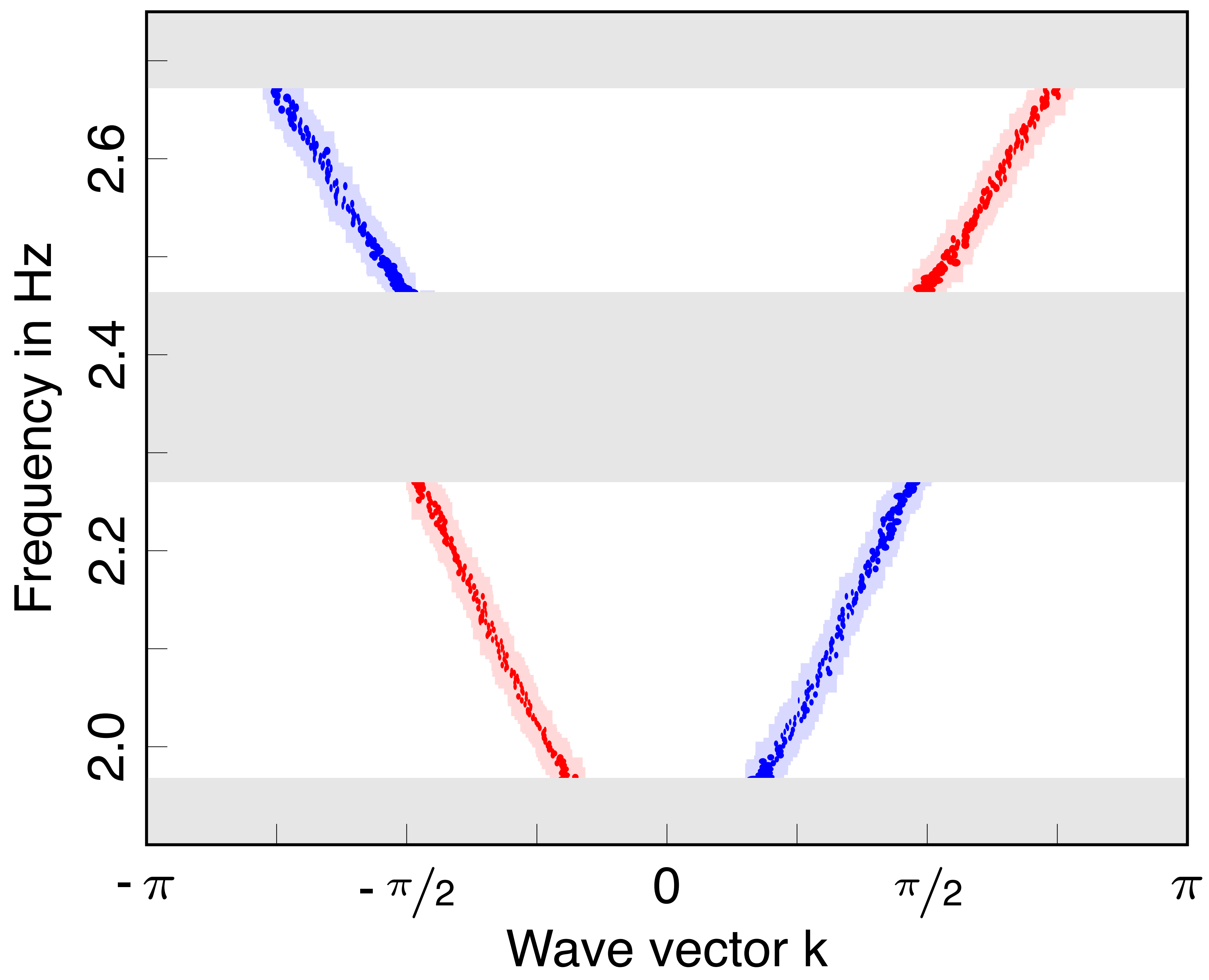


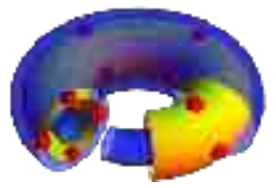




Süsstrunk & SDH, Science (2015)

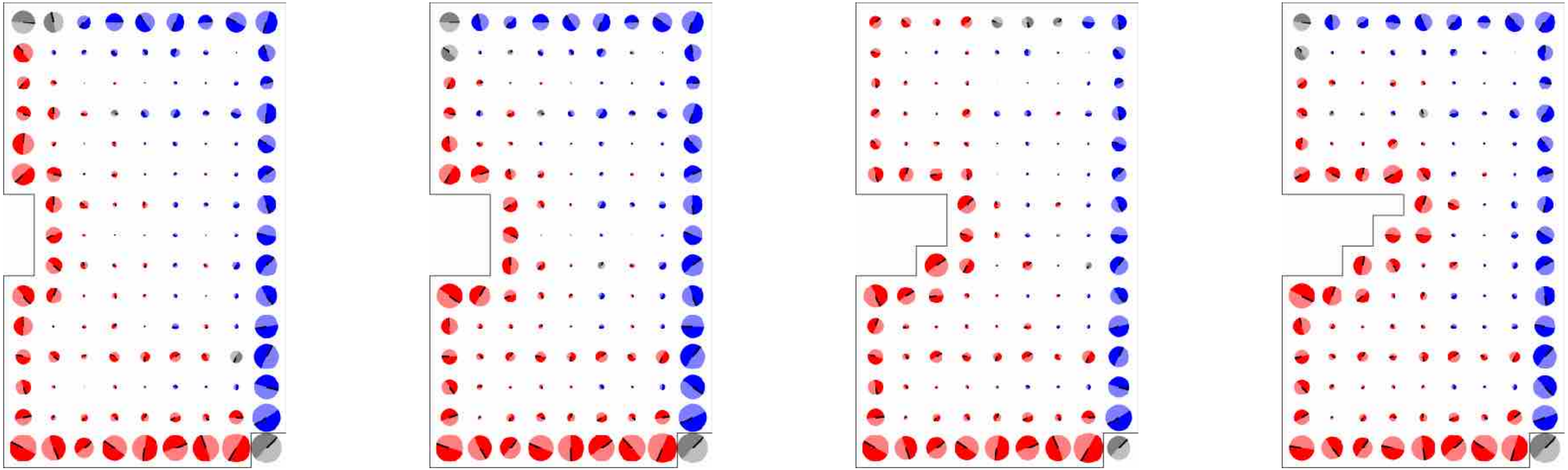
Helical edge modes



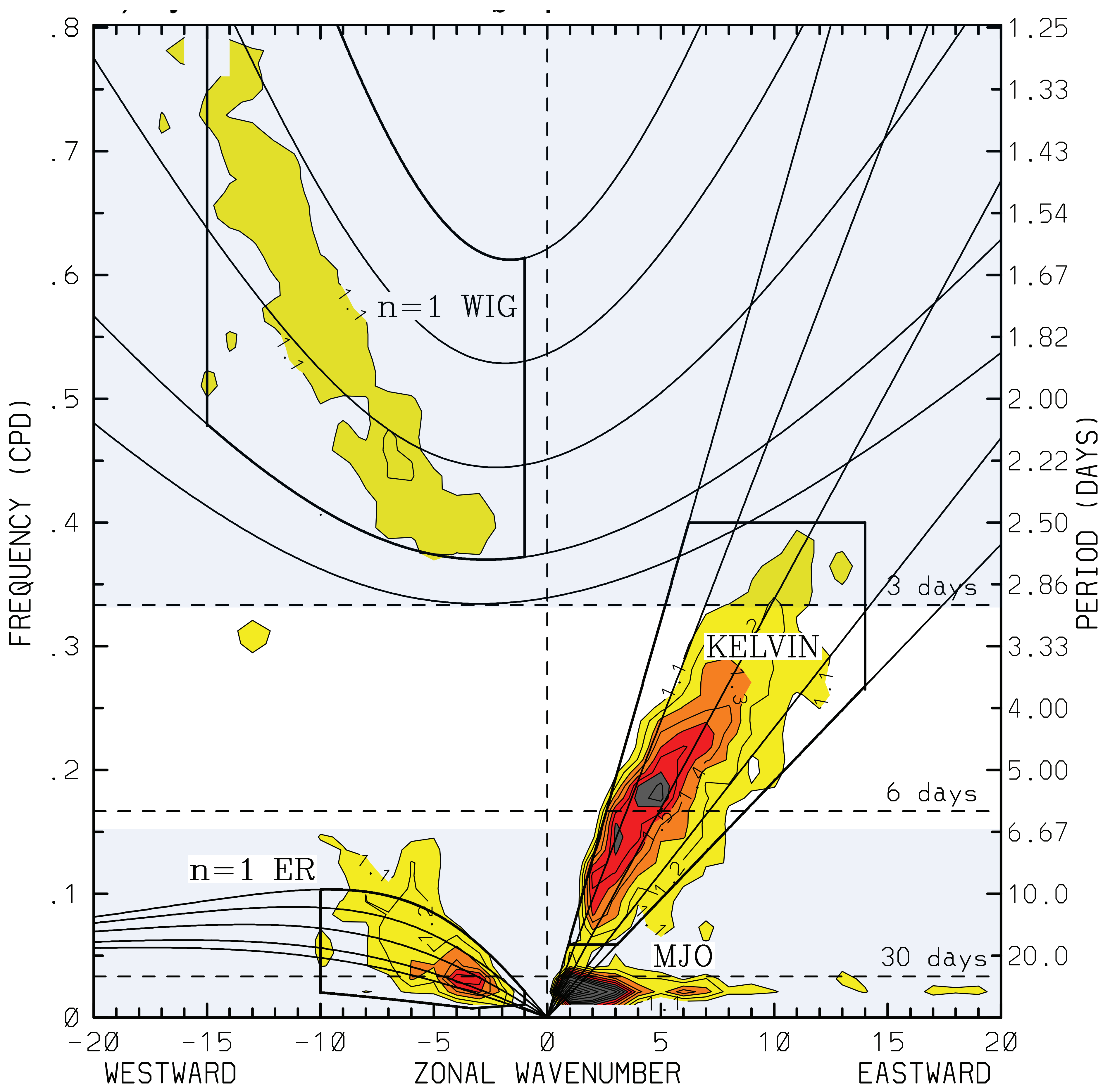


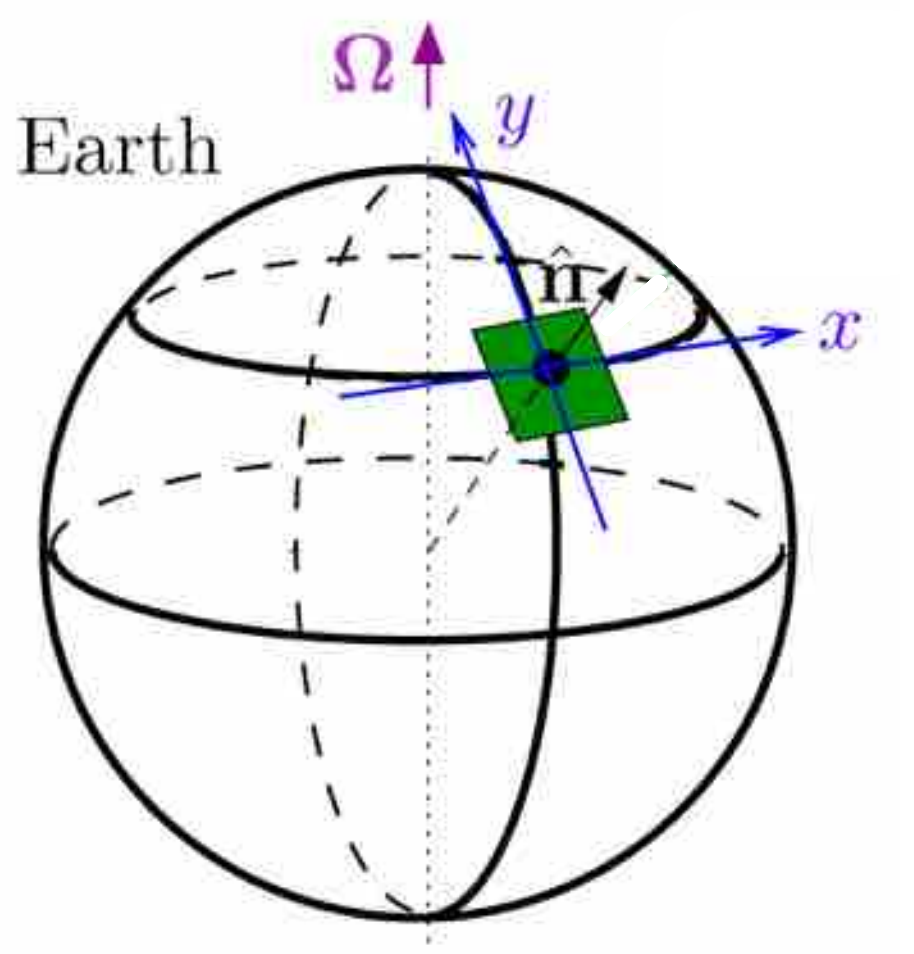
Süsstrunk & SDH, Science (2015)

Not “just” a whispering gallery mode



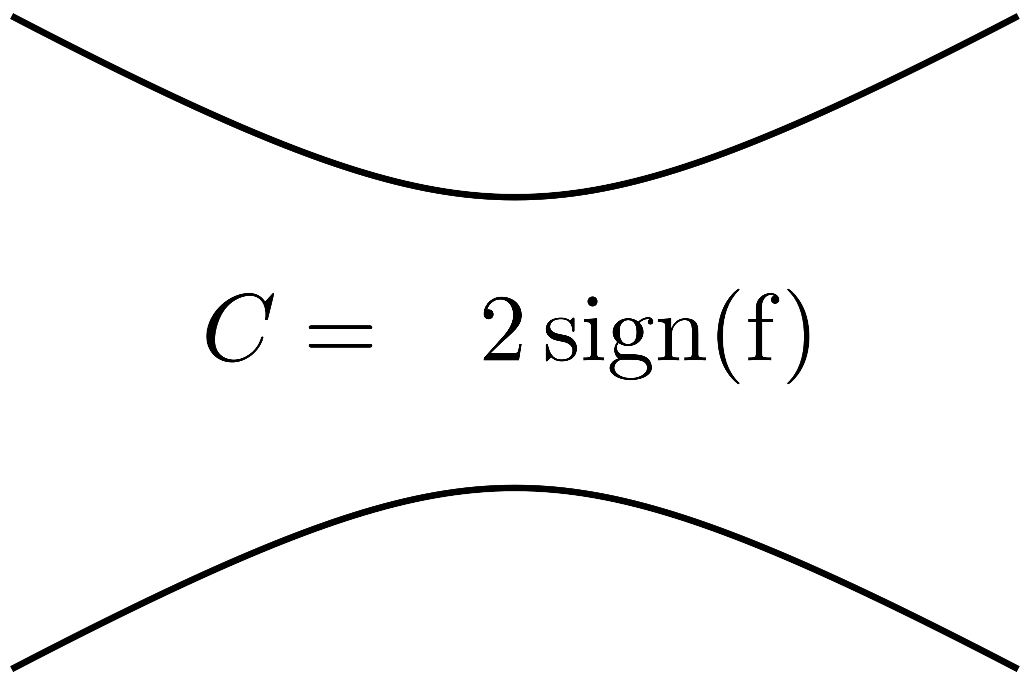
Süsstrunk & SDH, Science (2015)



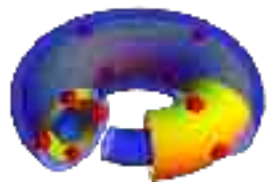


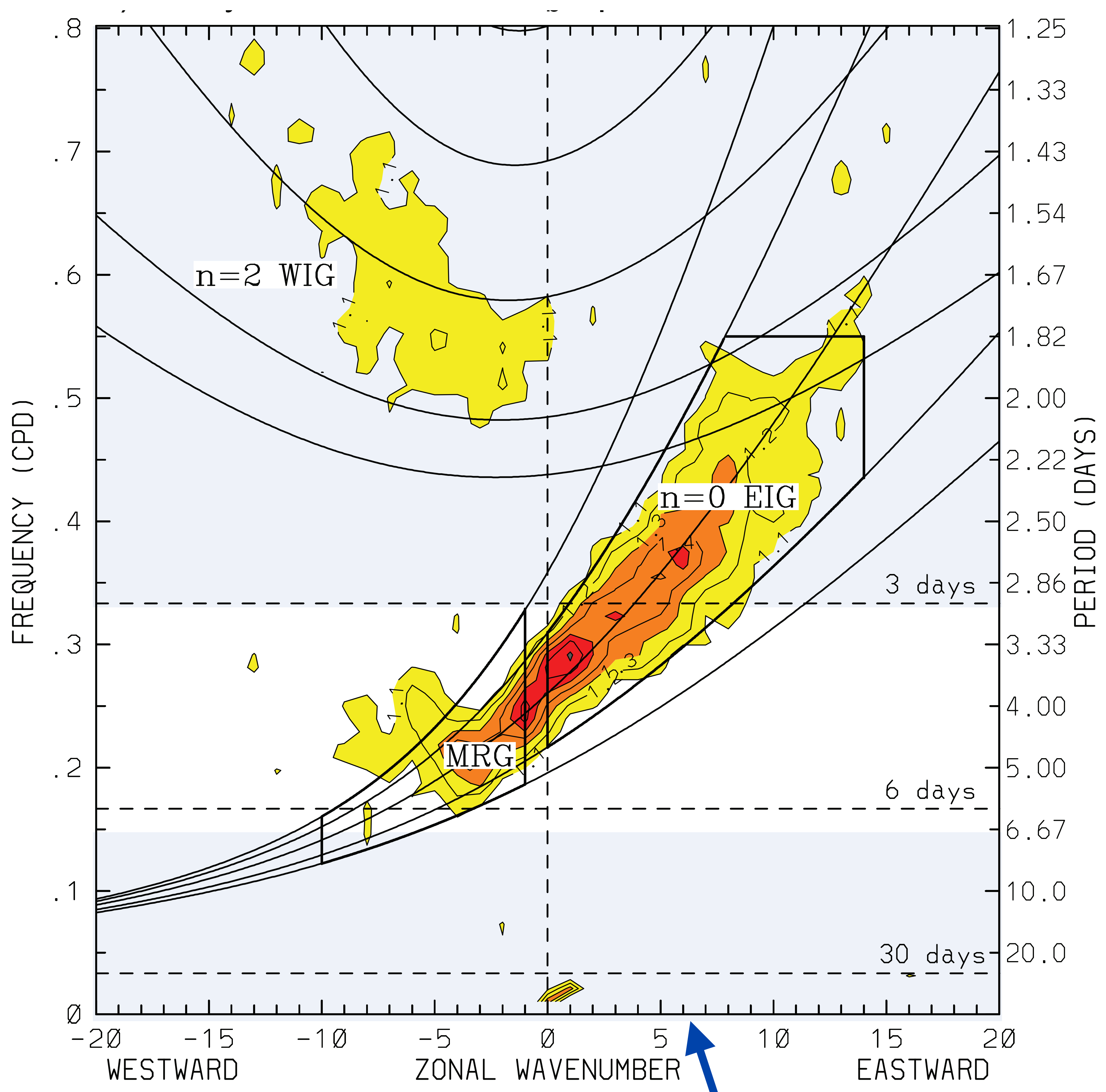
$$\omega_{\pm}(\mathbf{k}) = \pm\sqrt{f^2 + c^2k^2}$$

$$\omega_0(\mathbf{k}) = 0 \qquad f = \hat{\mathbf{n}} \cdot \Omega$$



$$C = \quad 2\,\mathrm{sign(f)}$$





λ=6000 km

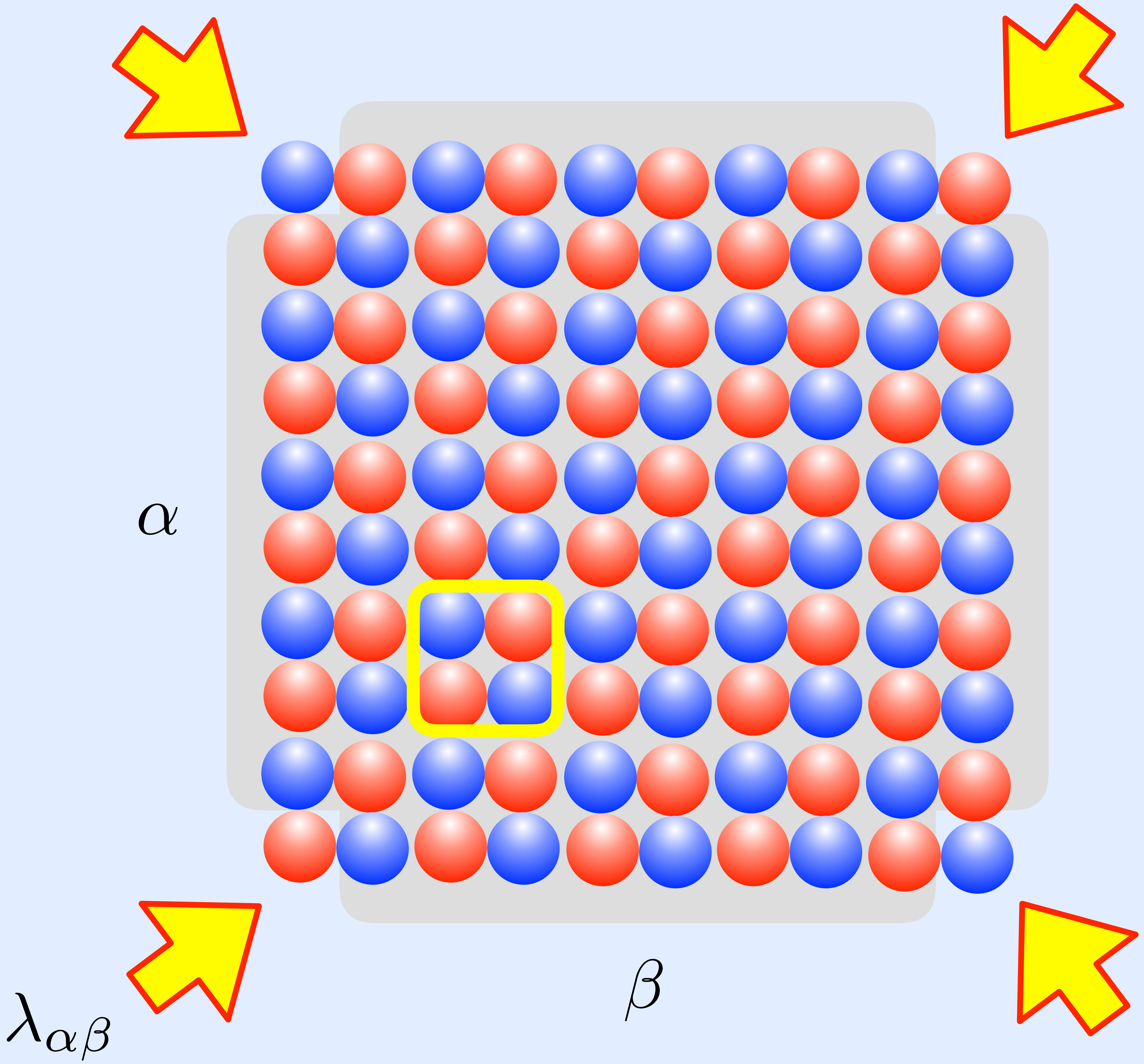
Wheeler, J. Atmos. Sci. (2000)
Delplace, Science (2017)

