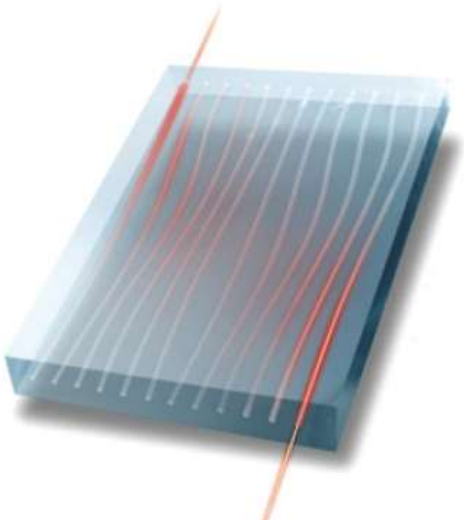
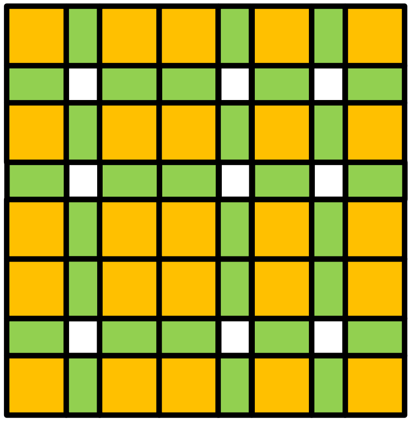
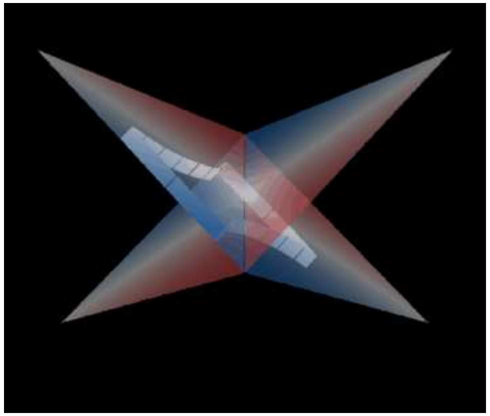
Topological pumps and topological quasicrystals







PRL **109**, 106402 (2012); PRL **109**, 116404 (2012); *PRL* **110**, 076403 (2013); *PRL* **111**, 226401 (2013); PRB **91**, 064201 (2015); PRL **115**, 195303 (2015), PRA **93**, 043827 (2016), PRB **93**, 245113 (2016), Nat. Phys. **12**, 350 (2016); Nat. Phys. **12**, 624 (2016); Nature **553**, 55 (2018), Nature **553**, 59 (2018) arXiv:1802.04173, arXiv:1804.01871, arXiv:1805.05209 & arXiv:1805.10670

QSIT Quantum Science and Technology

FNSNF

ETH zürich

Outline

Quantum Hall effect ⇔ Topological pumps

Photonic topological pump (1D)

Atomic pumps

Quasiperiodic systems

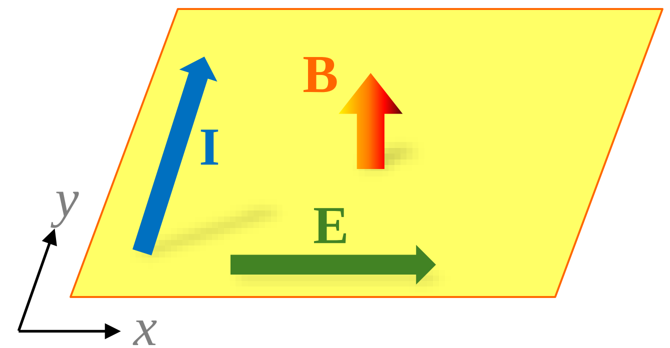
Four-dimensional QHE

Quantum Hall effect

- Landau gauge:

$$\mathcal{H} = \frac{p_x^{\,2}}{2m_a} + \frac{1}{2} m_a \omega_c^{\,2} \left(x - \frac{\hbar k_y}{m_a \omega_c} \right)^2$$

$$\psi_{p_y,n}(x,y) = e^{ip_y y}(\ldots)e^{-(x-p_y l^2)}$$



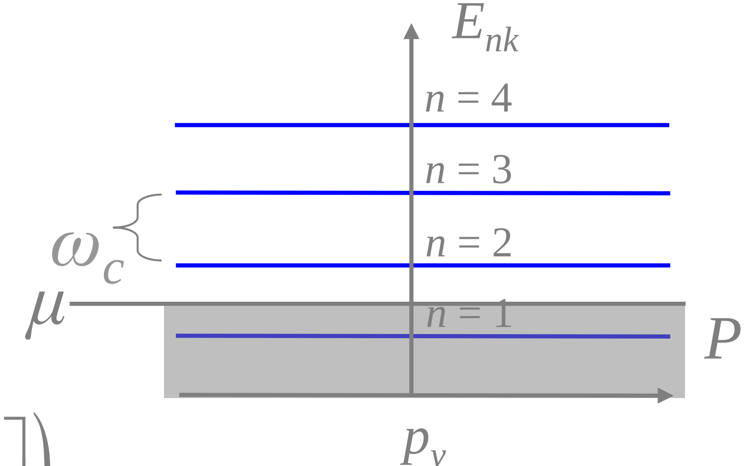
- Quantized Hall conductance

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

- Chern number

$$\nu = \int_0^{2\pi} \mathrm{d}\,\mathrm{p}_x \int_0^{2\pi} \mathrm{d}\,\mathrm{p}_y \Omega(\mathrm{p}_x, \mathrm{p}_y)$$

$$\Omega(\mathrm{p}_x, \mathrm{p}_y) = \frac{1}{2\pi i} \mathrm{Tr}\left(P \left[P_{p_x}, P_{p_y} \right] \right)$$



$$P = \sum_n |\psi_n\rangle\langle\psi_n|$$

Quantum Hall effect

- Landau gauge:

$$\mathcal{H} = \frac{p_x^{\ 2}}{2m_a} + \frac{1}{2} m_a \omega_c^{\ 2} \left(x - \frac{\hbar k_y}{m_a \omega_c} \right)^2$$

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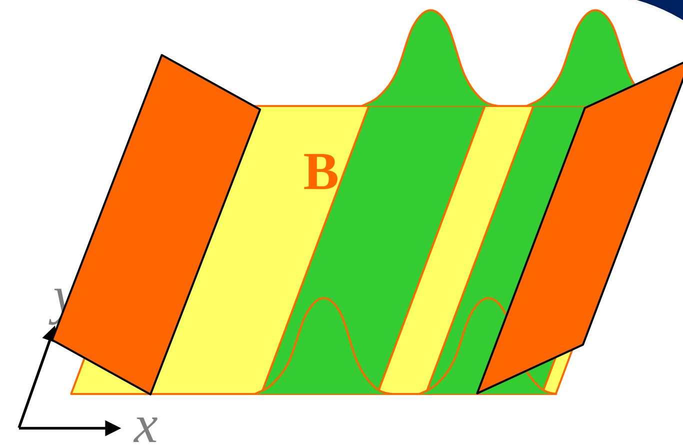
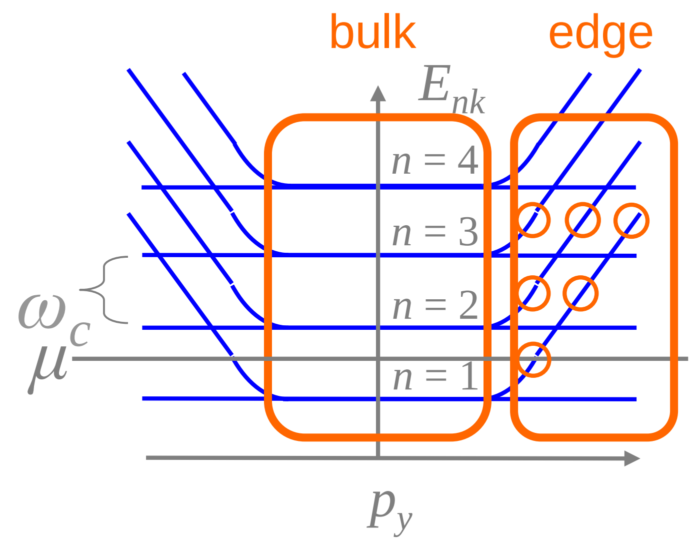


- Confining potential:

  Chiral edge modes

- Chern number

  Number of chiral edge modes



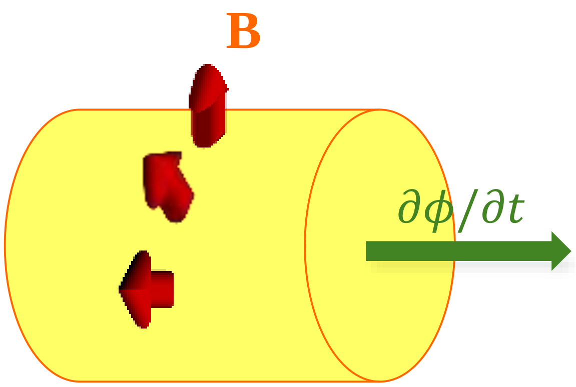
See Yasuhiro's talk and references therein

Laughlin/Halperin's argument

- Landau gauge:

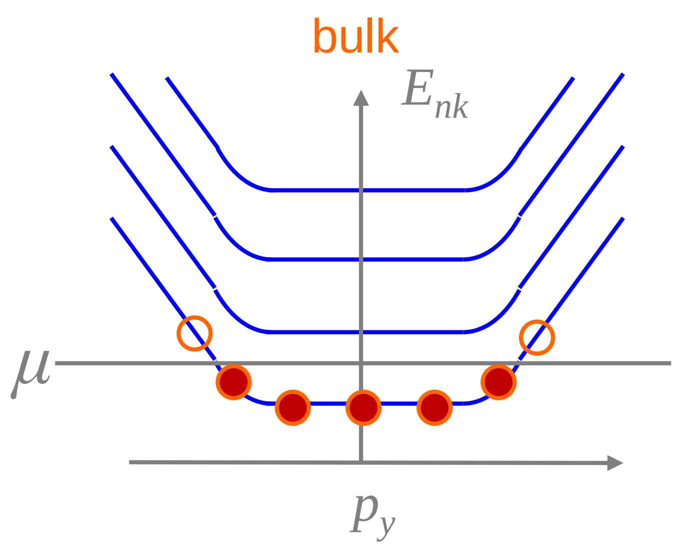
$$\mathcal{H} = \frac{p_x^{\ 2}}{2m_a} + \frac{1}{2} m_a \omega_c^{\ 2} \left(x - \frac{\hbar k_y}{m_a \omega_c} \right)^2$$

$$\psi_{p_y,n}(x,y) = e^{ip_y y} (\ldots) e^{-(x - p_y l^2)}$$



- Chern number

  Number of charges moved over a pump cycle



Laughlin/Halperin's argument

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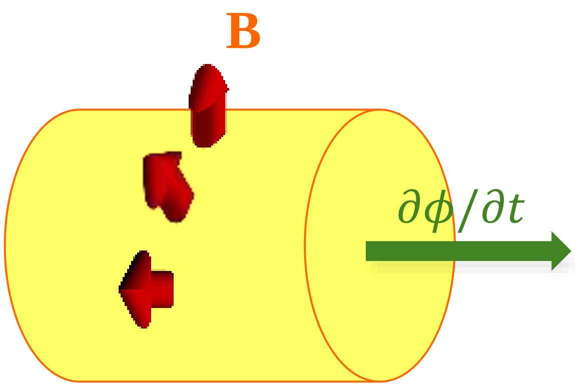

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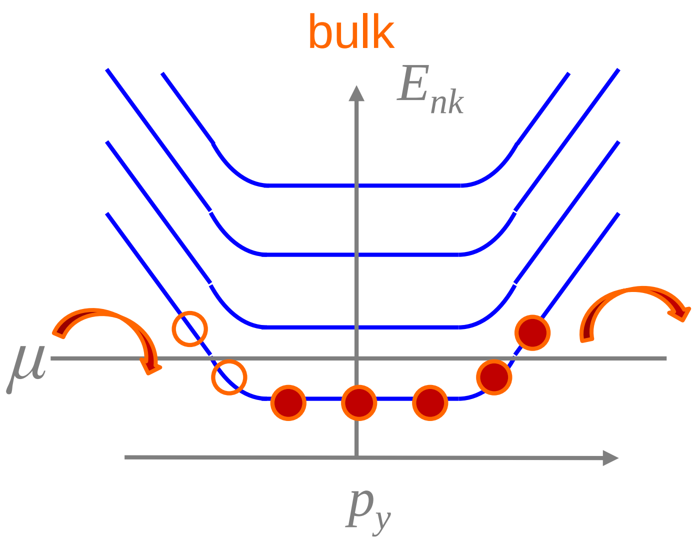
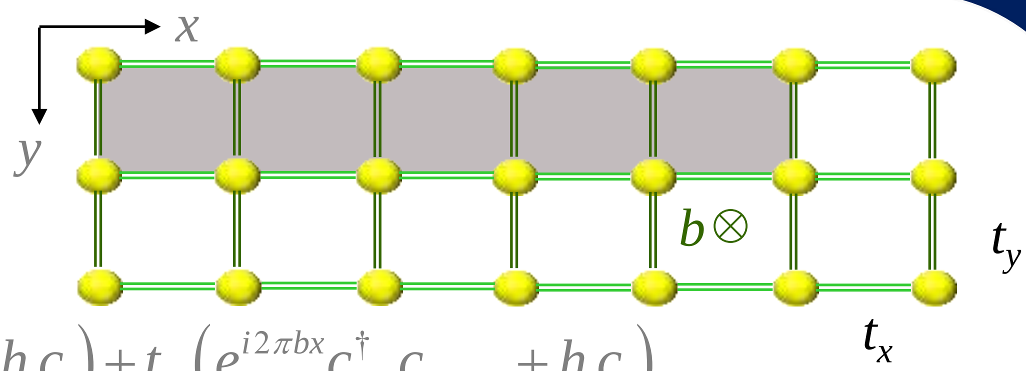