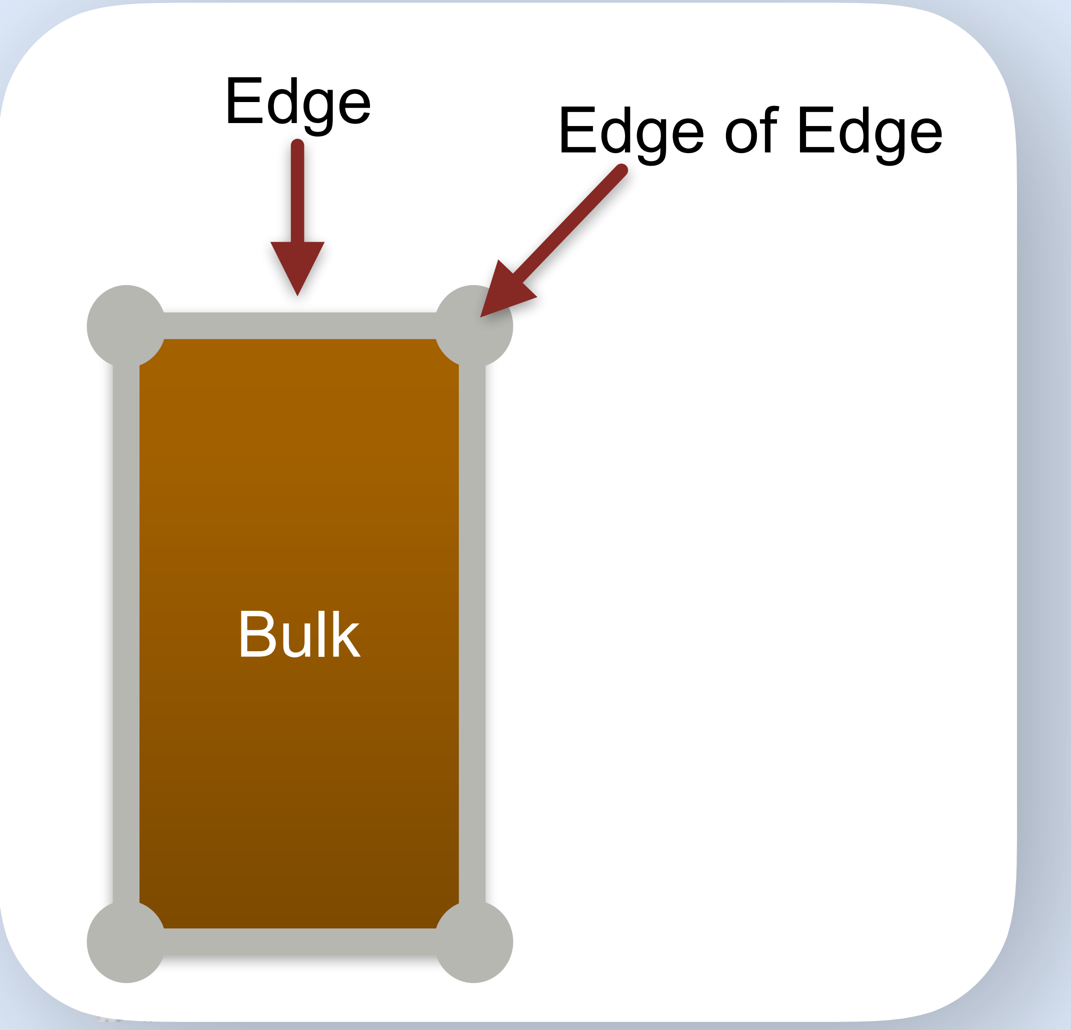
Quadrupole based topological system

$$\rho = \frac{1}{2}\partial_i \partial_j q_{ij}$$

$$\sigma^{\alpha} = -\partial_j (n_i^{\alpha} q_{ij})$$

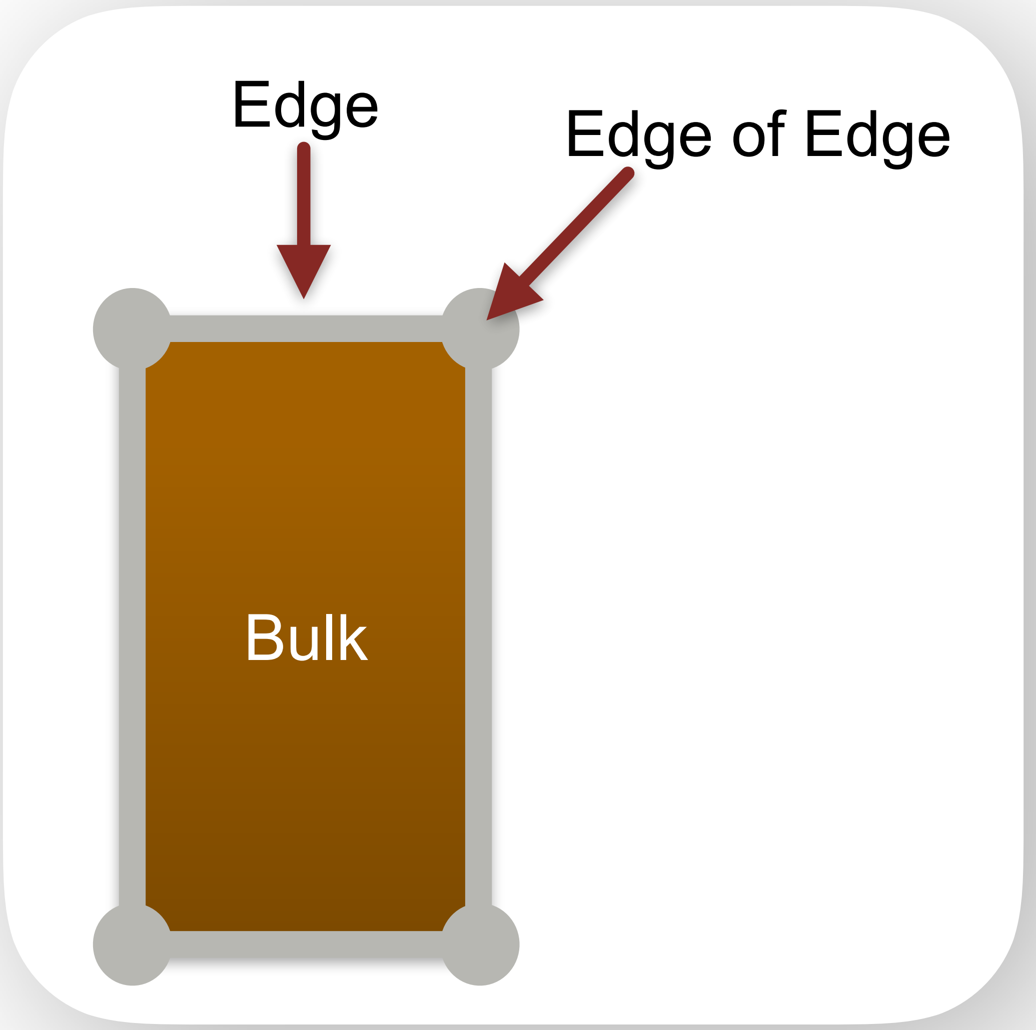
$$\lambda^{\alpha\beta} = n_i^{\alpha} n_j^{\beta} q_{ij}$$





Benalcazar et al., Science (2017)

Benalcazar et al. Science (2017)
Schindler et al. Sci. Adv. (2018)
Trifunovic and Brouwer, arXiv (2018)
Schindler et al. Nature Phys. (2018)

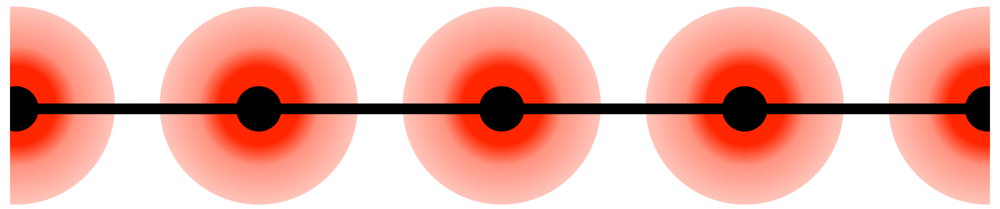


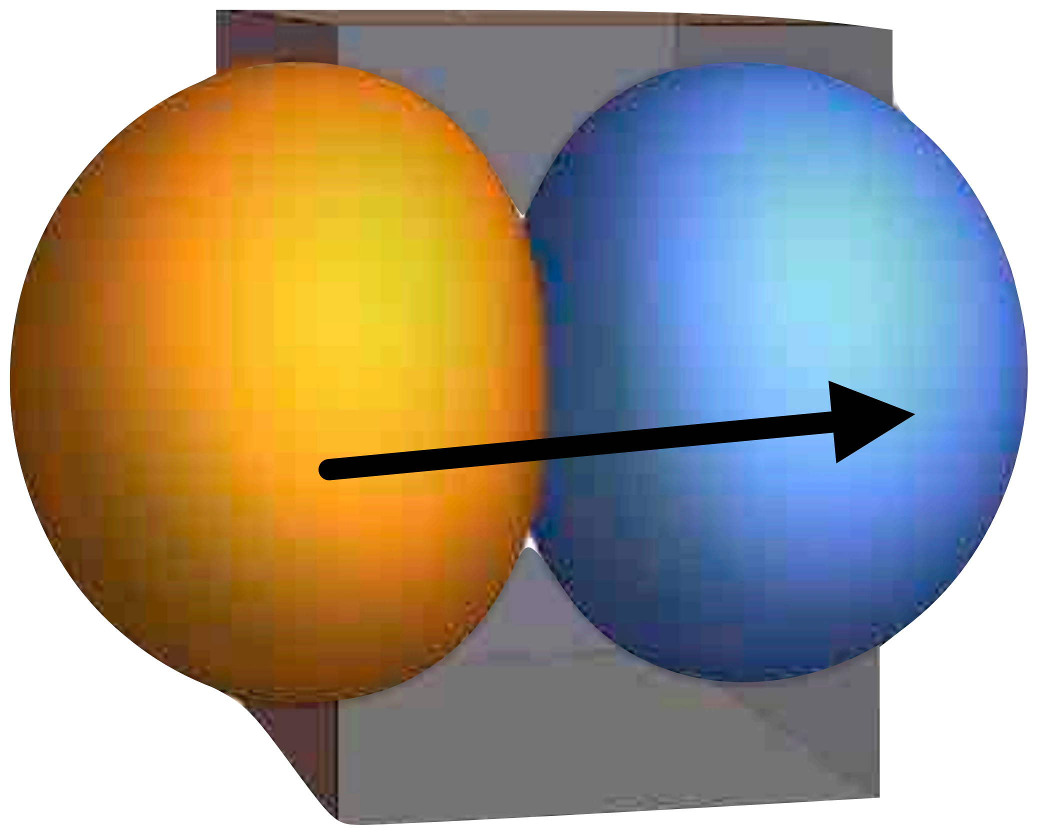
A higher-order topological system

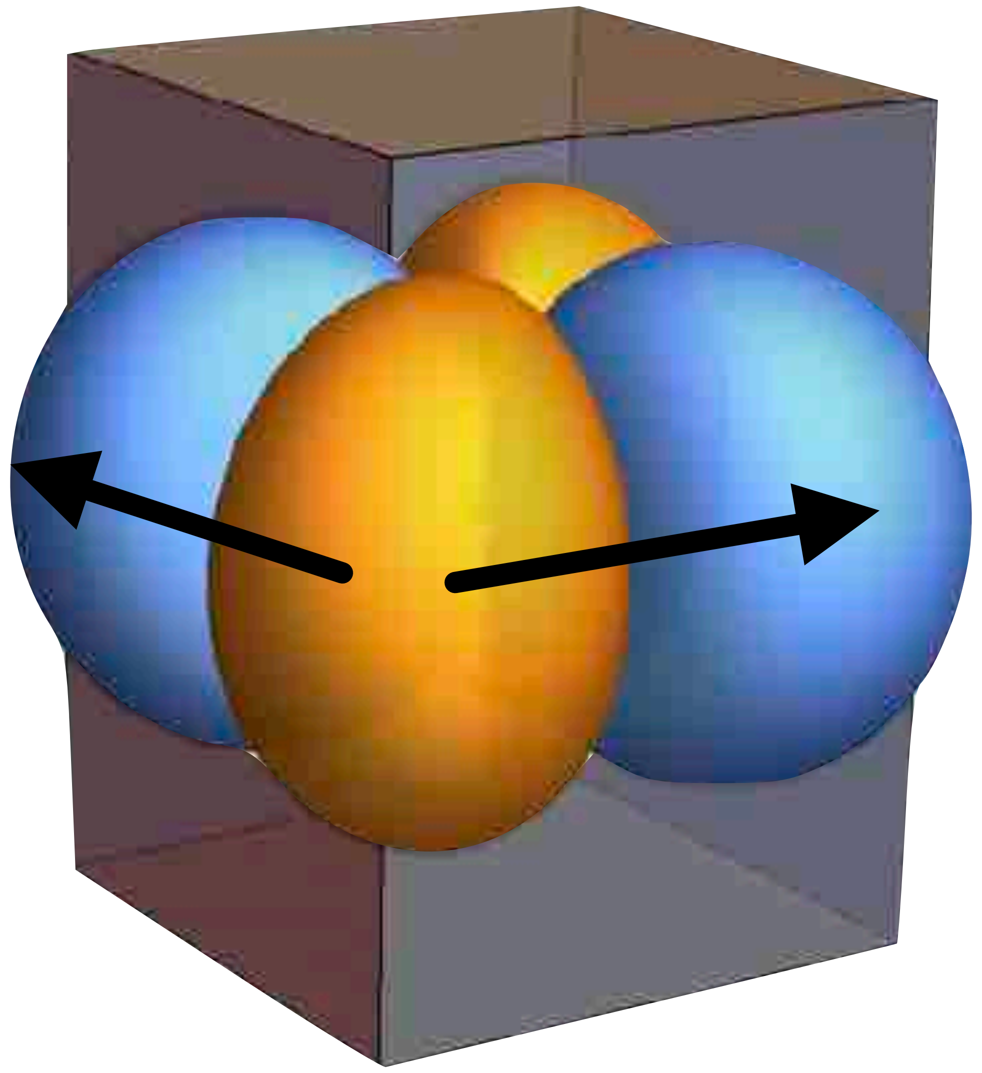
From dipoles to quadrupoles

$$p_\alpha = -\frac{1}{4\pi^2}\int_K d\mathbf{k}\,\mathrm{Tr}[\mathcal{A}_{\alpha,\mathbf{k}}]$$

$$\mathcal{A}_{\alpha,\mathbf{k}} = -i\langle u^m_{\mathbf{k}}|\partial_{k_\alpha}|u^n_{\mathbf{k}}\rangle$$







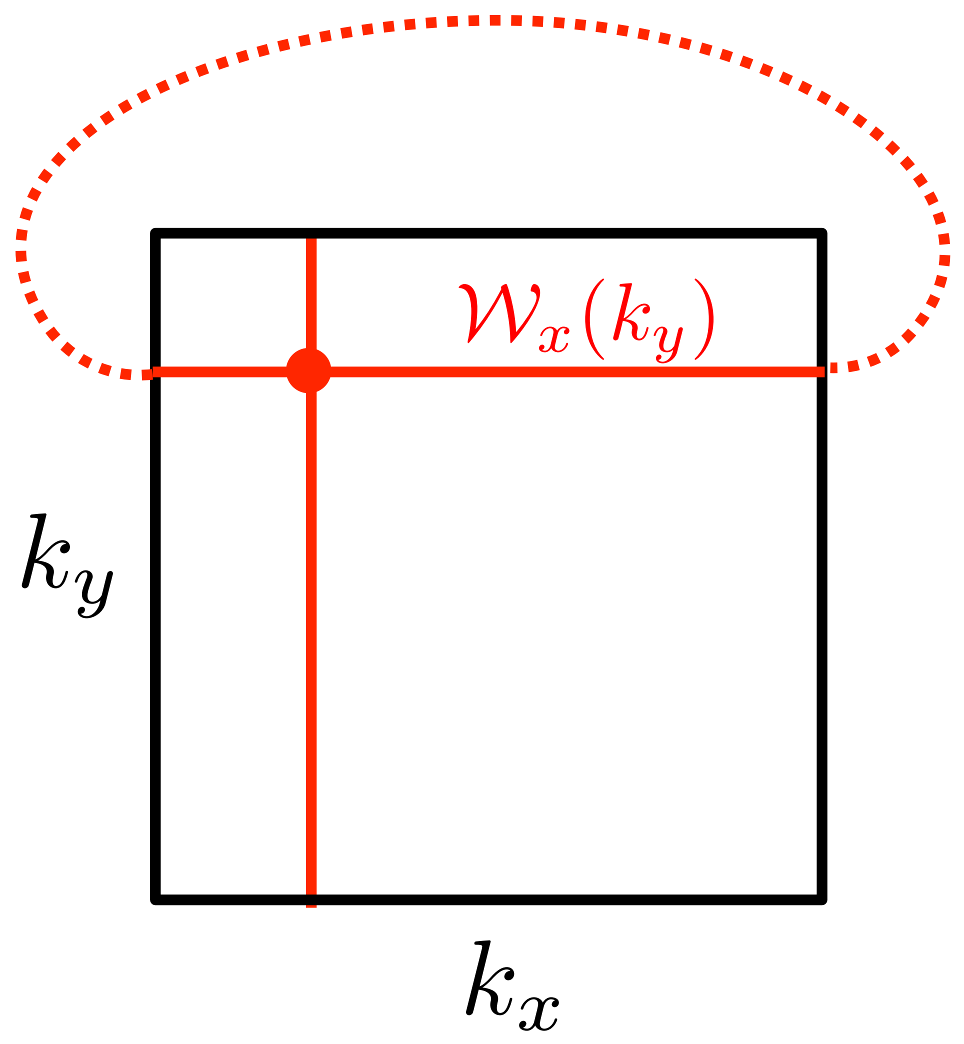
With, e.g., inversion symmetry:

$$p_\alpha = \begin{cases} 0 & \text{trivial} \\ 1/2 & \text{non-trivial} \end{cases}$$

Benalcazar et al. Science (2017)

Formalizing the quadrupole moment: Wilson loops

$$\mathcal{W}_x(k_y) = \mathrm{T}_{\mathcal{C}} \exp\left(-i \oint_{\mathcal{C}} dk_x \langle u^n_{\mathbf{k}} | \partial_{k_x} | u^m_{\mathbf{k}} \rangle\right)$$