Hofstadter model



- Hamiltonian:

$$\mathcal{H} = \sum_{x,y} t_x \left(c^\dagger_{\mathrm{x,y}} c_{x+1,\mathrm{y}} + h.c. \right) + t_y \left(e^{i2\pi bx} c^\dagger_{\mathrm{x,y}} c_{\mathrm{x,y}+1} + h.c. \right)$$

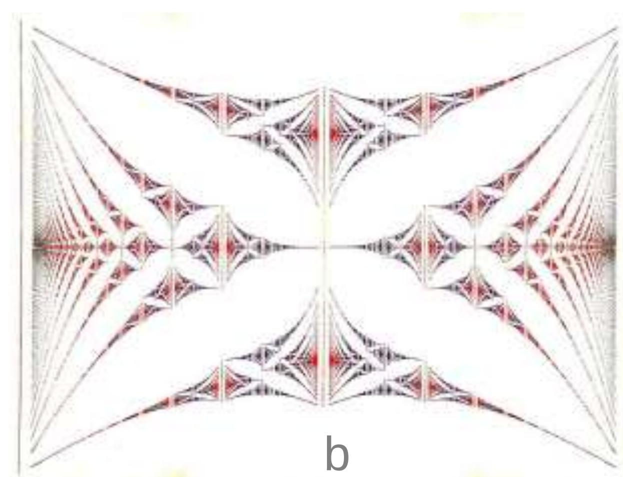
$$= \sum_{\mathrm{x},k_y} t \left(c^\dagger_{\mathrm{x},k_y} c_{x+1,k_y} + h.c. \right) + 2t_y \cos(2\pi bx + k_y) c^\dagger_{\mathrm{x},k_y} c_{\mathrm{x},k_y}$$

- Spectrum:

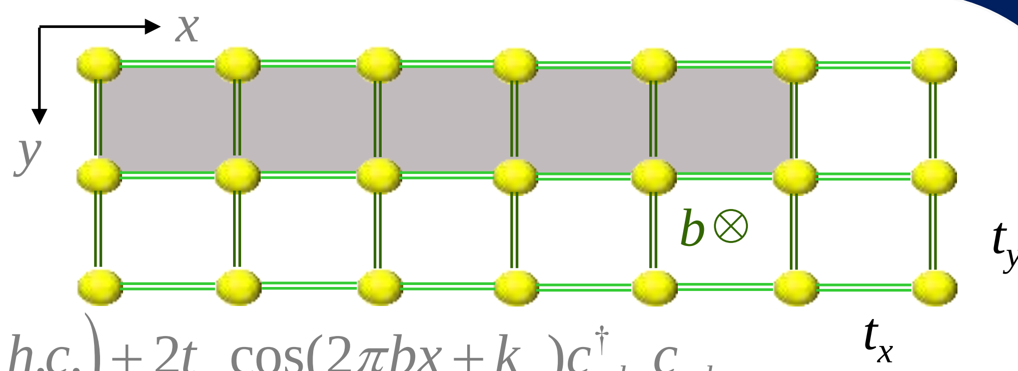
  Harper, PPSL A **68**, 874 (1955)
  Azbel, JETP **19**, 634 (1964)
  Hofstadter, PRB **14**, 2239 (1976)

$$b = p/q \qquad q\text{-}1 \text{ gaps}$$

$$c_{\mathrm{x,y}} = \sum_{k_y} e^{ik_y y} c_{\mathrm{x}}$$



Hofstadter model



- Hamiltonian:

$$\mathcal{H} = \sum_{x,k_y} t_x \left(c^\dagger_{x,k_y} c_{x+1,k_y} + h.c. \right) + 2t_y \cos(2\pi b x + k_y) c^\dagger_{x,k_y} c_{x,k_y}$$

- Quantized Hall conductance:

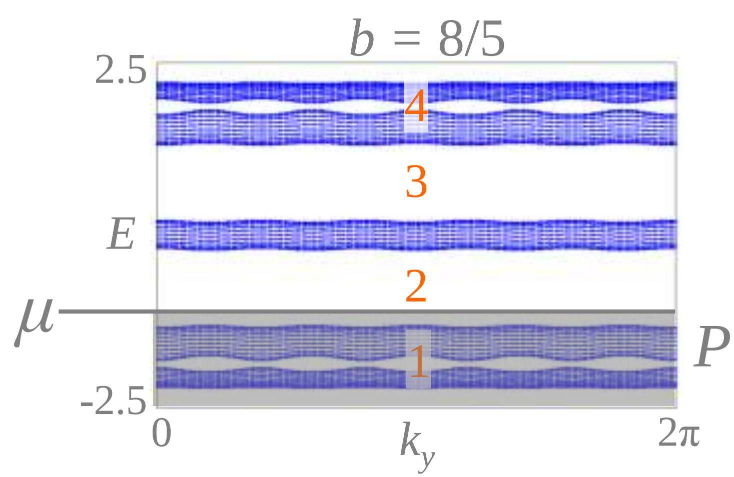
TKNN, PRL **49**, 405 (1982)

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

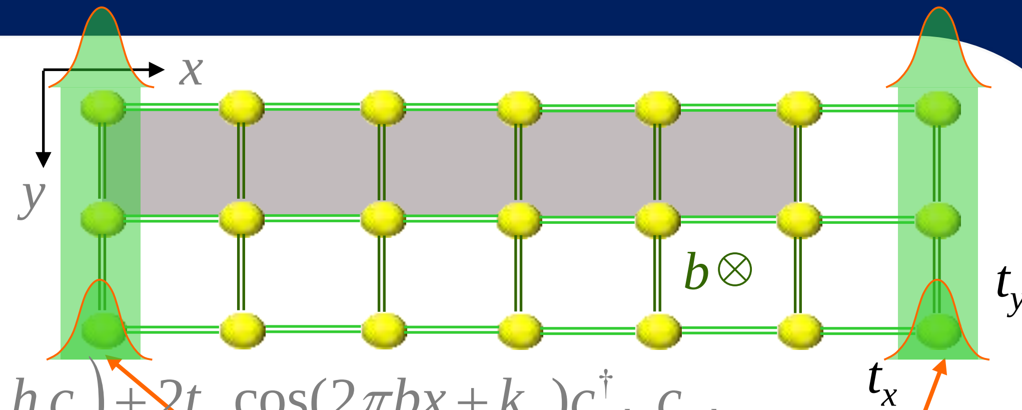
- Chern numbers:

$$\nu = \int_0^{2\pi} d\,p_x \int_0^{2\pi} d\,p_y \Omega(p_x, p_y)$$

$$\Omega(p_x, p_y) = \frac{1}{2\pi i} \mathrm{Tr}\left(P \left[P_{p_x}, P_{p_y} \right] \right)$$