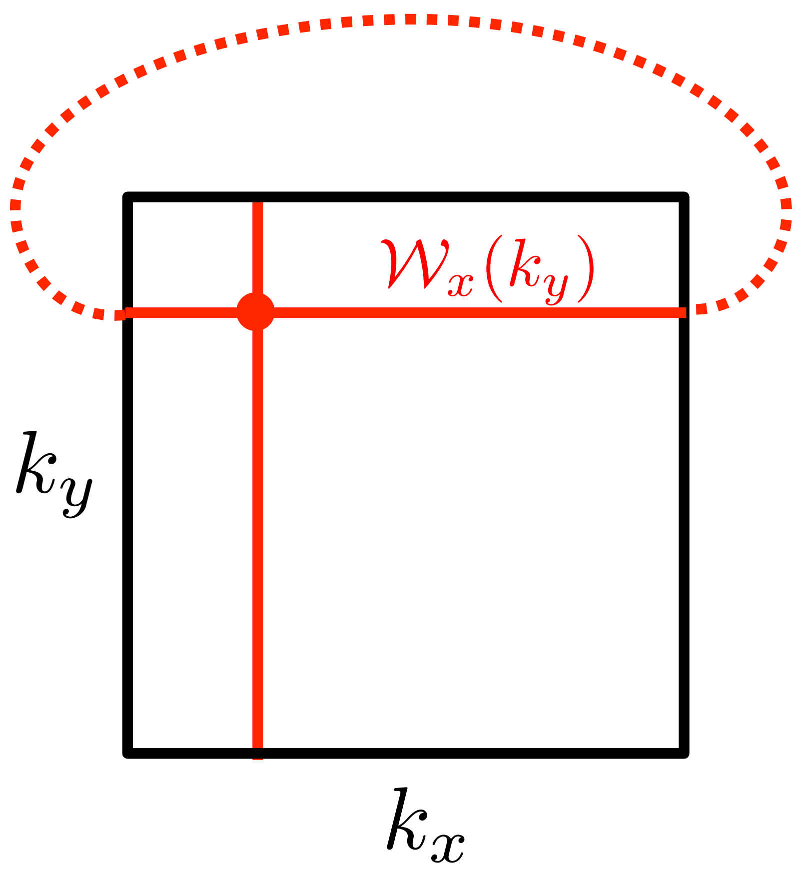
$$p_\alpha = -\frac{1}{4\pi^2} \int_K d\mathbf{k}\, \mathrm{Tr}[\mathcal{A}_{\alpha,\mathbf{k}}]$$

$$\mathcal{A}_{\alpha,\mathbf{k}} = -i \langle u^m_{\mathbf{k}} | \partial_{k_\alpha} | u^n_{\mathbf{k}} \rangle$$

Benalcazar et al. Science (2017)

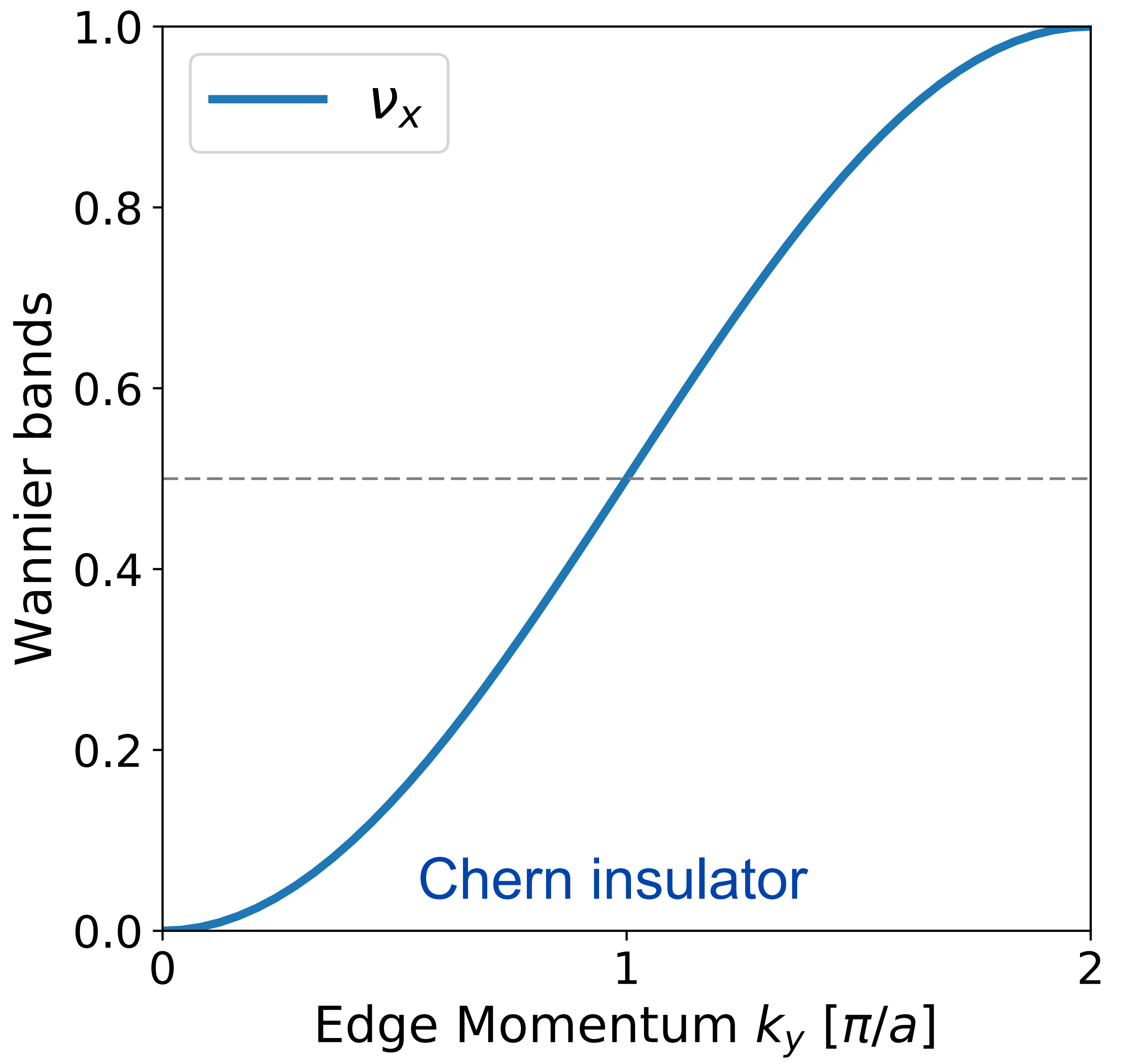
Benalcazar et al. PRB 2017

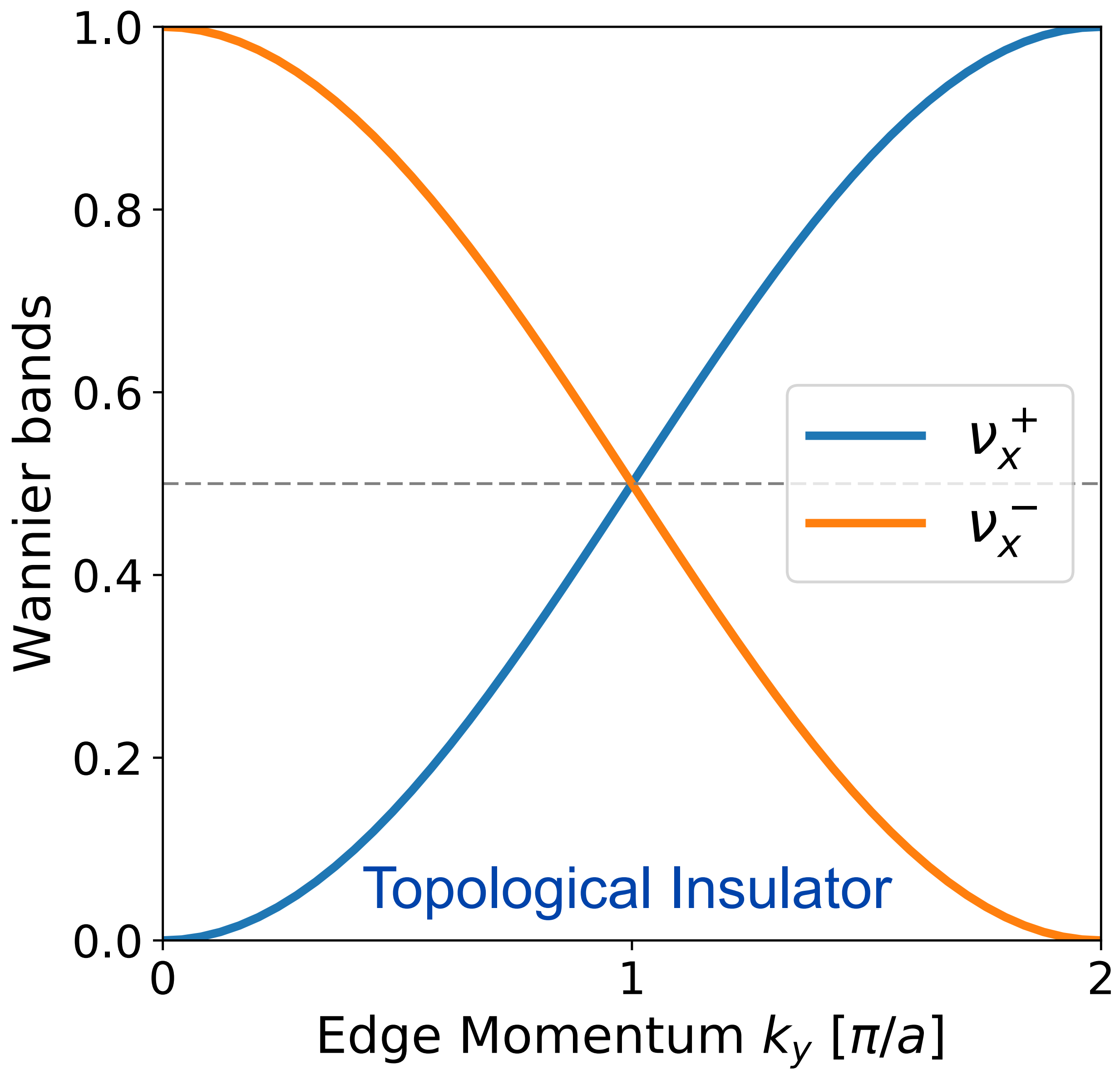
Wilson loops encode the surface physics

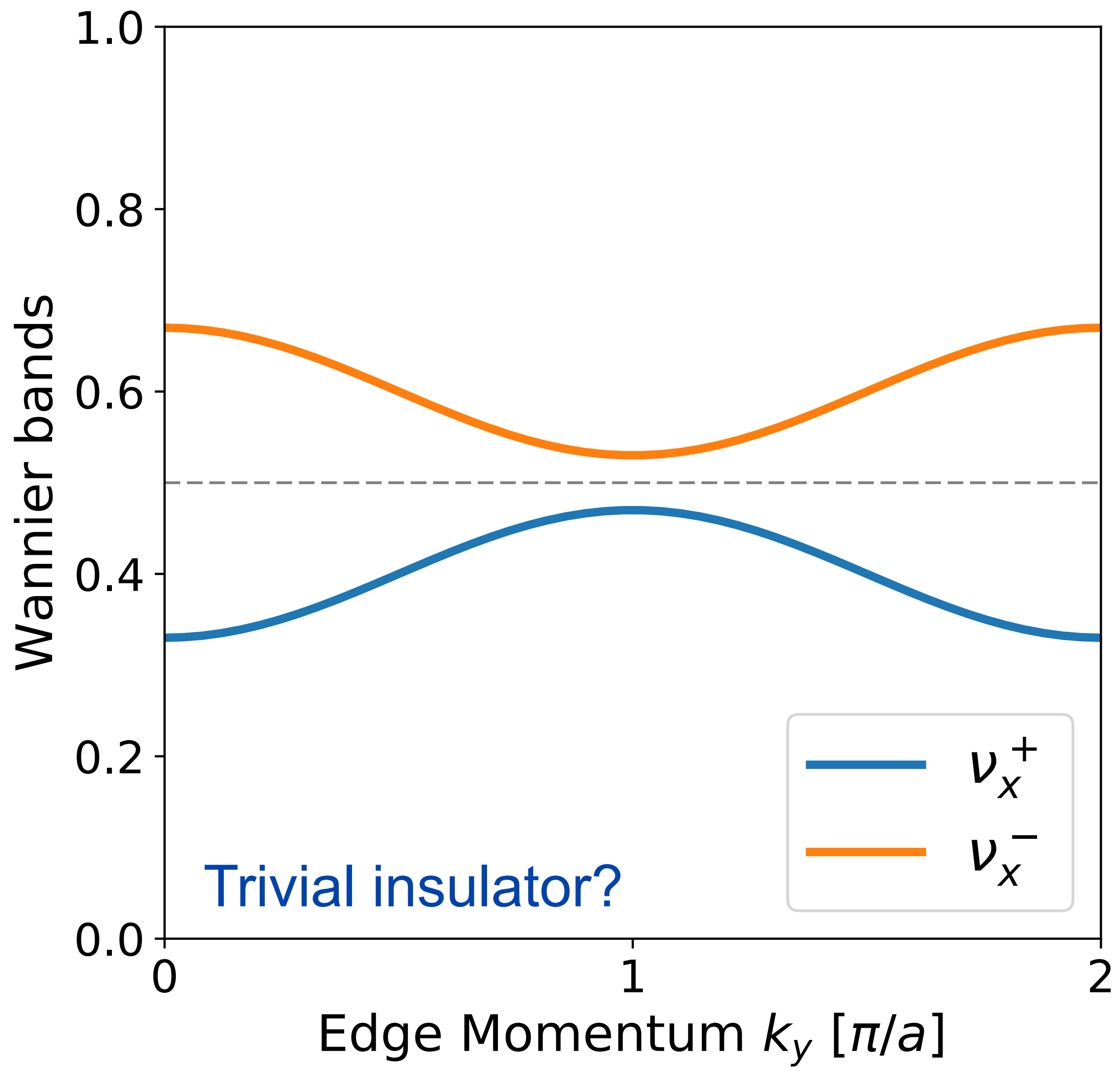


$$\mathcal{W}_x(k_y) = \mathrm{T}_{\mathcal{C}} \exp\left(-i \oint_{\mathcal{C}} dk_x \langle u^n_{\mathbf{k}} | \partial_{k_x} | u^m_{\mathbf{k}} \rangle \right)$$

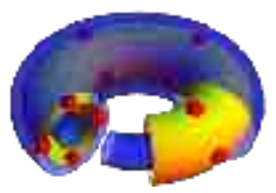
$$\text{eigenvalues: } \exp[2\pi i \nu_x^{\pm}(k_y)], \quad \text{eigenvectors: } \mathbf{v}^{\pm}$$







Fidkowski et al. PRL (2011)



Nested Wilson loops capture the quadrupole

$$\mathcal{W}_x(k_y) = \mathrm{T}_{\mathcal{C}} \exp\left(-i \oint_{\mathcal{C}} dk_x \langle u^n_{\mathbf{k}} | \partial_{k_x} | u^m_{\mathbf{k}} \rangle\right)$$

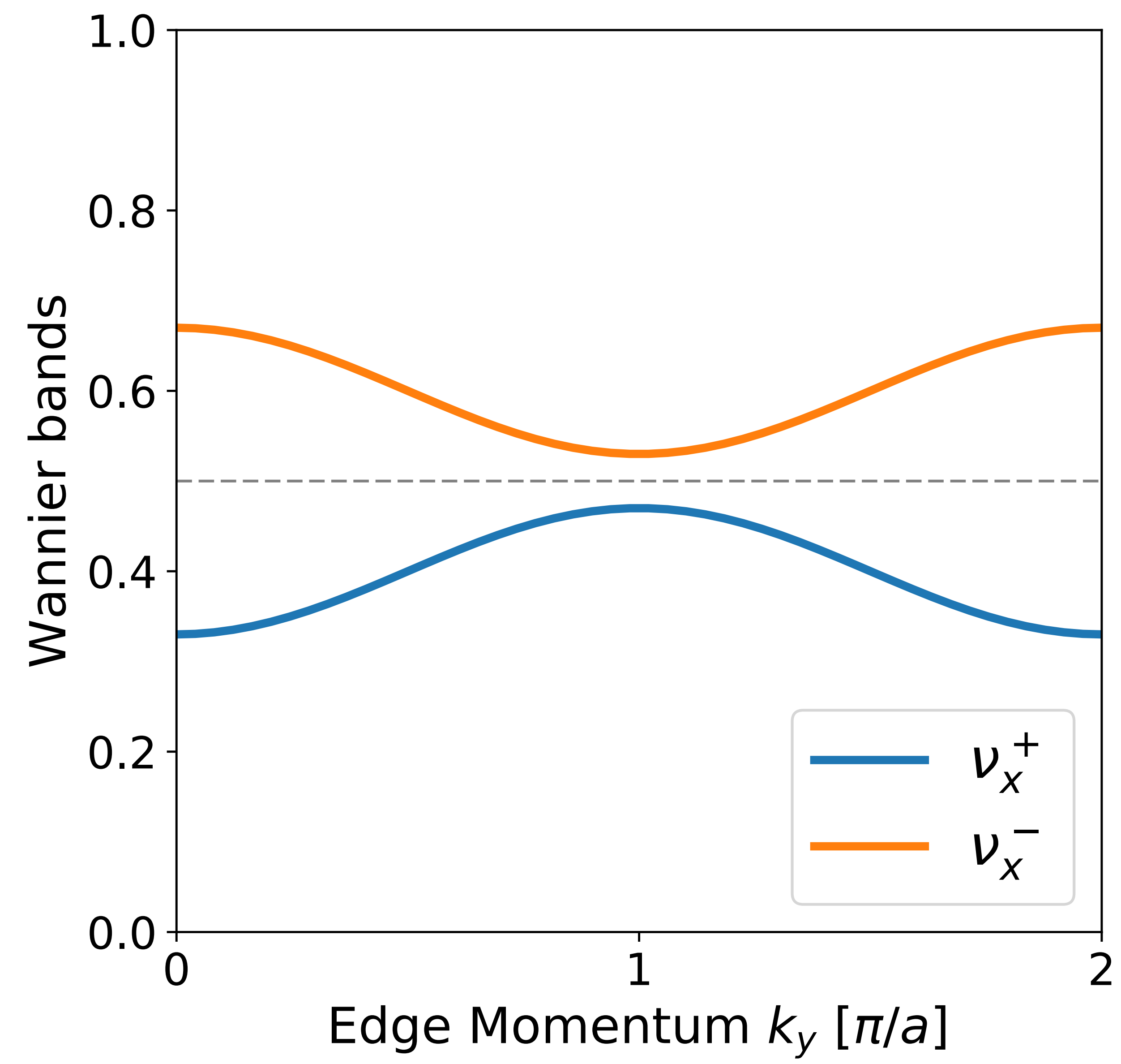
eigenvalues: $\exp[2\pi i \nu_x^{\pm}(k_y)]$, eigenvectors: $\mathbf{v}^{\pm}$

$$|\tilde{w}^{\pm}_{x,\mathbf{k}}\rangle = \sum_m v^{\pm}_m |u^m_{\mathbf{k}}\rangle$$

$$\tilde{p}^{\pm}_{\alpha} = -\frac{1}{4\pi^2} \int_K d\mathbf{k}\, \mathrm{Tr}[\tilde{\mathcal{A}}^{\pm}_{\alpha,\mathbf{k}}]$$

$$\tilde{\mathcal{A}}^{\pm}_{\alpha,\mathbf{k}} = -i \langle \tilde{w}^{\pm}_{\mathbf{k}} | \partial_{k_\alpha} | \tilde{w}^{\pm}_{\mathbf{k}} \rangle$$

split the filled bands

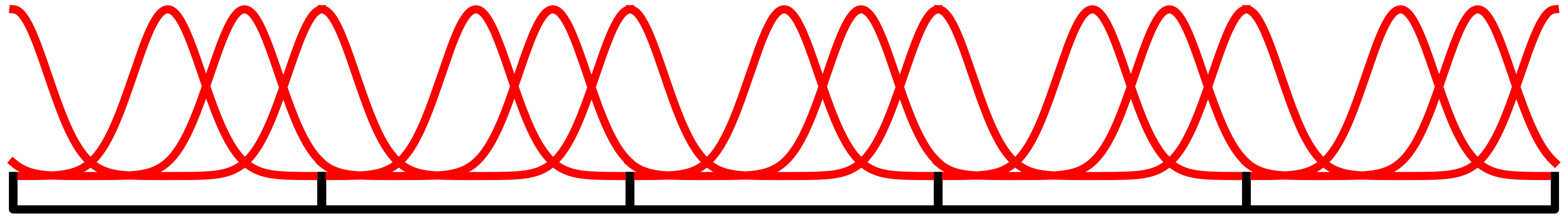


Fidkowski et al. PRL (2011)
Benalcazar et al. Science (2017)