Hofstadter model



- Hamiltonian:

$$\mathcal{H}=\sum_{x,k_y} t_x\left(c^\dagger_{\mathrm{x},k_y}c_{x+1,k_y}+h.c.\right)+2t_y\cos(2\pi bx+k_y)c^\dagger_{\mathrm{x},k_y}c_{\mathrm{x},k_y}$$

- Quantized Hall conductance:

TKNN, PRL **49**, 405 (1982)

$$\sigma_{xy}=\nu_r\frac{e^2}{h}$$

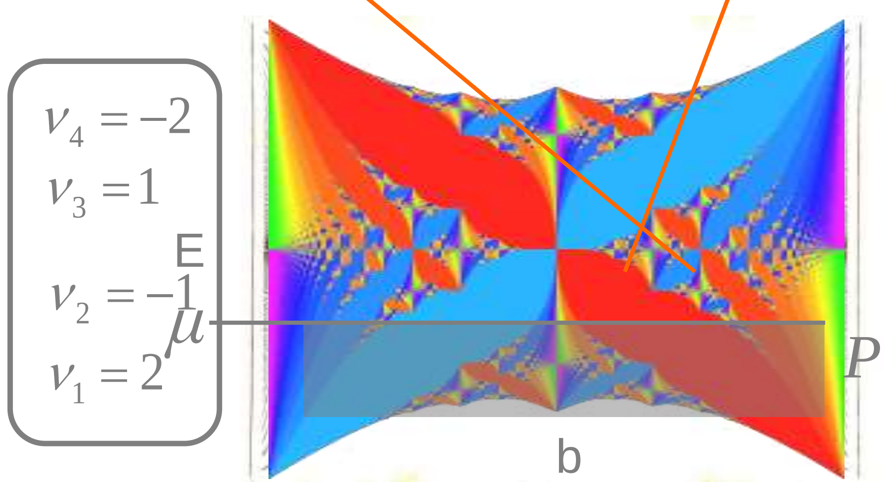
- Chern numbers:

$$b=p/q$$

$$r=p\nu_r+qm_r$$

$$|\nu_r|<\frac{q}{2}$$

D. Osadchy and J. E. Avron, J. Math. Phys. **42**, 5665 (2001)



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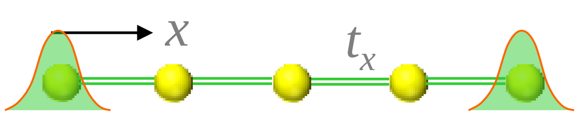
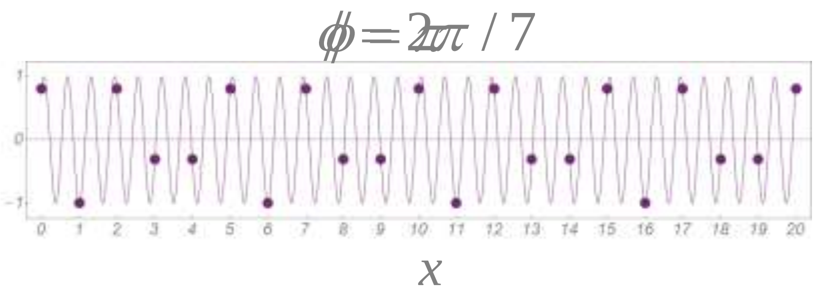
Topological pump (1+1)

- Harper model as a topological pump:

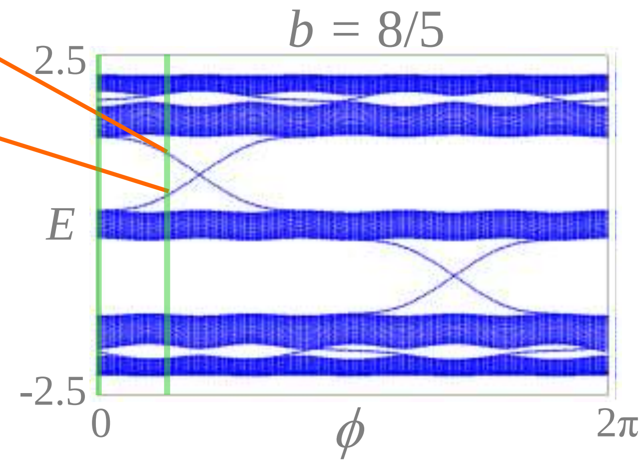
  D. J. Thouless, Phys. Rev. B **27**, 6083 (1983)

$$H(\phi) = \sum_x t_x \left(c_x^\dagger c_{x+1} + h.c. \right) + 2t_y \cos(2\pi b x + \phi) c_x^\dagger c_x$$





- Spectrum:



Topological edge states of a 2D crystal

⇩

Boundary states of a 1D topological pump

Outline

Quantum Hall effect ⇔ Topological pumps

Photonic topological pump (1D)

Atomic pumps

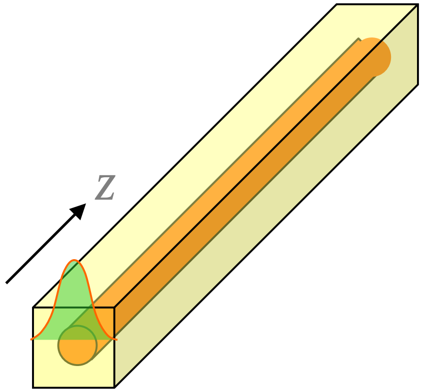
Quasiperiodic systems

Four-dimensional QHE

Photonic waveguide array

- A waveguide:

$$i\partial_z \psi = V\psi$$

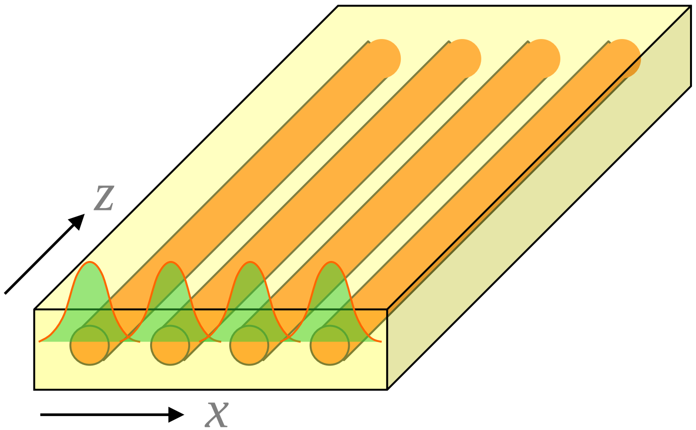


- A photonic crystal:

Lahini *et al.*, PRL **103,** 013901 (2009)

$$i\partial_z \psi_x = t_x \psi_{x+1} + t_{x-1}\psi_{x-1} + V_x \psi_x$$

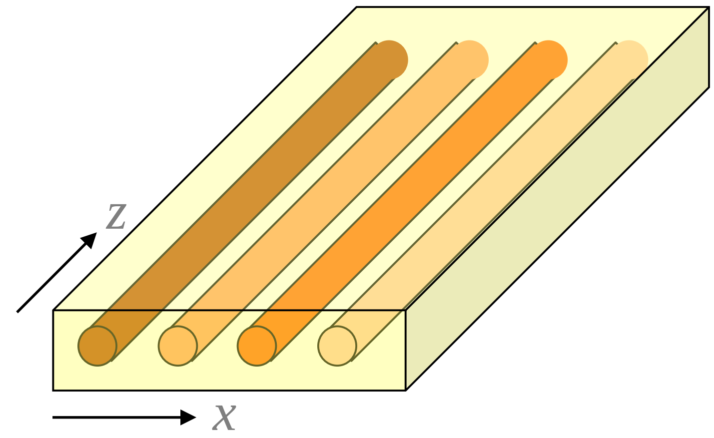
tight binding model
propagation replaces time



Photonic waveguide array

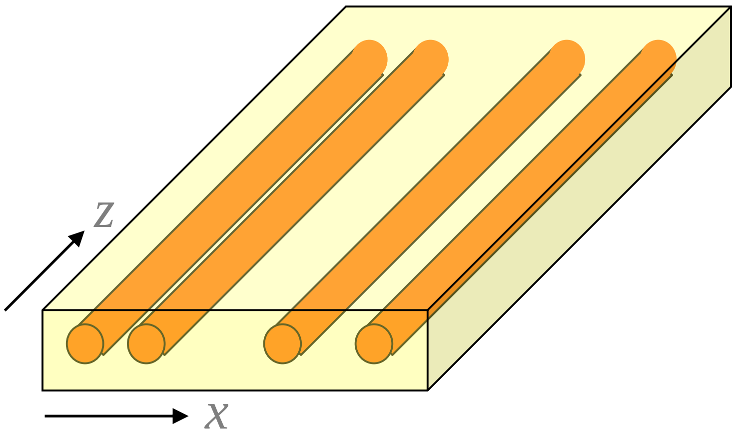
- Diagonal modulation:

$$i\partial_z \psi_x = t(\psi_{x+1} + \psi_{x-1}) + V_x \psi_x$$



- Off-diagonal modulation:

$$i\partial_z \psi_x = t_x \psi_{x+1} + t_{x-1} \psi_{x-1}$$



Experiment

- Setup:

  input: single site

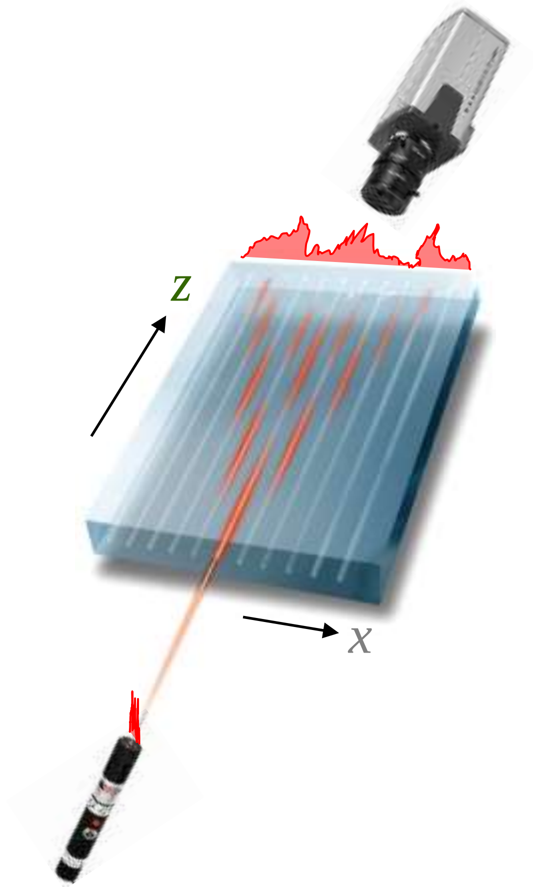
  output: distribution

- Evolution:

  overlap with eigenstates

  expansion according to eigenstates

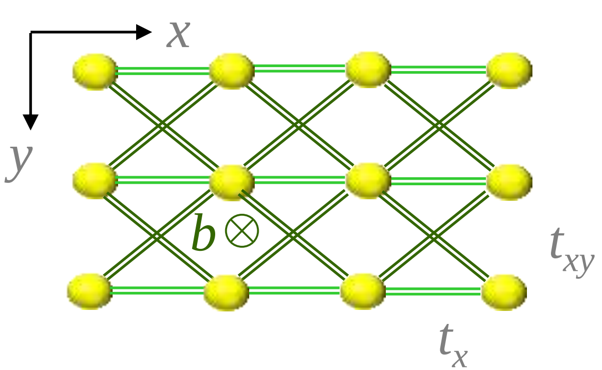
- No chemical potential

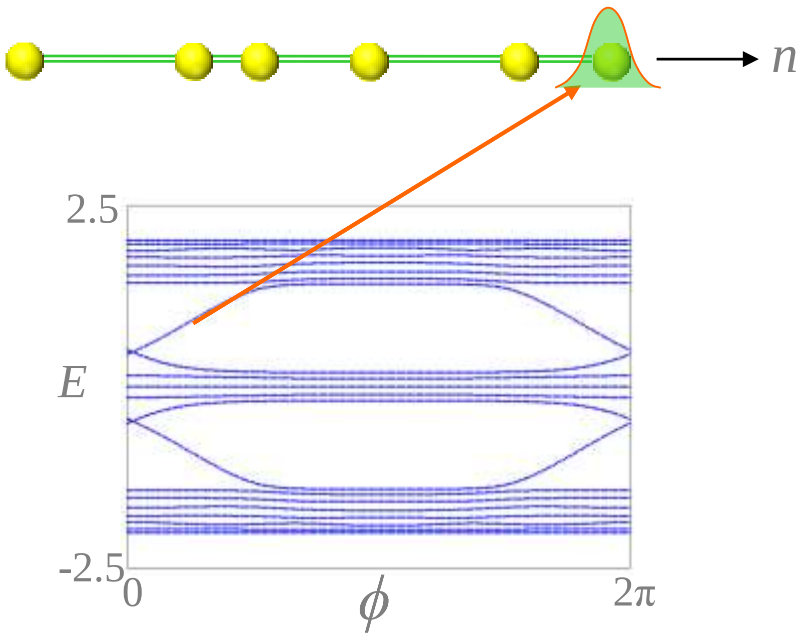


Generalizations – Other 1D models

- Off-diagonal Harper model:

$$H(\phi) = \sum_x \left[t_x + 2t_{xy}\cos(2\pi b x + \phi) \right] c^\dagger_{x+1} c_x + h.c.$$