Quantizing the quadrupole moment: symmetries

$$p_\alpha = -\frac{1}{4\pi^2}\int_K d\mathbf{k}\,\mathrm{Tr}[\mathcal{A}_{\alpha,\mathbf{k}}]$$

$$\mathcal{A}_{\alpha,\mathbf{k}} = -i\langle u^m_{\mathbf{k}}|\partial_{k_\alpha}|u^n_{\mathbf{k}}\rangle$$



With, e.g., inversion symmetry:

$$p_\alpha = \begin{cases} 0 & \text{trivial} \\ 1/2 & \text{non-trivial} \end{cases}$$

Symmetries

$$[M_x, H] = [M_y, H] = 0$$

$$[M_x, M_y] \neq 0$$

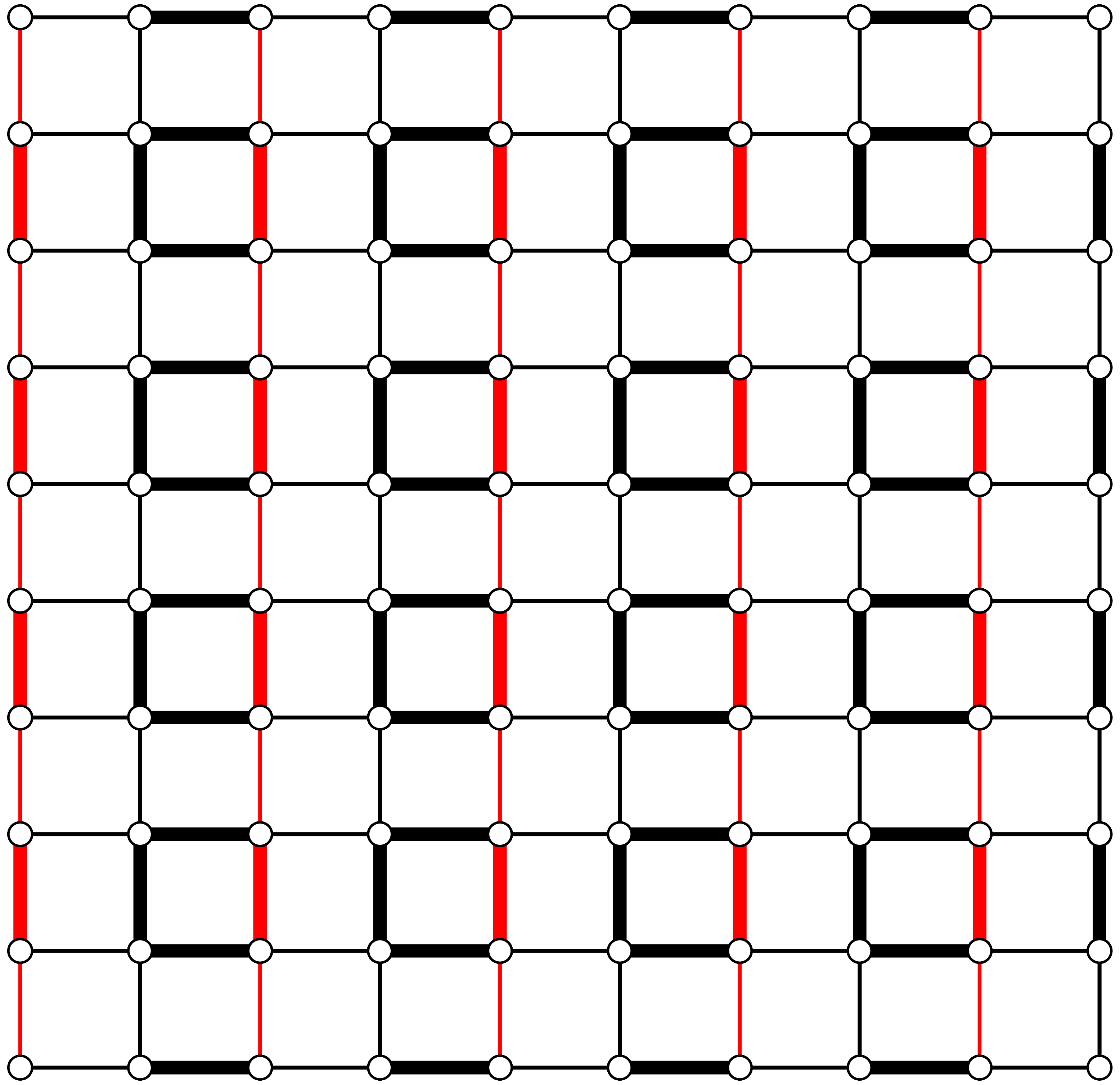
$$\Downarrow$$

$$(\tilde{p}^{\pm}_y, \tilde{p}^{\pm}_x) = (1/2, 1/2)$$

Benalcazar et al. Science (2017)

Benalcazar et al. PRB 2017

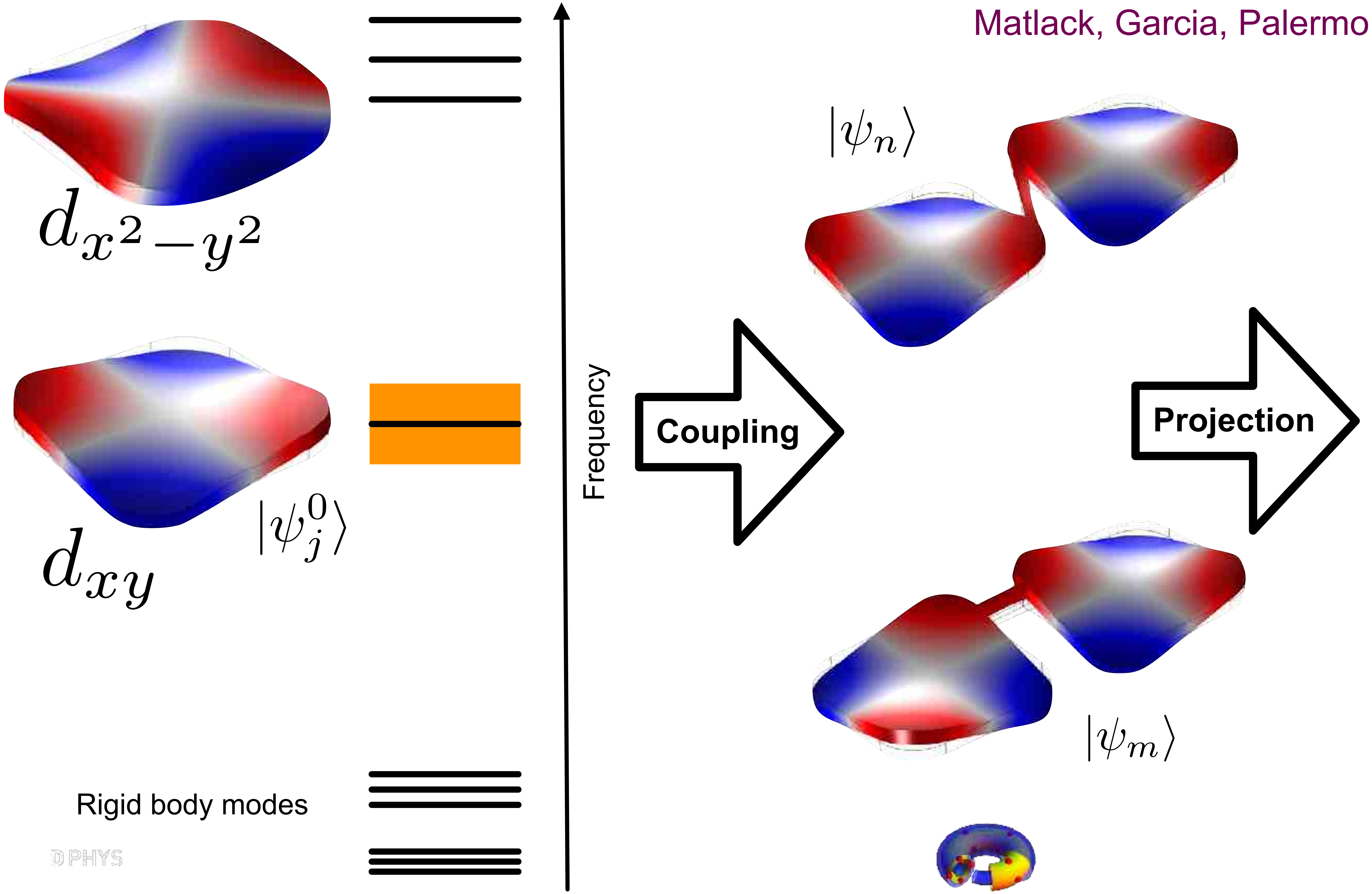
A model



Benalcazar et al. Science (2017)

Design principle: Perturbative metamaterials

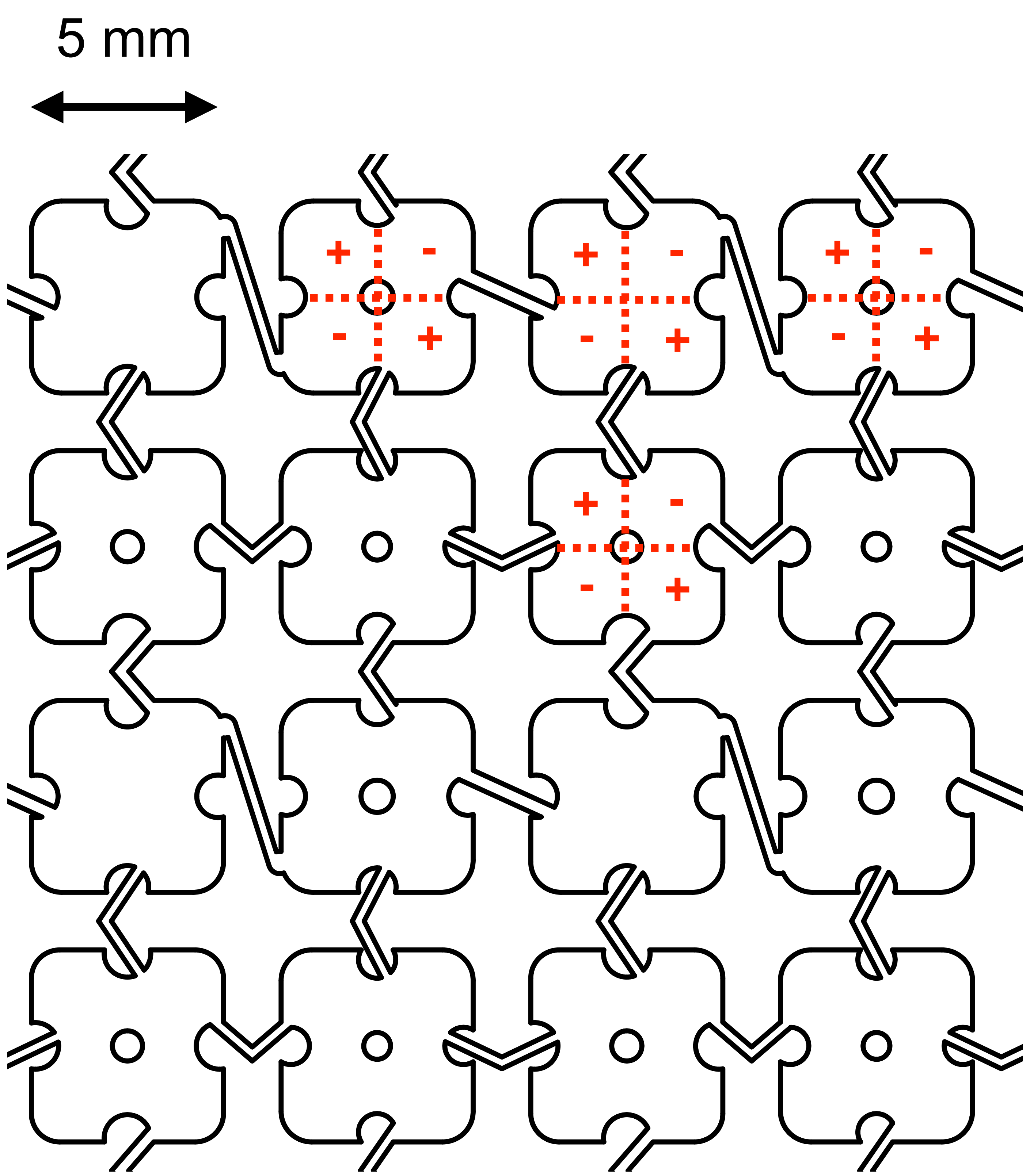
Matlack, Garcia, Palermo, SDH and Daraio, Nature Mat. 2018

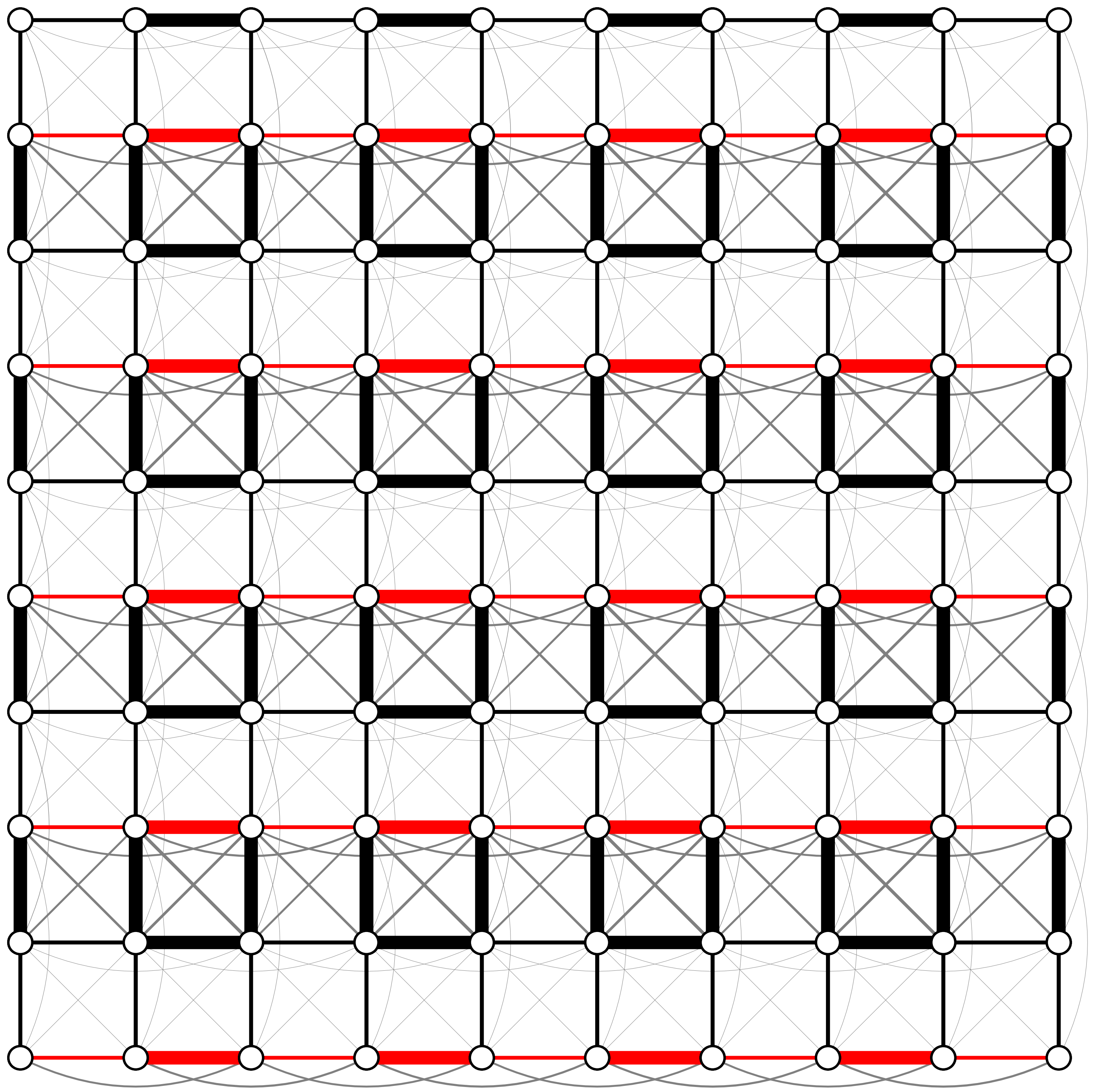


$$\mathcal{H}_{ij} = \sum_m \langle \psi_i^0 | \psi_m \rangle \nu_m \langle \psi_m | \psi_j^0 \rangle$$

ETH zürich

Design and tight-binding approximation





Silicon wafer, [100], 350μm thickness, TTV < 1μm

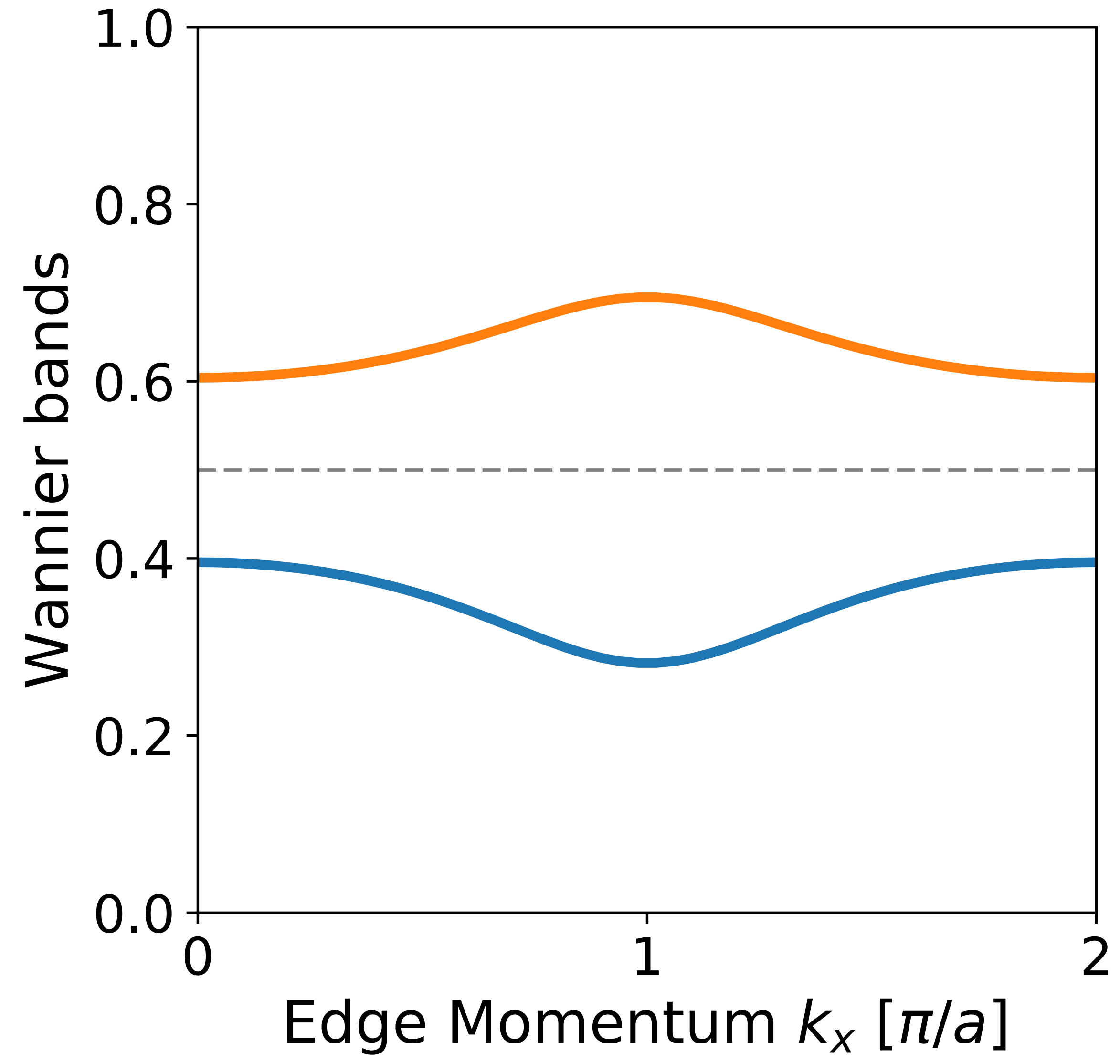
Matlack, Serra-Garcia, Palermo, SDH and Daraio, Nature Mat. 2018

Serra-Garcia et al. Nature (2018)

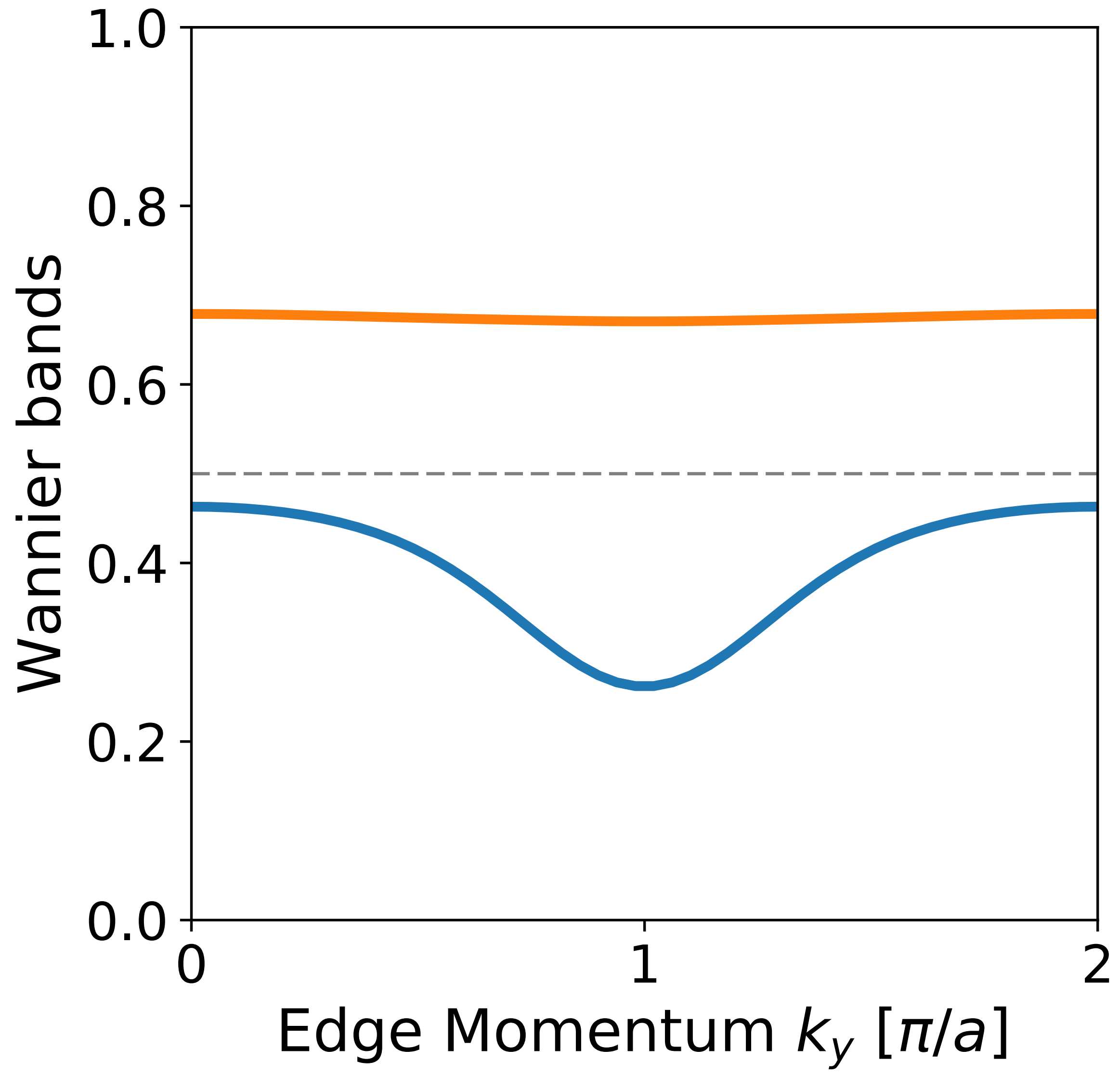
Assessment of design quality

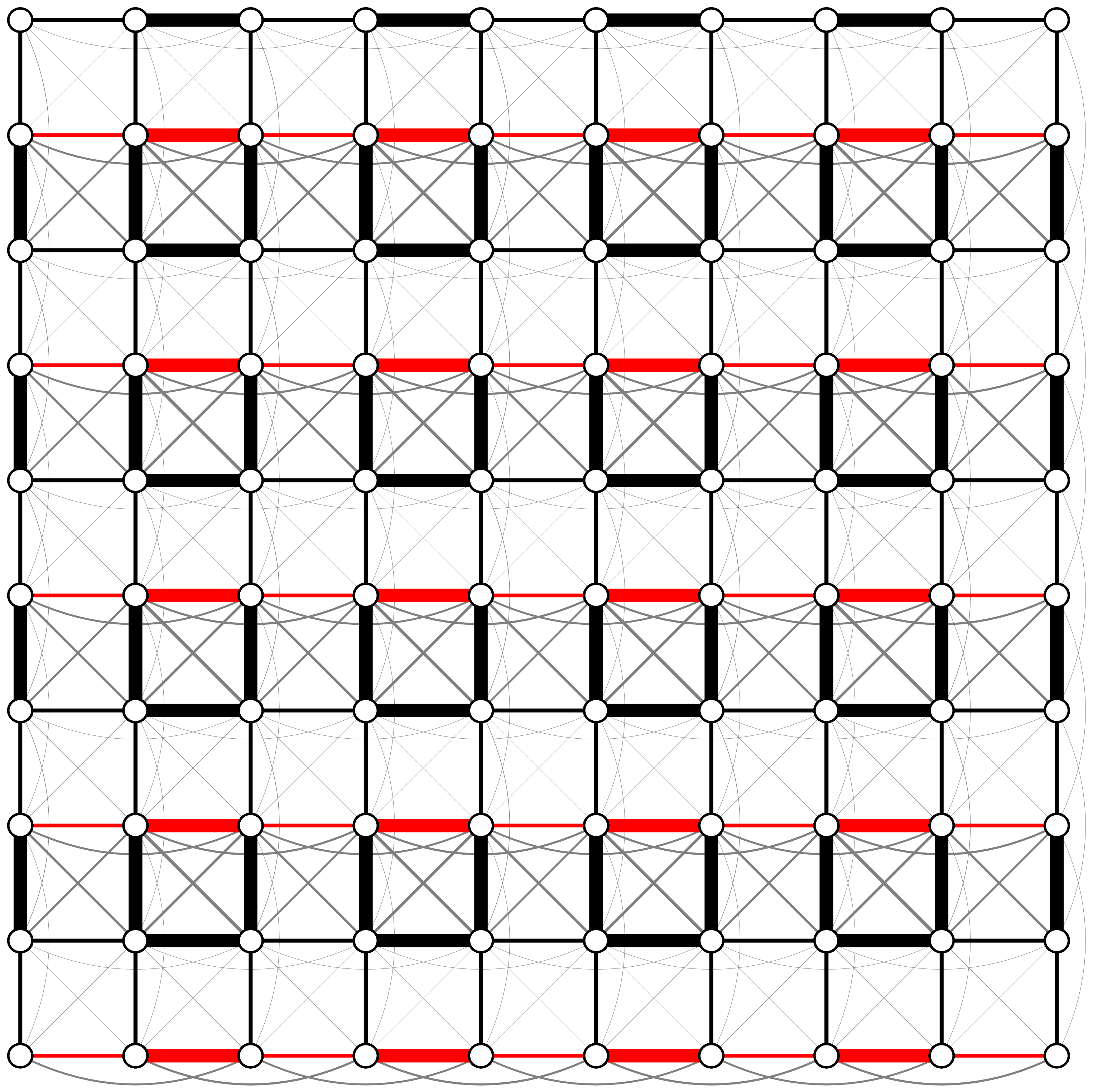
$$(p_x, p_y) = (0.06, 0.01)$$

$$(\tilde{p}_y^+, \tilde{p}_y^-) = (0.58, 0.57)$$



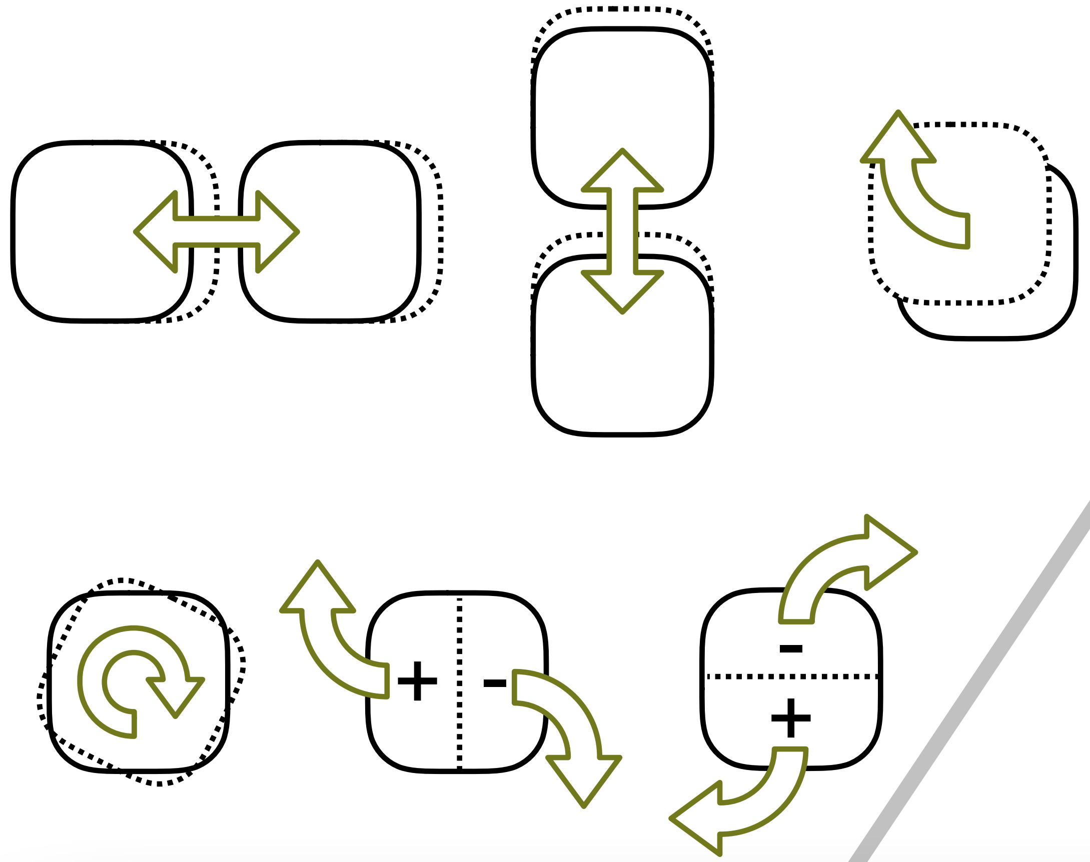
$$(\tilde{p}_x^+, \tilde{p}_x^-) = (0.50, 0.50)$$

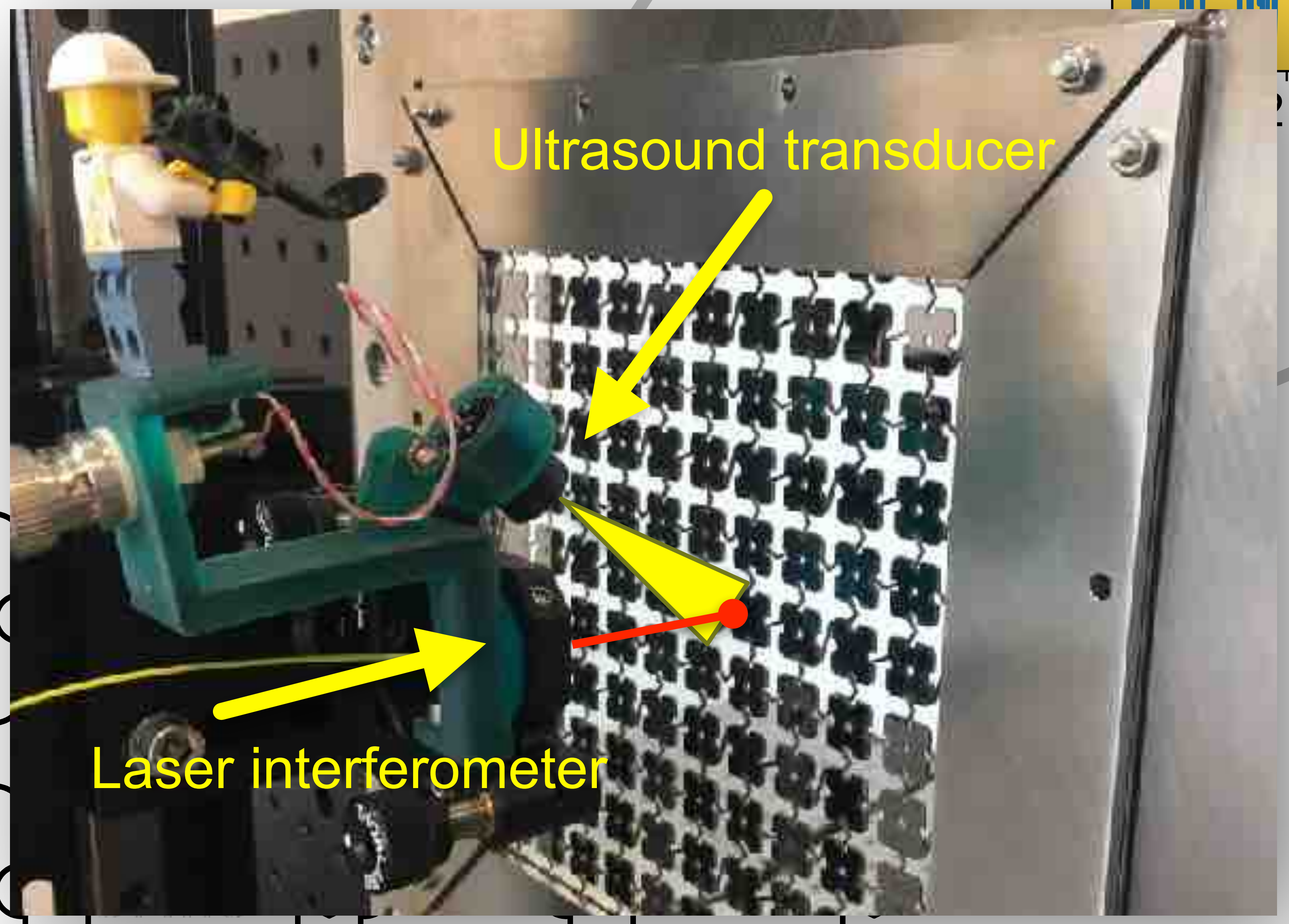


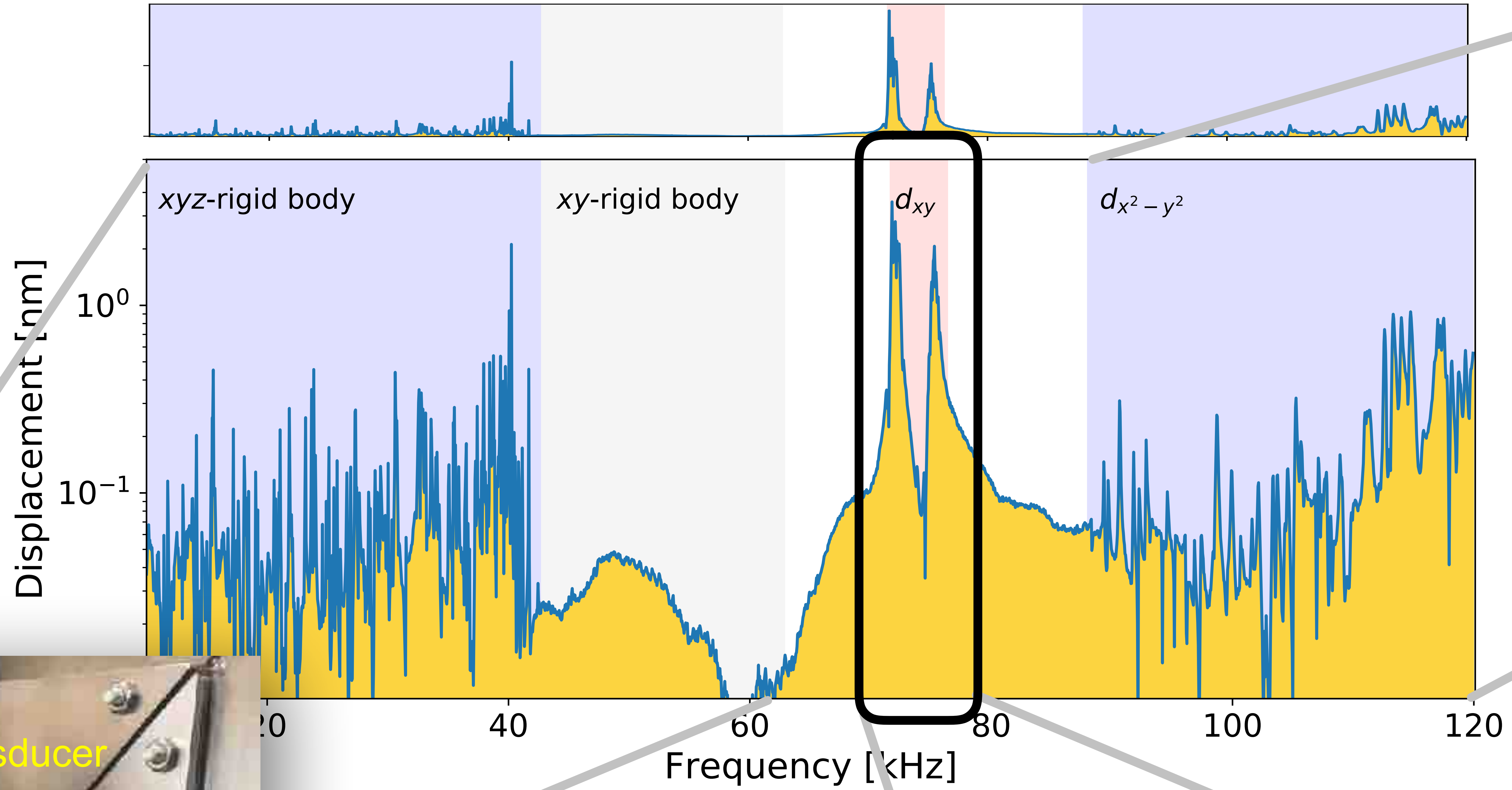


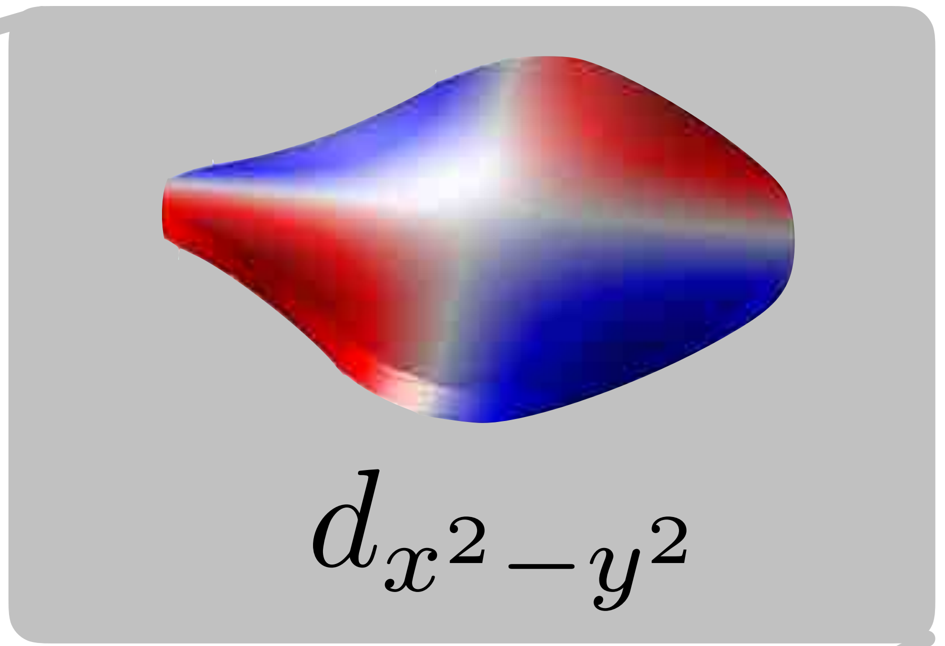
Serra-Garcia et al. Nature (2018)