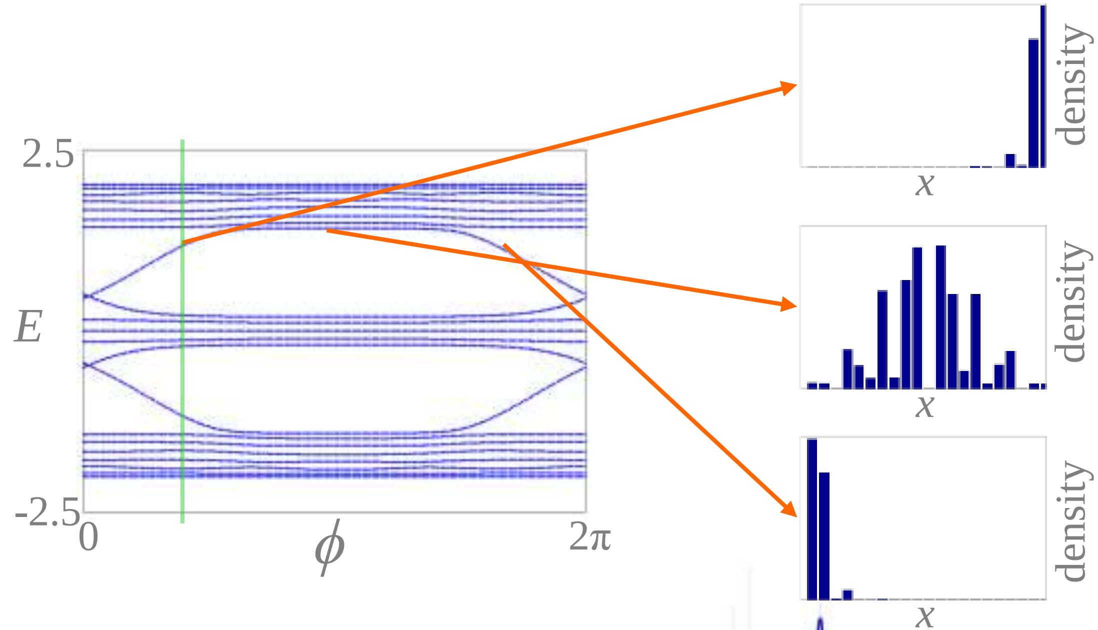




Phys. Rev. Lett. **109**, 106402 (2012)

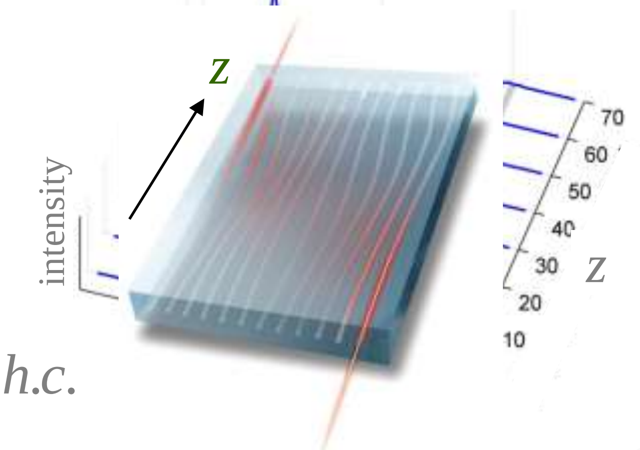
Experiment 1: adiabatic pumping (1D+1)

- Pumping:



- Adiabatic pumping: $H\left(\phi(z)\right)$

  off-diagonal



$$H(\phi) = \sum_x \left[t_x + 2t_{xy} \cos(2\pi b x + \phi) \right] c_{x+1}^{\dagger} c_x + h.c.$$

Outline

Quantum Hall effect ⇔ Topological pumps

Photonic topological pump (1D)

Atomic pumps

Quasiperiodic systems

Four-dimensional QHE

Quasiperiodic models

Systems with long-range order but no translation symmetry.

- 1D Fibonacci chain:

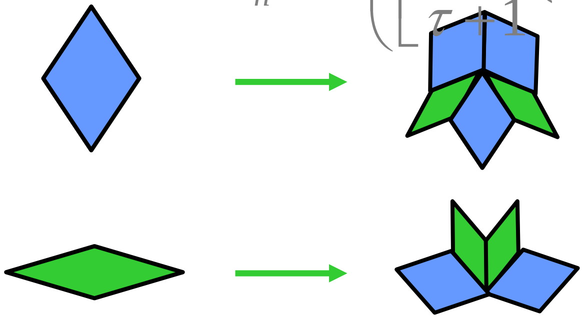
  L $\rightarrow$ LS

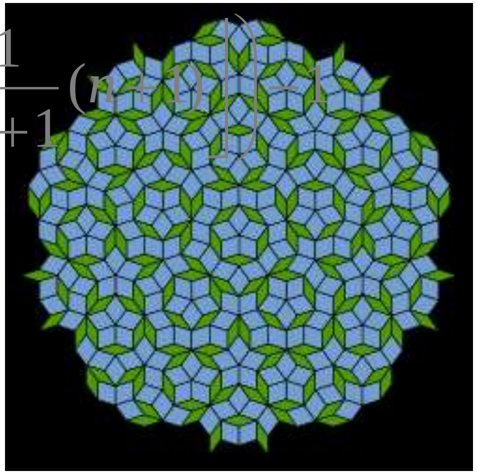
  S $\rightarrow$ L

L
LS
LSL
LSLLS
LSLLSLSL

- 2D Penrose tiling:

$$d_n = 2\left(\left\lfloor \frac{1}{\tau+1}(n+2)\right\rfloor - \left\lfloor \frac{1}{\tau+1}(n+1)\right\rfloor\right) - 1$$





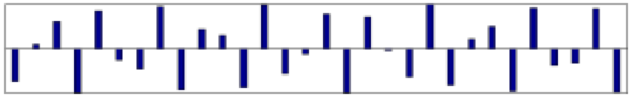
Long-range order vs. translations

- Harper (or Aubry-André) model:

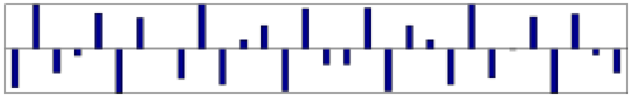
$$H(\phi)=\sum_n t\left(c_n^\dagger c_{n+1}+h.c.\right)+2t'\cos(2\pi bn+\phi)c_n^\dagger c_n$$

- Long-range order vs. translations

$$\cos(2\pi bn) \rightarrow n=1..30$$



$$\cos(2\pi bn) \rightarrow n=101..130$$



The order determines the system up to translations of the origin (which have no effect on bulk properties).

Y. E. Kraus, Y. Lahini, Z. Ringel, M. Verbin, and **OZ**, Phys. Rev. Lett. **109**, 106402 (2012)

Long-range order vs. translations

- Harper (or Aubry-André) model:

$$H(\phi) = \sum_n t\left(c_n^\dagger c_{n+1} + h.c.\right) + 2t'\cos(2\pi bn + \phi)c_n^\dagger c_n$$

- Translations vs. ϕ

b = p/q: $(2\pi bn) \bmod 2\pi \in \left\{0, 2\pi \cdot \frac{1}{q}, 2\pi \cdot \frac{2}{q}, \ldots\right\}$

irrational b: $(2\pi bn) \bmod 2\pi \in [0, 2\pi]$

For the latter ϕ has no effect on bulk properties

Y. E. Kraus, Y. Lahini, Z. Ringel, M. Verbin, and **OZ**, Phys. Rev. Lett. **109**, 106402 (2012)

Long-range order vs. translations

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$$H(\phi)=\sum_n t\left(c_n^\dagger c_{n+1}+h.c.\right)+2t'\cos(2\pi bn+\phi)c_n^\dagger c_n$$

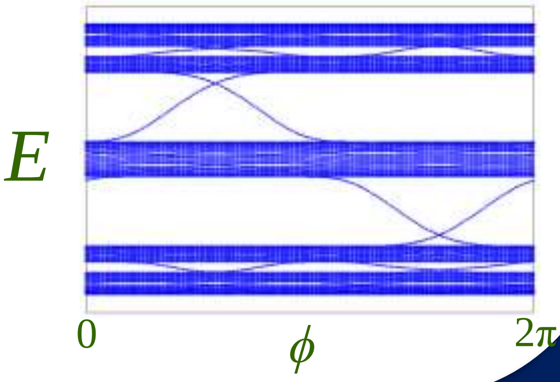
- Translations vs. ϕ

$\phi \longrightarrow \phi+\varepsilon$ *is equivalent to* $n \rightarrow n+n_\varepsilon$

$$H(\phi+\varepsilon)=T_\varepsilon H(\phi)T_\varepsilon^\dagger \;=T_\varepsilon\left(\sum_l E_l(\phi)\left|l\right\rangle\left\langle l\right|\right)T_\varepsilon^\dagger$$

$$=\sum_l E_l(\phi)\left|T_\varepsilon l\right\rangle\left\langle T_\varepsilon l\right|$$

independent of ϕ → *flat bulk bands*



Y. E. Kraus, Y. Lahini, Z. Ringel, M. Verbin, and **OZ**, Phys. Rev. Lett. **109**, 106402 (2012)

Long-range order vs. translations

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$$H(\phi) = \sum_n t\left(c_n^\dagger c_{n+1} + h.c.\right) + 2t'\cos(2\pi b n + \phi) c_n^\dagger c_n$$

- *Projector P:*

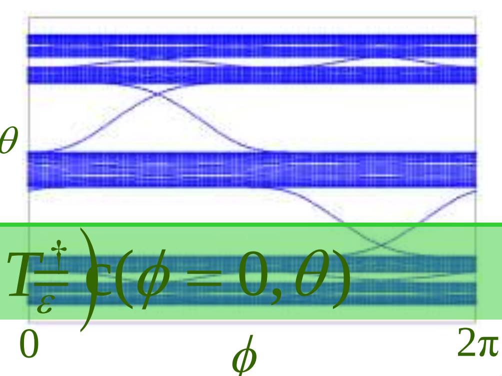
P ~ H $\quad P(\phi+\varepsilon) = T_\varepsilon P(\phi) T_\varepsilon^\dagger \qquad \partial_\phi P(\phi+\varepsilon) = T_\varepsilon \partial_\phi P(\phi) T_\varepsilon^\dagger$

- *Berry curvature*

$$\Omega(\phi,\theta) = \frac{1}{2\pi i}\mathrm{Tr}\left(P\left[\partial_\phi P, \partial_\theta P\right]\right)$$

$$\Omega(\phi,\varepsilon,\theta) = \frac{1}{2\pi i}\mathrm{Tr}\left(P(\phi+\varepsilon)\left[\partial_\phi P(\phi+\varepsilon), \partial_\theta P(\phi+\varepsilon)\right]\right)$$

$$= \frac{1}{2\pi i}\mathrm{Tr}\left(T_\varepsilon P(\phi)\left[\partial_\phi P(\phi), \partial_\theta P(\phi)\right] T_\varepsilon^\dagger\right) = \Omega(\phi=0,\theta)$$



Long-range order vs. translations

- Harper (or Aubry-André) model:

$$H(\phi) = \sum_n t\left(c_n^\dagger c_{n+1} + h.c.\right) + 2t'\cos(2\pi bn + \phi)c_n^\dagger c_n$$

- *Berry curvature*

$$\Omega(\phi,\theta) = \frac{1}{2\pi i}\mathrm{Tr}\left(P\left[\partial_\phi P, \partial_\theta P\right]\right) = \Omega(\theta)$$

- *Chern number:*

$$\nu = \int_0^{2\pi} \mathrm{d}\phi \int_0^{2\pi} \mathrm{d}\theta \Omega(\phi,\theta) = 2\pi \int_0^{2\pi} \mathrm{d}\theta \Omega(\theta)$$

- *Chern numbers can be associated with the whole family {H(ϕ)}.* *Thouless, PRB **27**, 6083 (1983)*

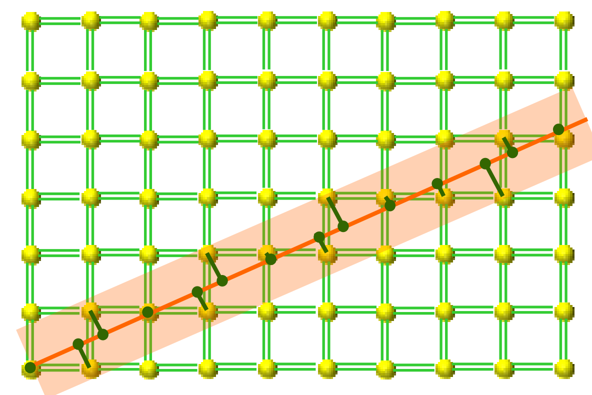
- **We can associate Chern numbers with any H(ϕ)!**

Y. E. Kraus, Y. Lahini, Z. Ringel, M. Verbin, and **OZ**, Phys. Rev. Lett. **109**, 106402 (2012)

Fibonacci-like chains

- Fibonacci quasicrystal:

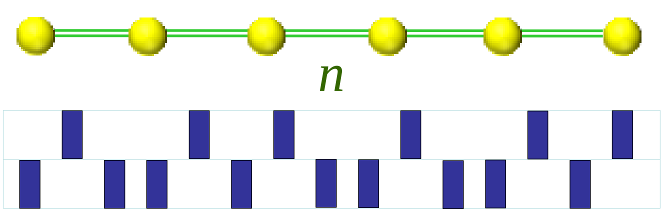
$$d_n = \pm 1 = 2\left(\left\lfloor \frac{1}{\tau+1}(n+2) \right\rfloor - \left\lfloor \frac{1}{\tau+1}(n+1) \right\rfloor\right) - 1$$



- Off-diagonal Fibonacci

$$H = \sum_n [t + t' d_n] c_{n+1}^\dagger c_n + h.c.$$

- Diagonal Fibonacci



$$H = \sum_n t(c_{n+1}^\dagger c_n + h.c.) + t' d_n c_n^\dagger c_n$$

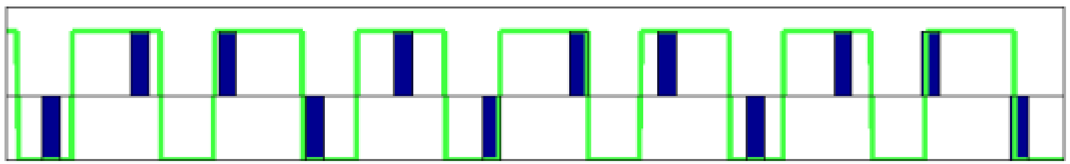
Y. E. Kraus and **OZ**, Phys. Rev. Lett. **109**, 116404 (2012);