Embedding



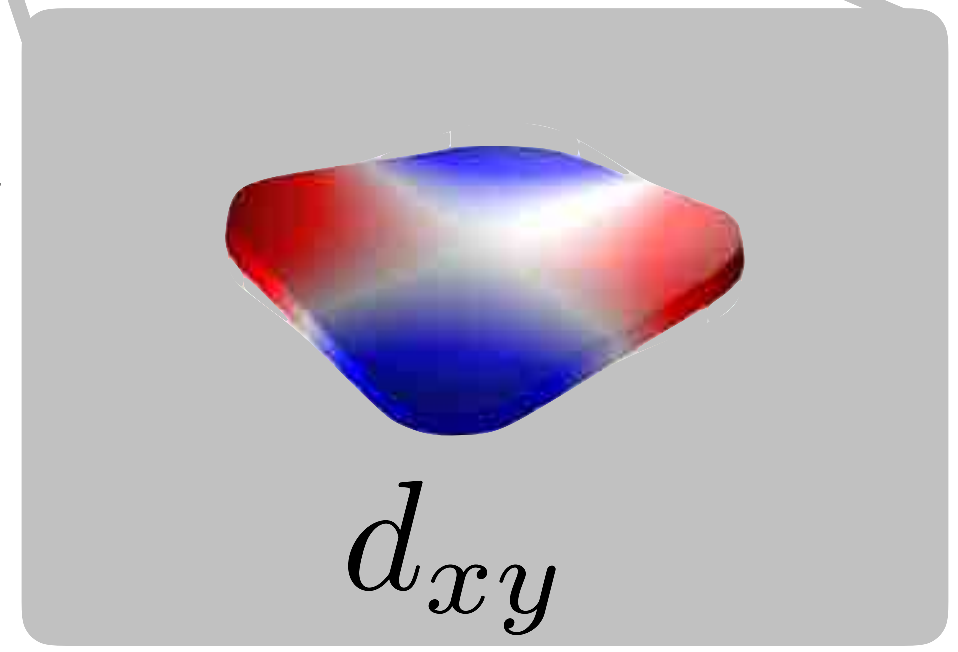


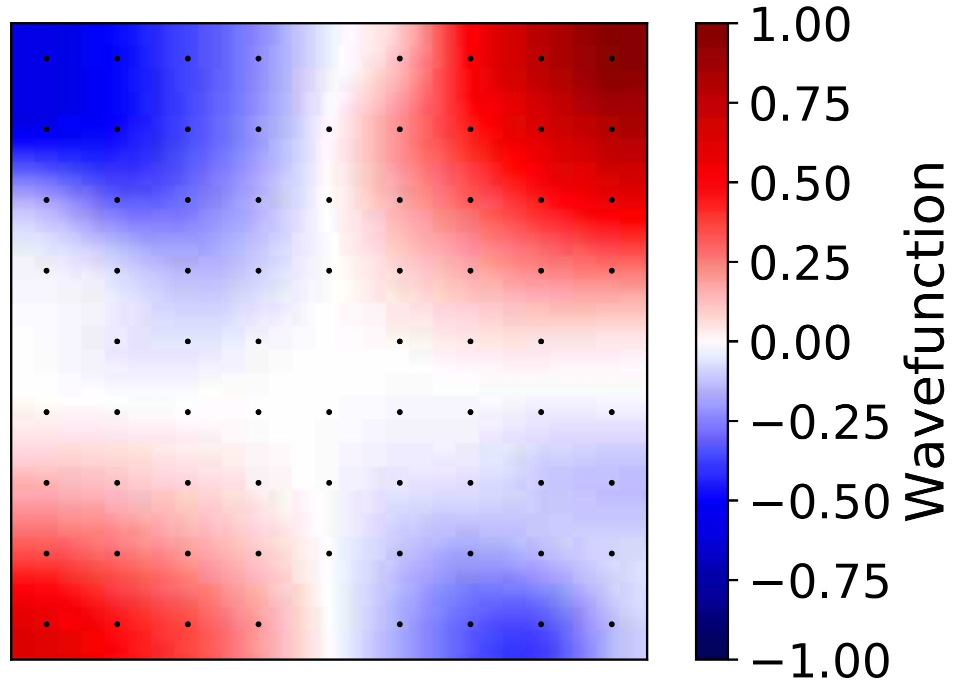


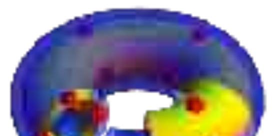


$\Delta z(\omega) \propto \psi_i^2(\omega) \sim 20\,\mathrm{pm}$

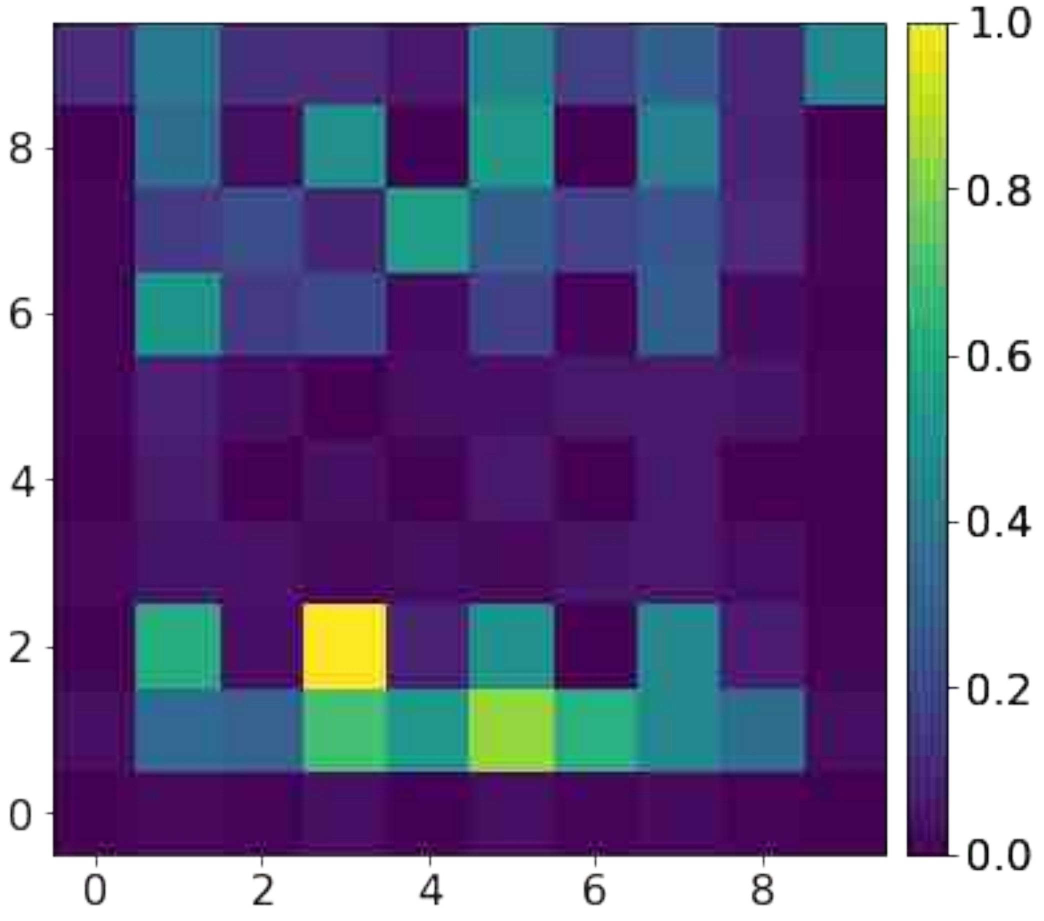
Energy $\propto \Delta z^2(\omega)$

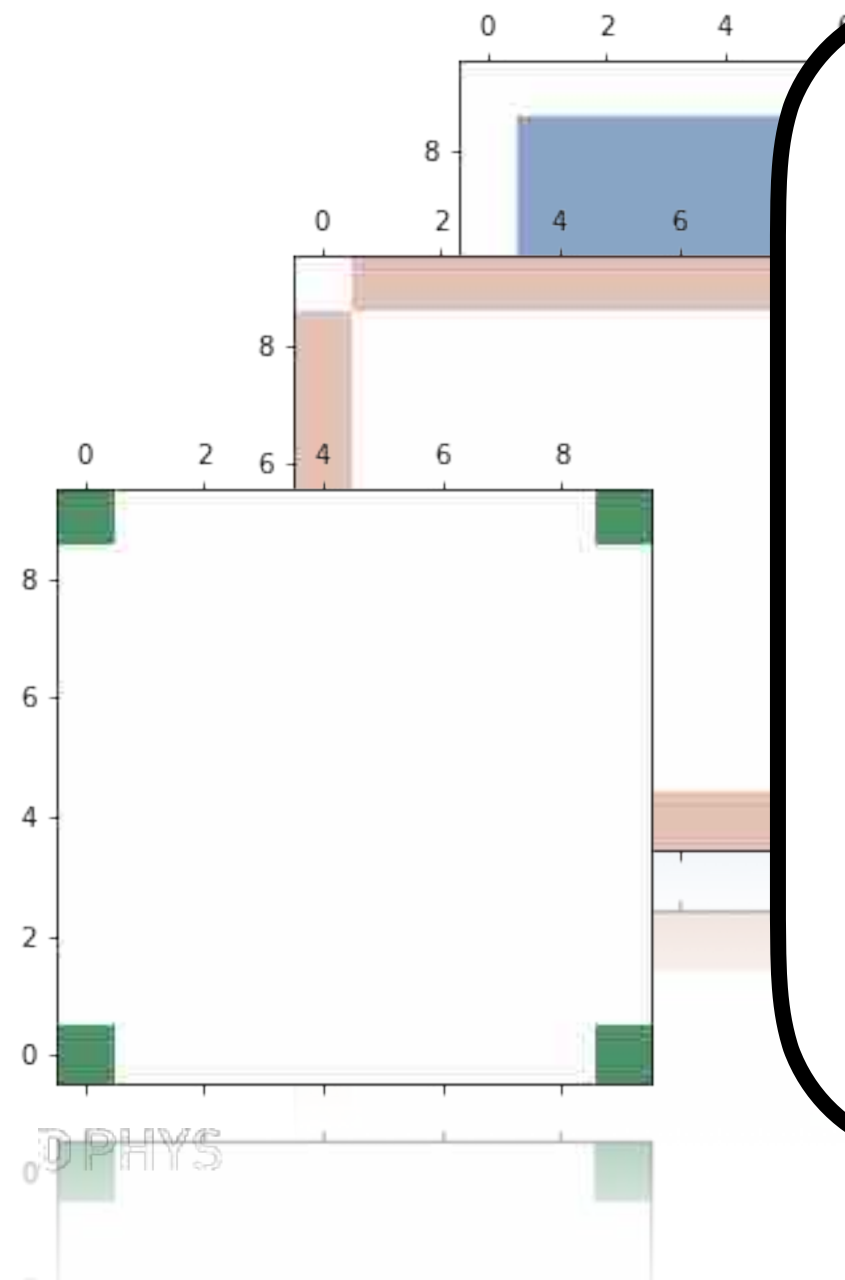


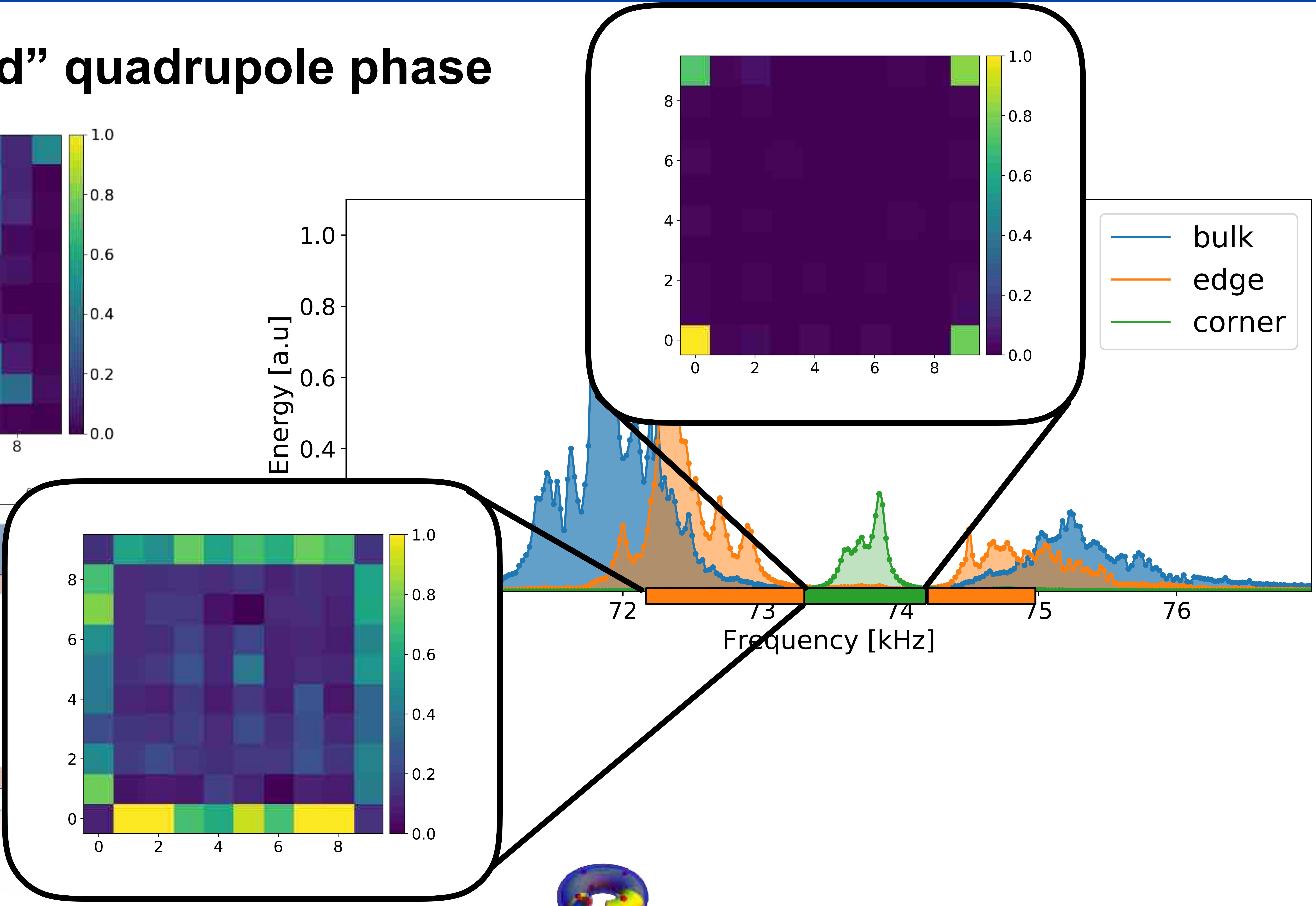




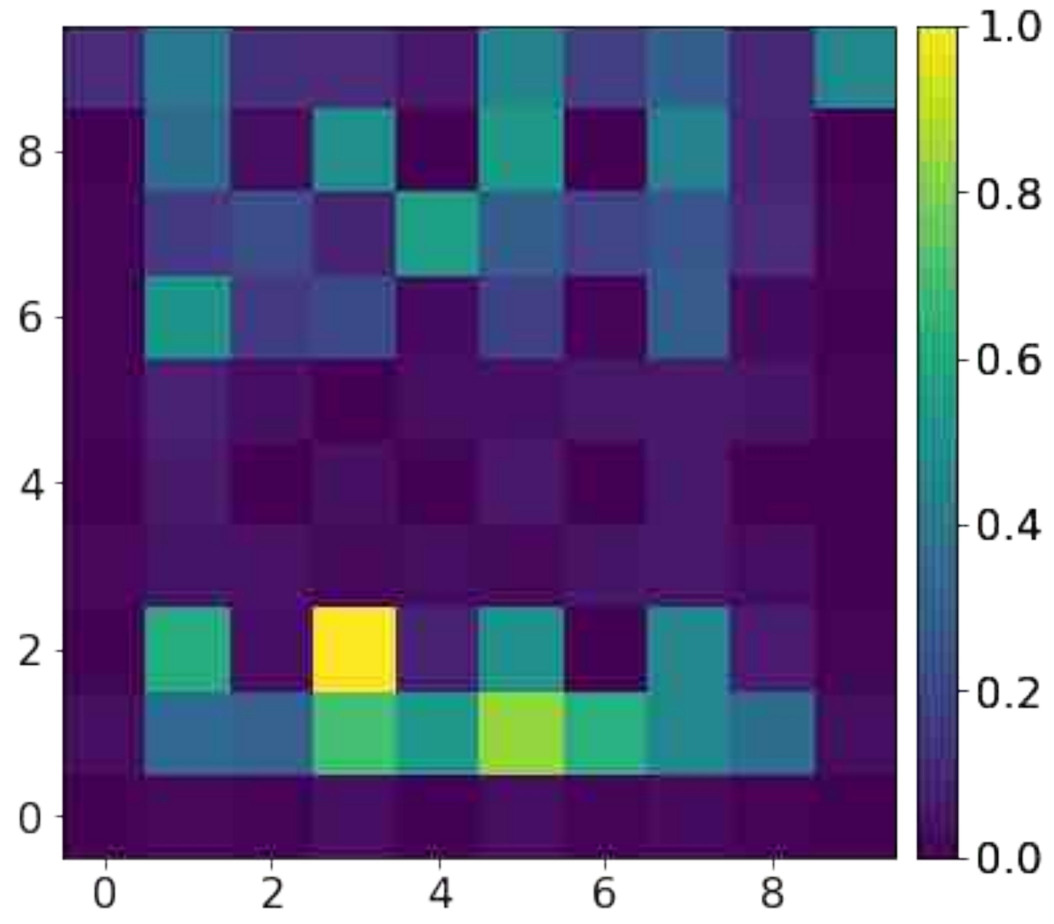
“Quantized” quadrupole phase

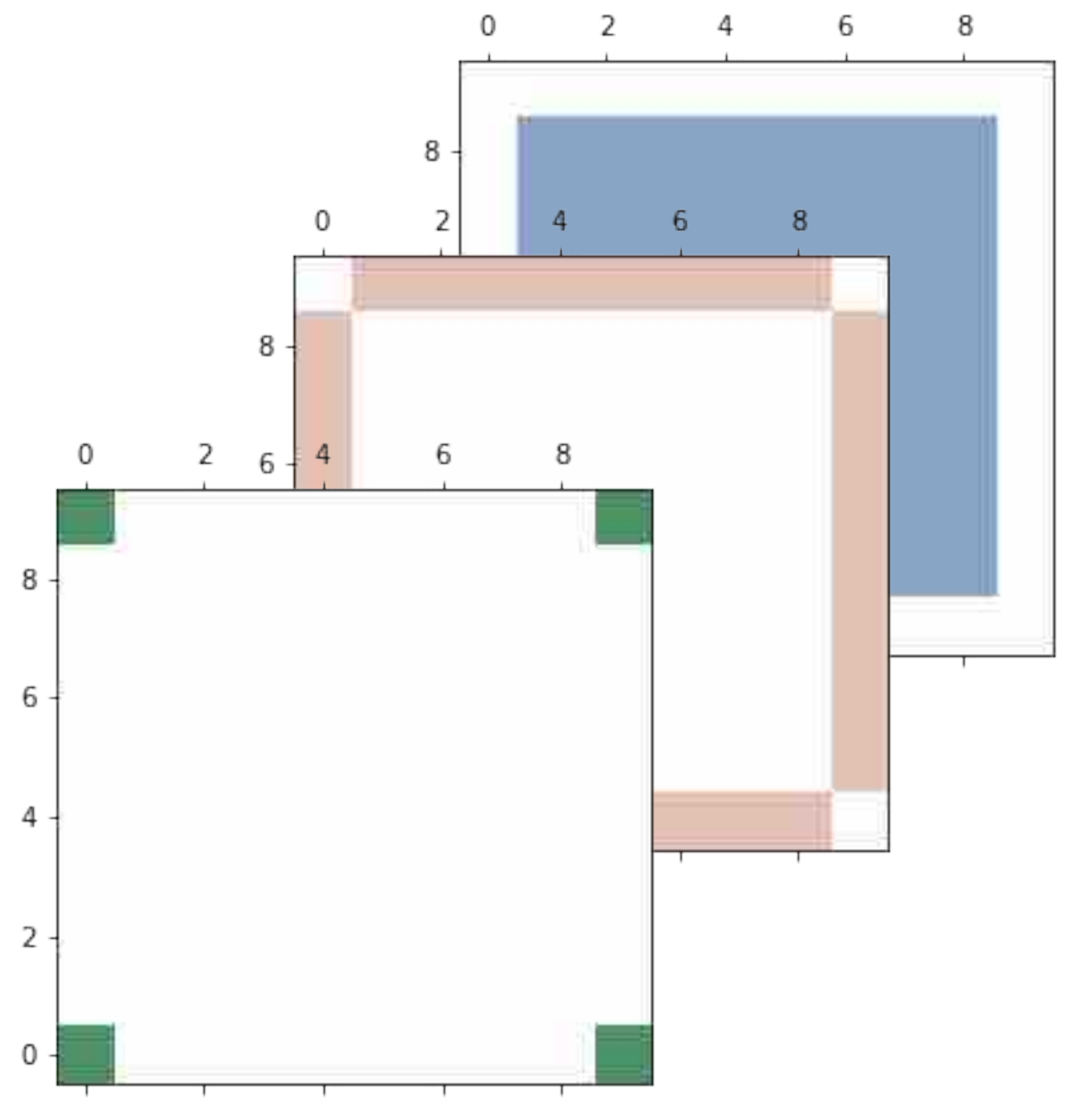


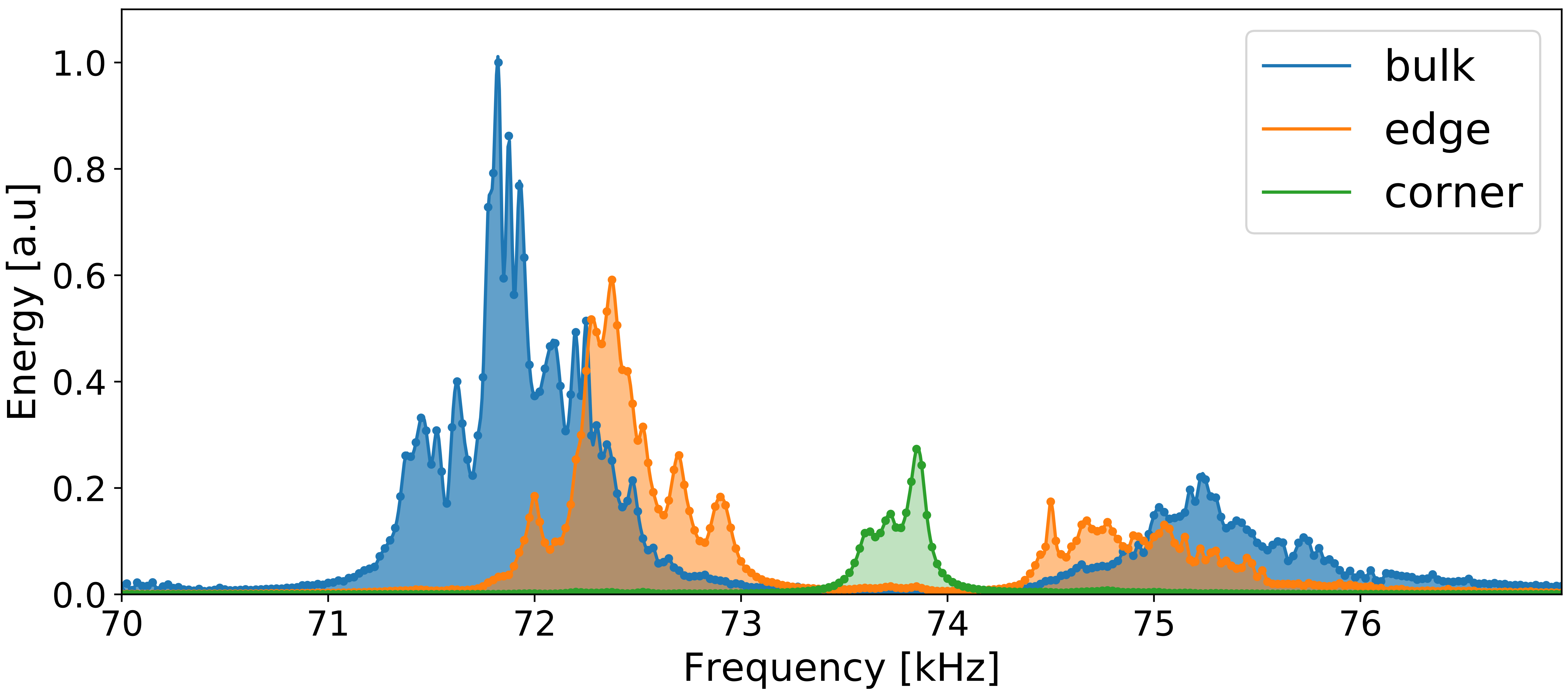




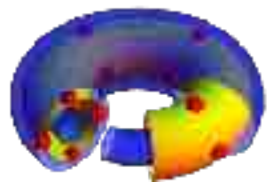
Trivial phase





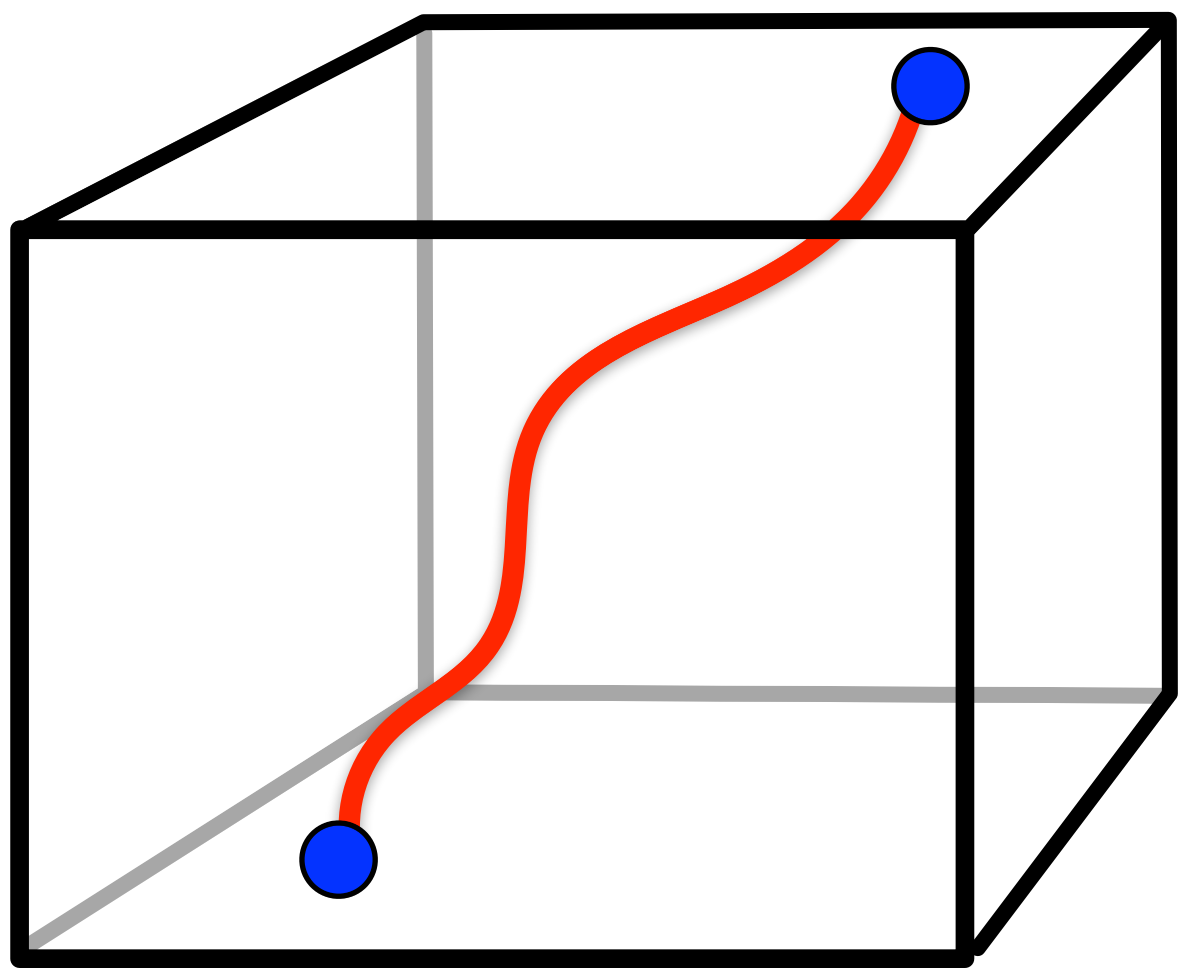


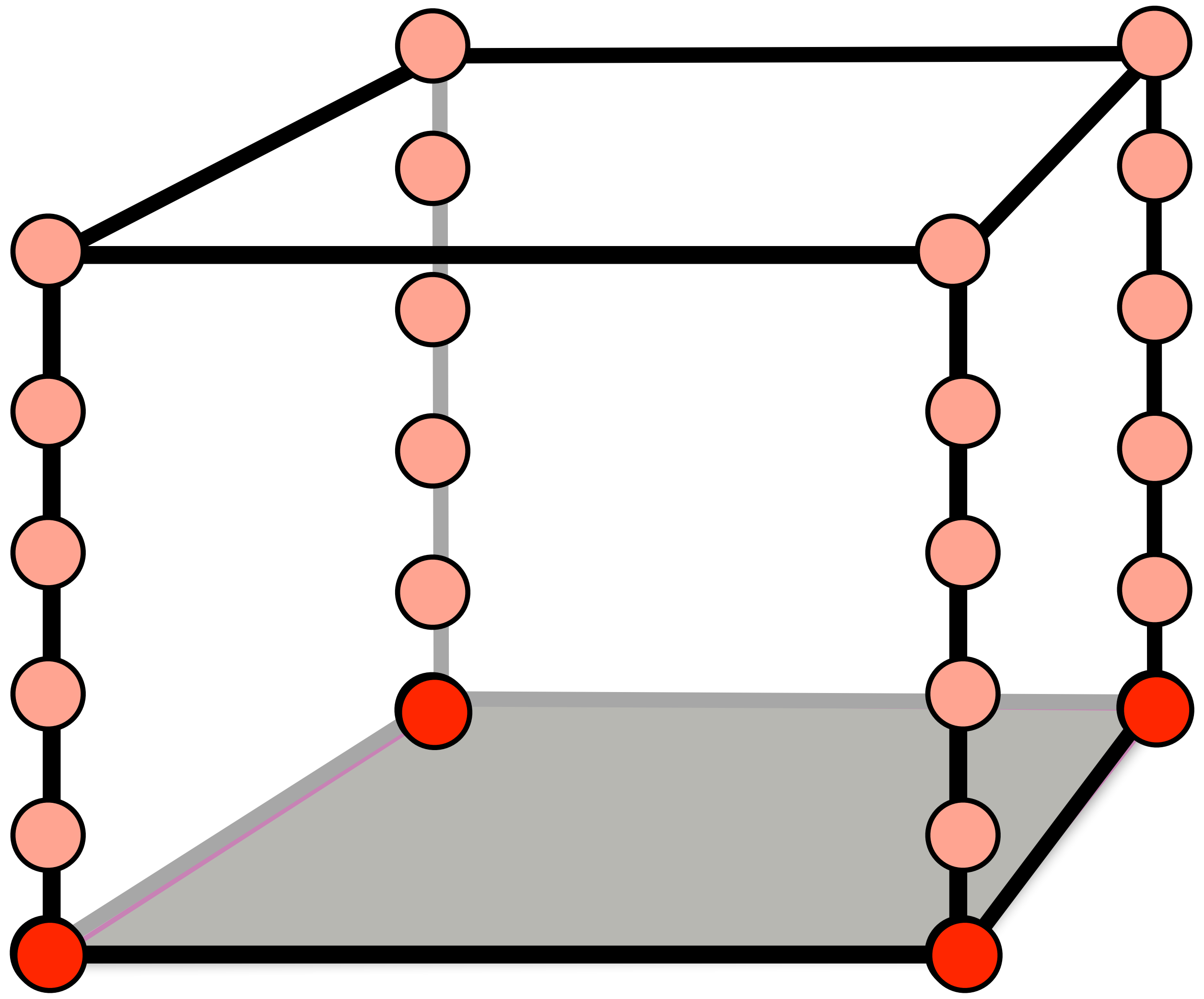
Serra-Garcia et al. Nature (2018)



Peterson et al. Nature (2018)

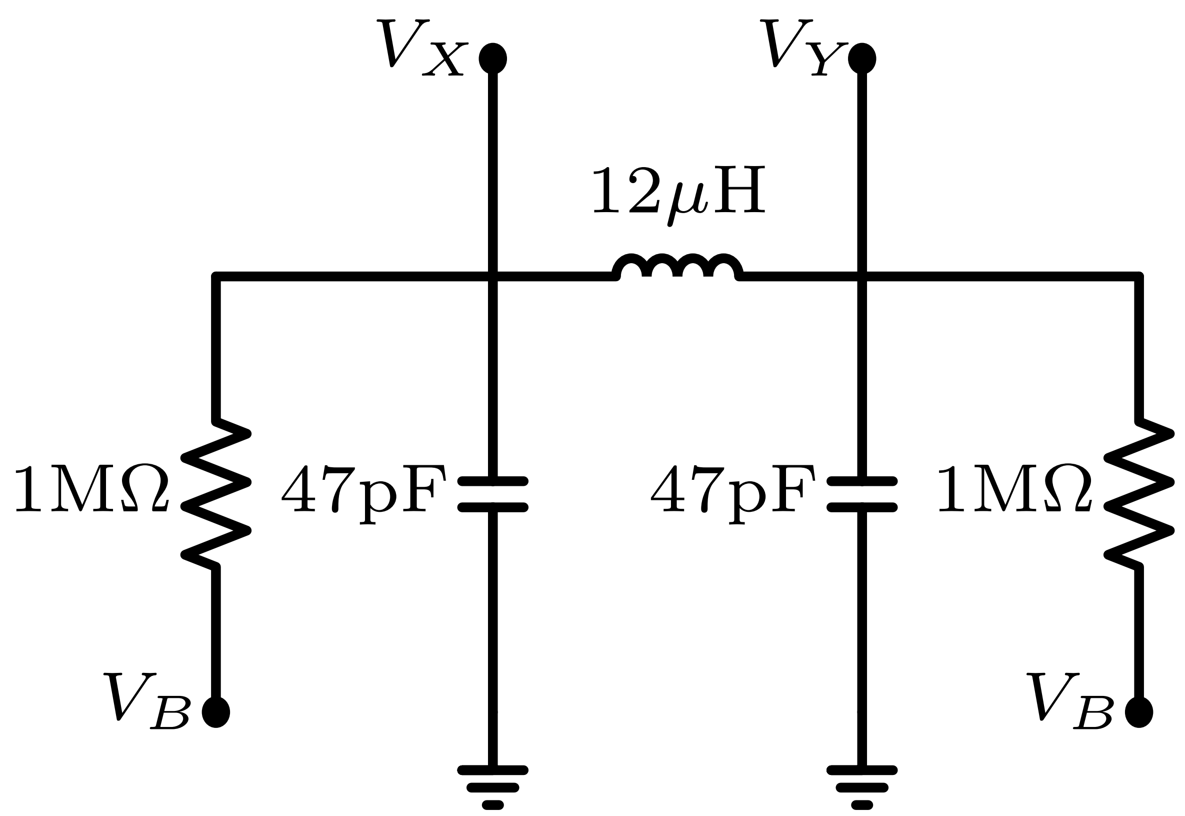
Potential for wave guiding

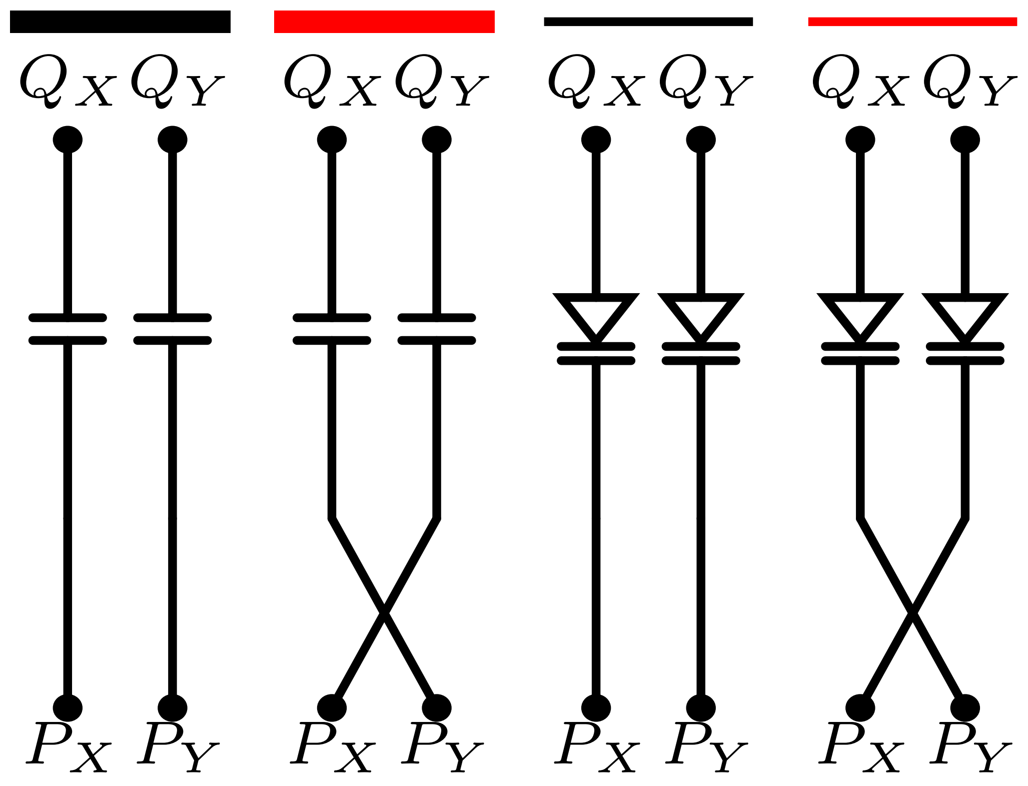




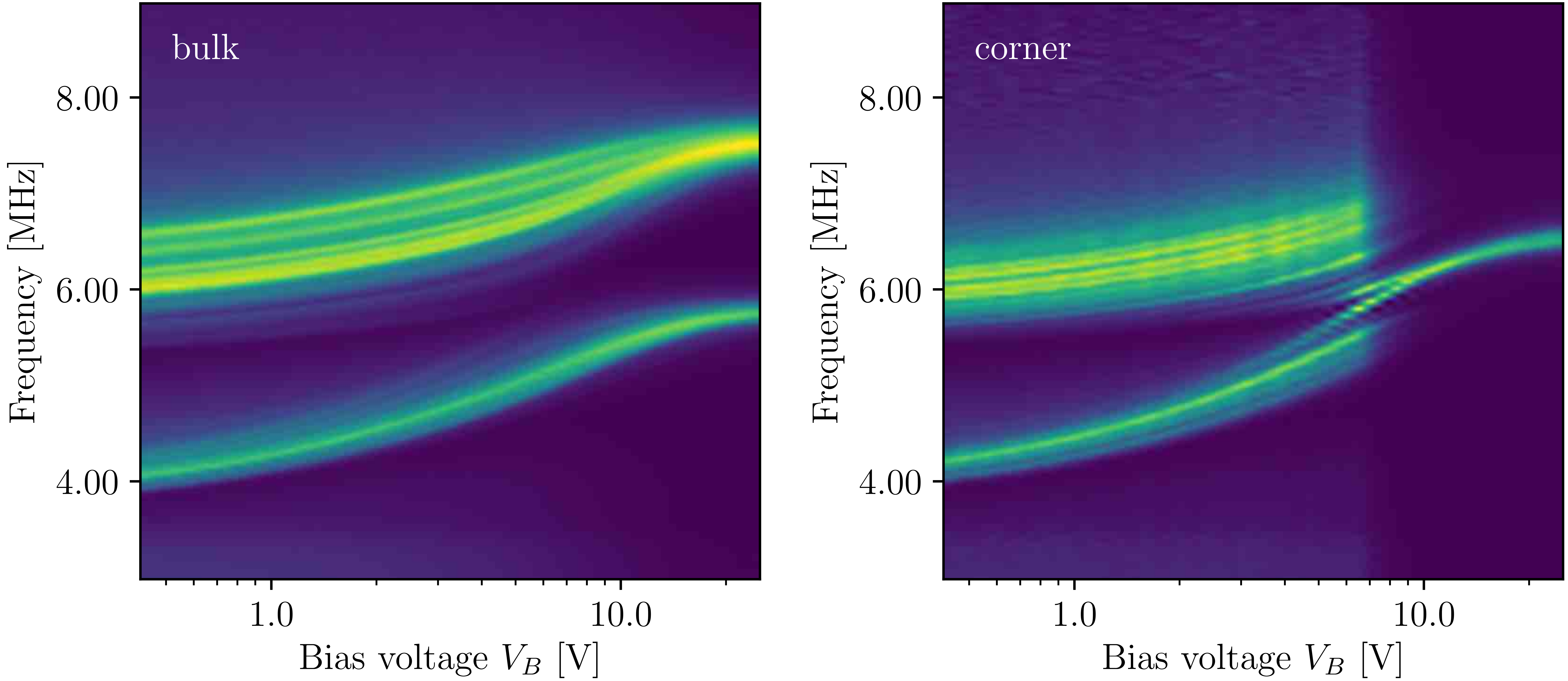
An LC-version

LC Quadrupole

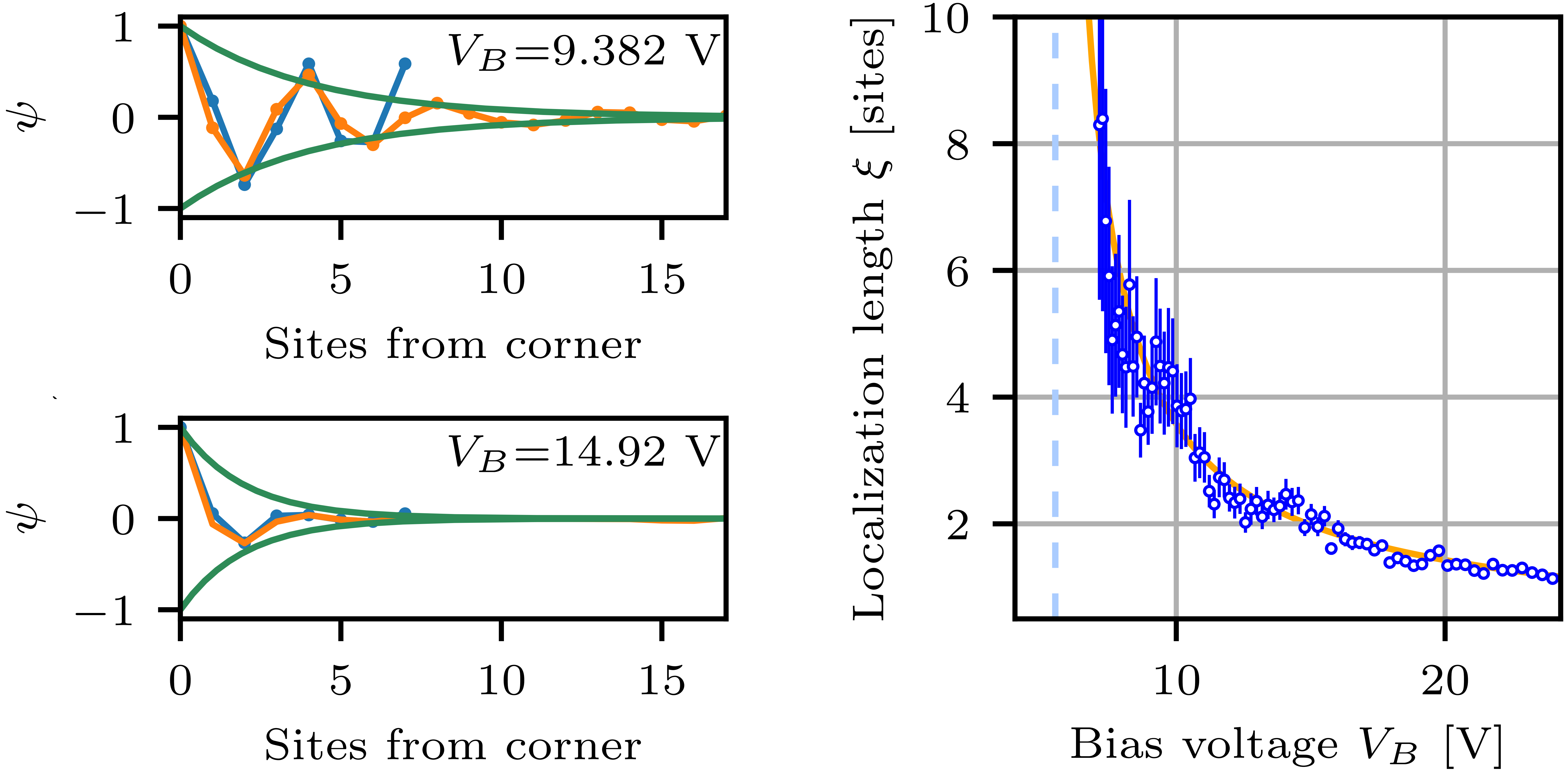




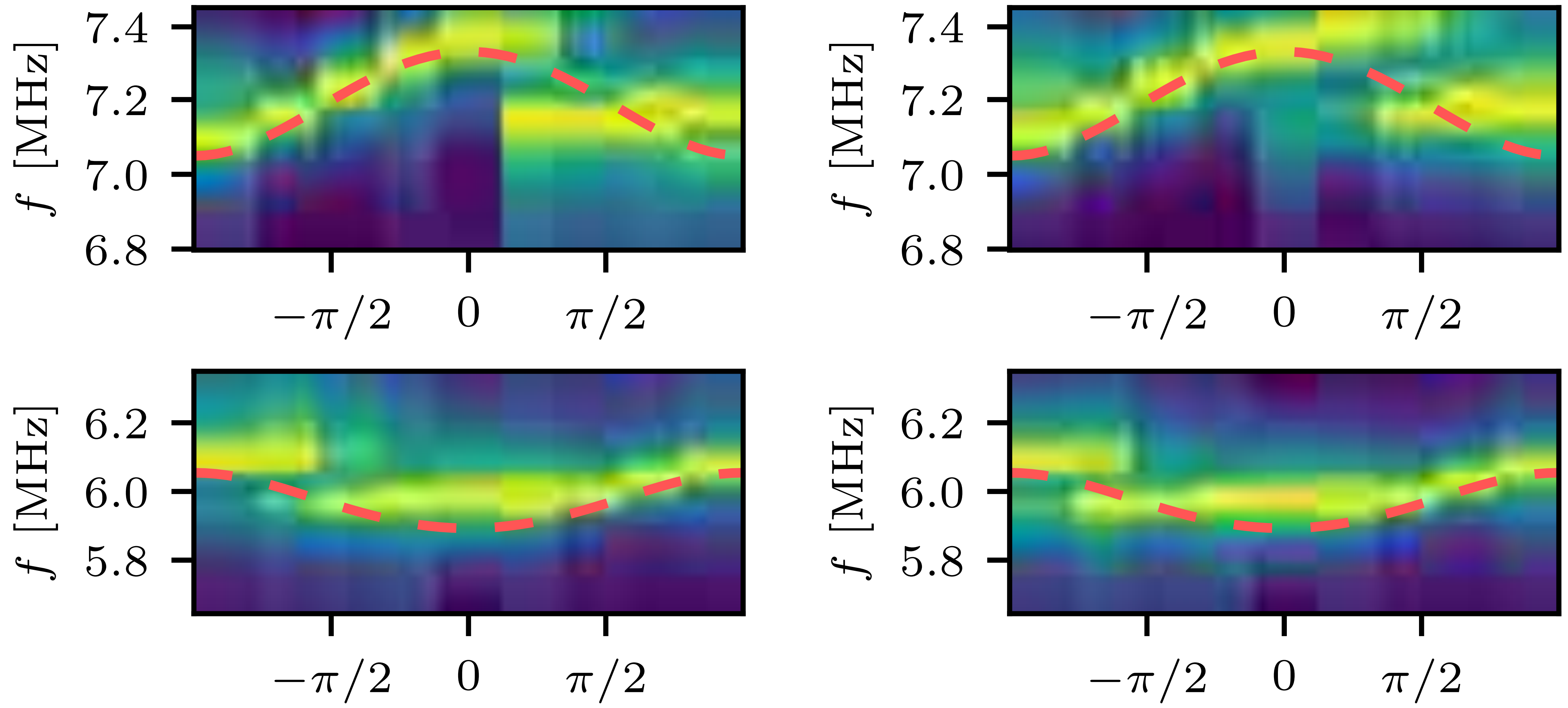
Quadrupole phase transition



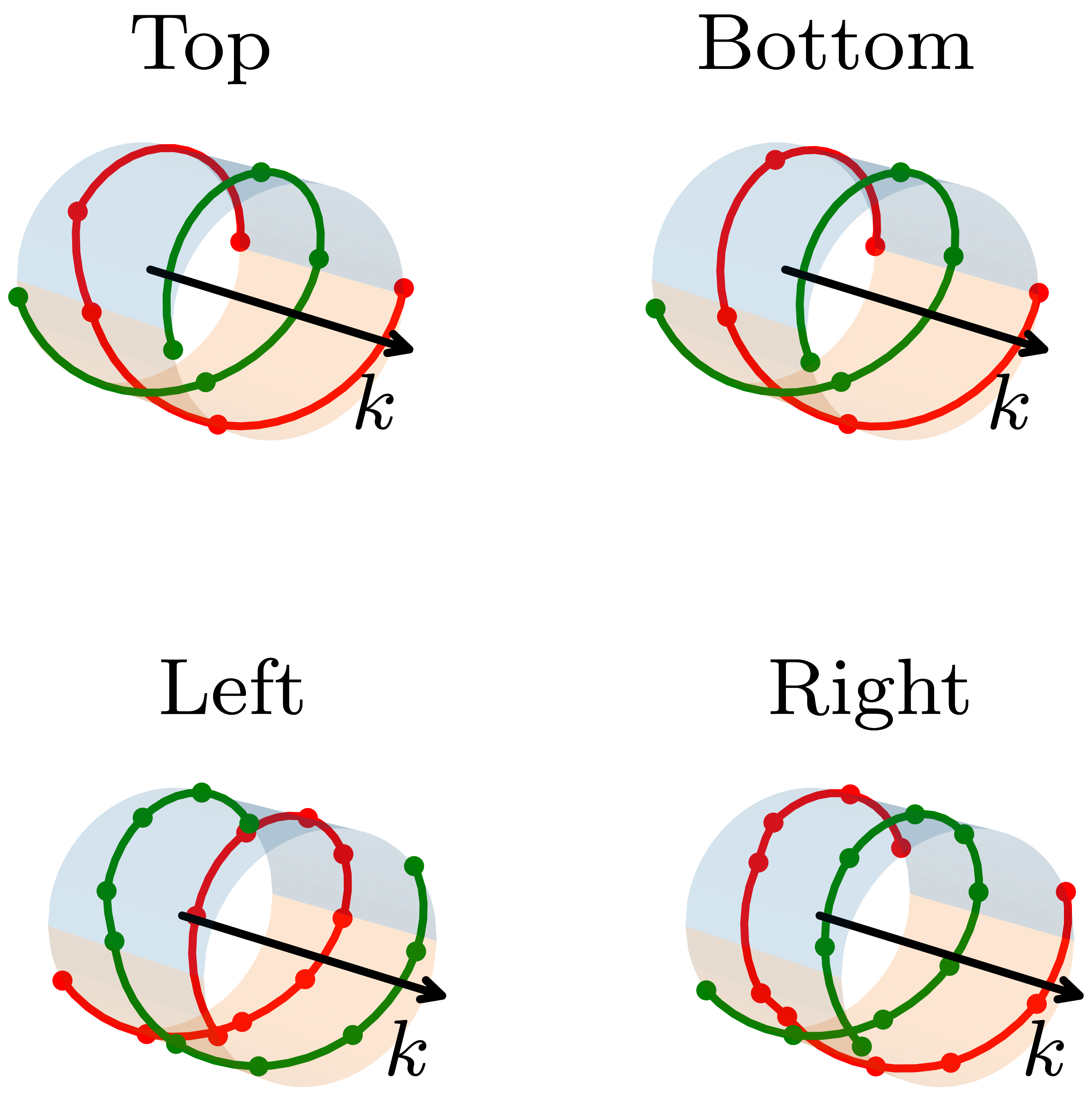
Localization length



Edge physics and Berry phase



A B



$$\varphi_k = \arg(\psi_k^A / \psi_k^B)$$

The electrons can do it too!

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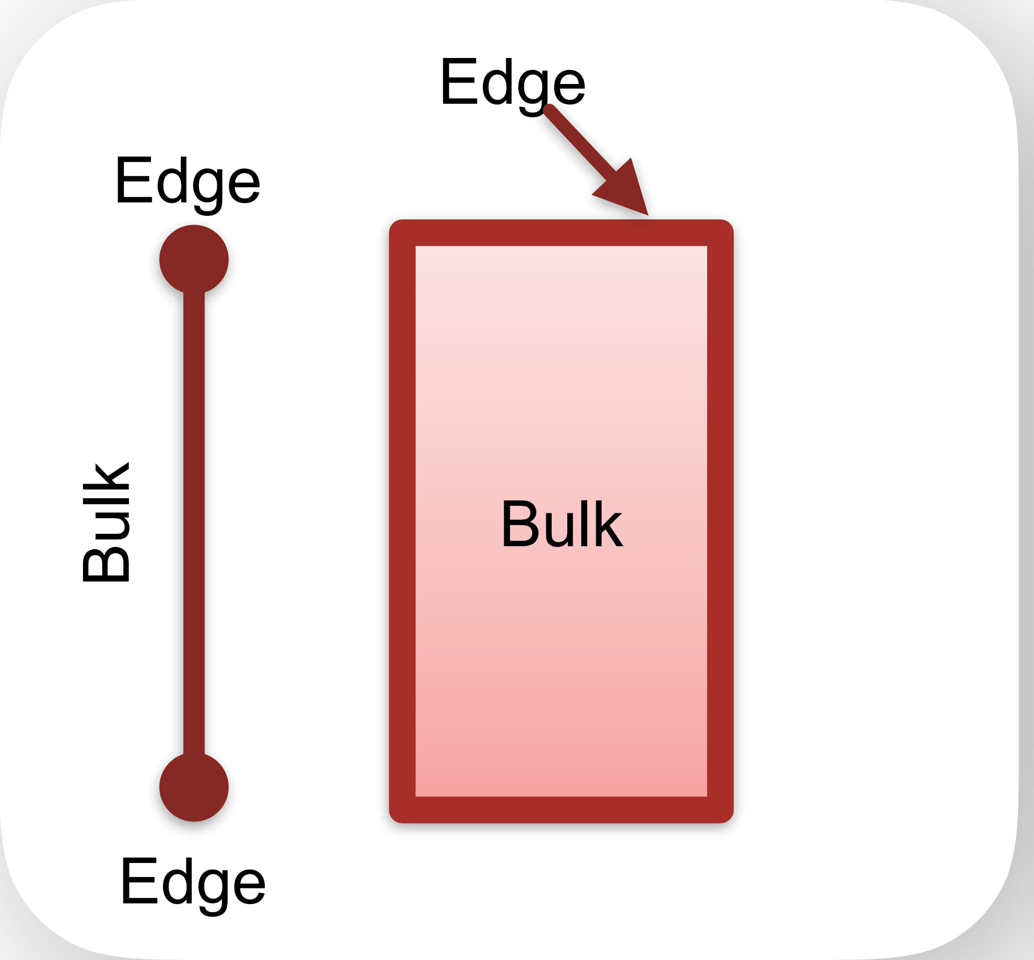
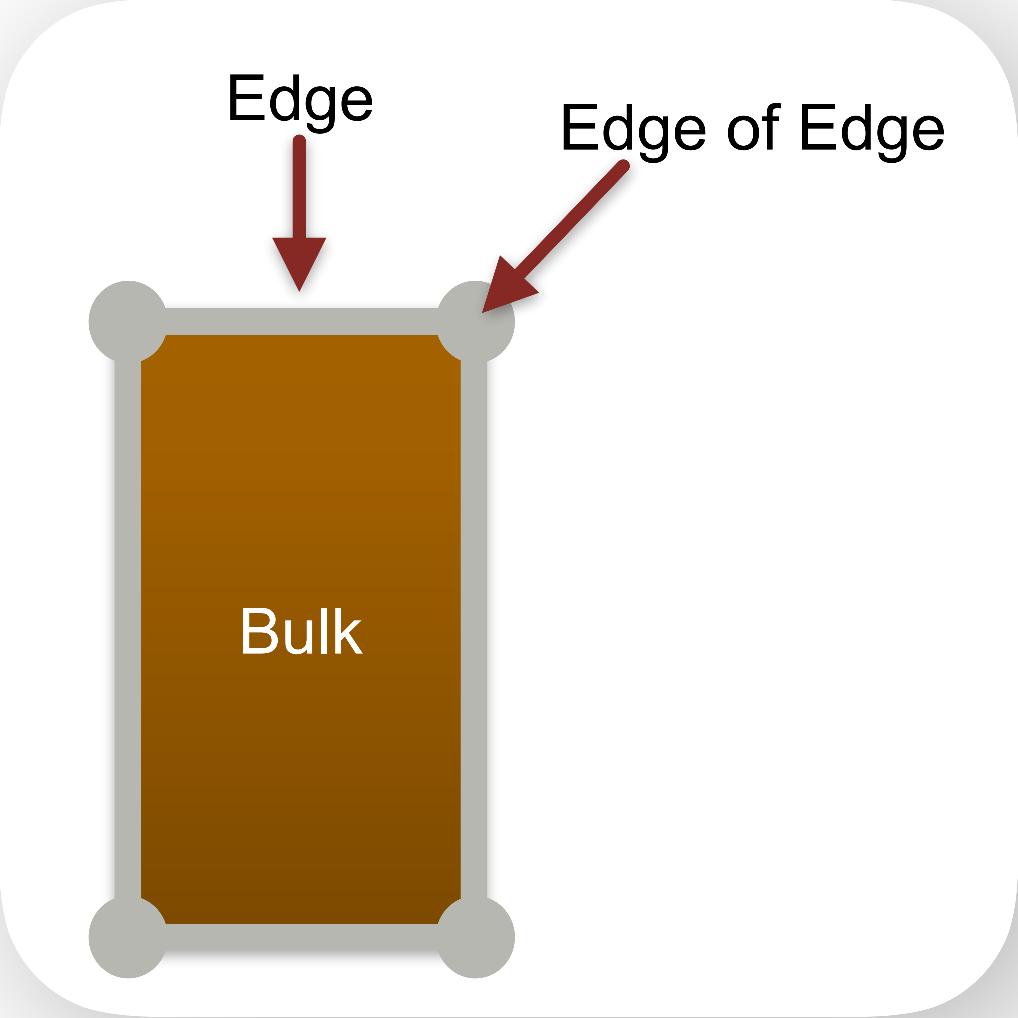
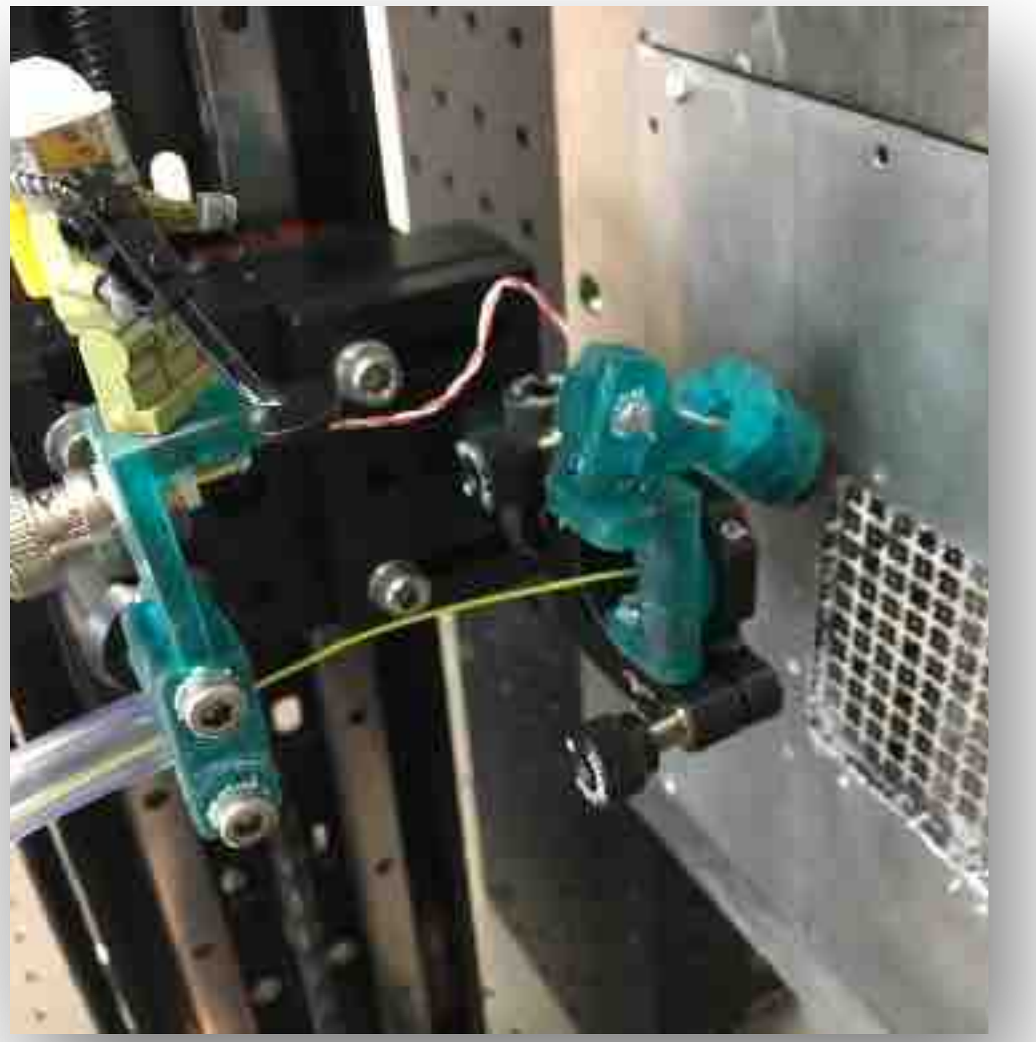
Chiara Daraio, Caltech







Science **349,** 47 (2015)
Nature Phys. **12**, 621 (2016)
PNAS **113**, E4767 (2016)
NJP **19**, 015013 (2017)
Adv. Mater. 1700540 (2017)
Nature (2018)
Nature Materials (2018)