Fibonacci-like chains

- Fibonacci quasicrystal:

$$\bar{d}_n(\beta) = \frac{1}{\tanh(\beta)} \tanh\left\{\beta\left[\cos\left(2\pi b n + 3\pi b + \phi\right) - \cos\left(\pi b\right)\right]\right\}$$

$$d_n = 2\left(\left\lfloor \frac{1}{\tau+1}(n+2) \right\rfloor - \left\lfloor \frac{1}{\tau+1}(n+1) \right\rfloor\right) - 1$$



$\beta = 0$ $\beta \to \infty$

- $\beta \to 0$: Harper

$$\bar{d}_n(\beta \to 0) = \cos\left(2\pi b n + 3\pi b + \phi\right) - \cos\left(\pi b\right)$$

- $\beta \to \infty$: Fibonacci

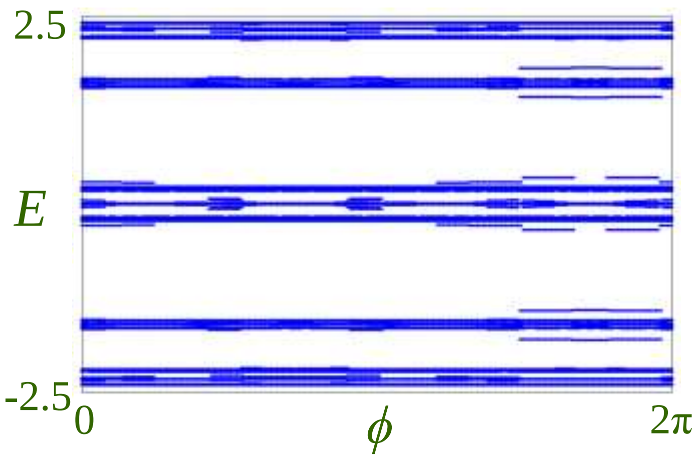
$$\bar{d}_n(\beta \to \infty) = 2\left(\left\lfloor b\left(n + 2 + \frac{\phi}{2\pi}\right) \right\rfloor - \left\lfloor b\left(n + 1 + \frac{\phi}{2\pi}\right) \right\rfloor\right) - 1$$

Y. E. Kraus and **OZ**, Phys. Rev. Lett. **109**, 116404 (2012);

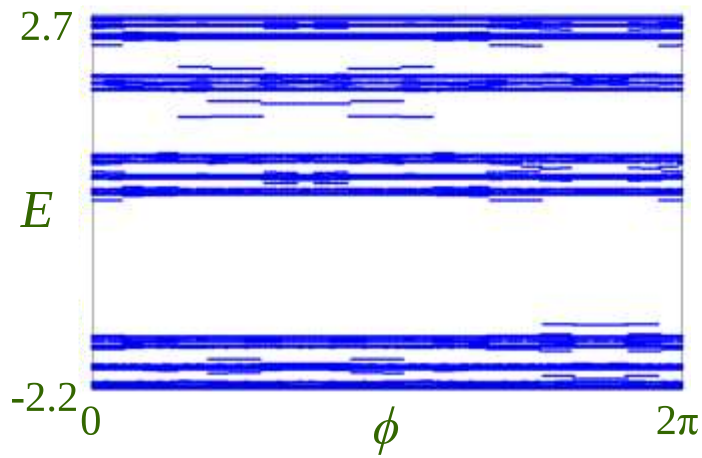
ETH zürich

Fibonacci-like chains

Off-diagonal:



Diagonal:



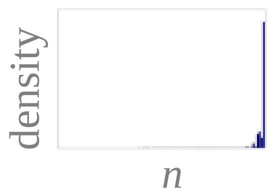
$$\beta = 0,\ 10,\ \to\infty$$

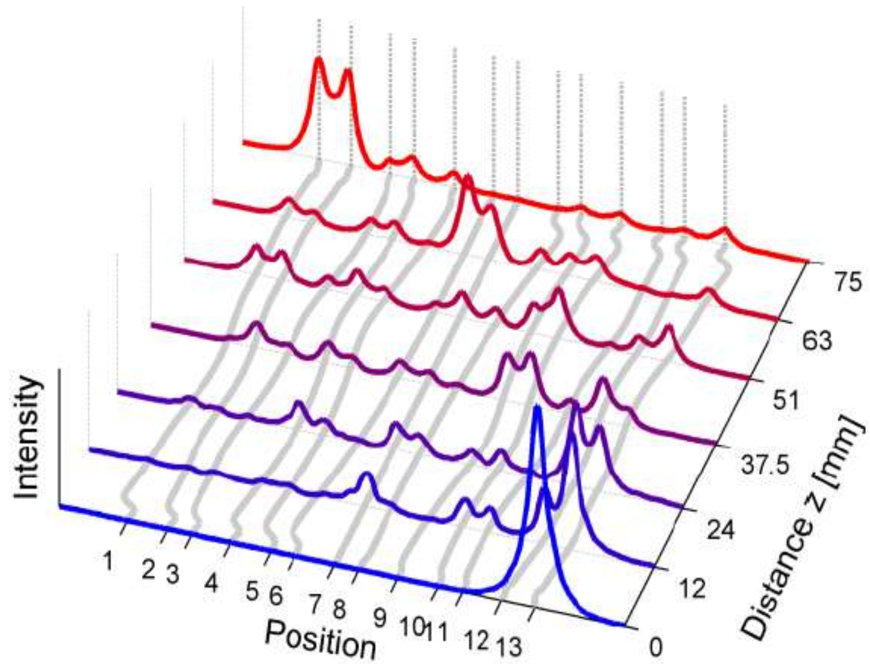
Topological boundary states.

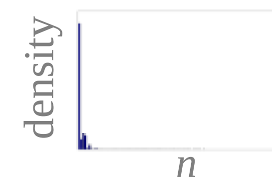
Zijlstra, Fasolino and Janssen, PRB 59, 302 (1999).
El Hassouani *et al.*, PRB 74, 035314 (2006).
Pang, Dong and Wang, J. Opt. Soc. Am. B 27, 2009 (2010).
Martínez and Molina, PRA 85, 013807 (2012).

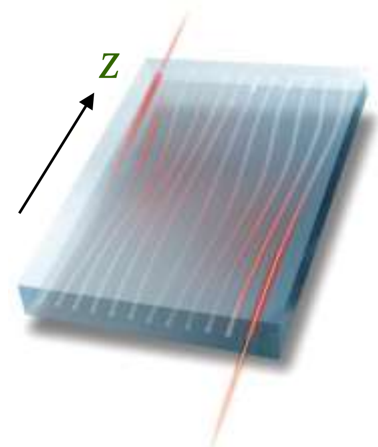
Y. E. Kraus and **OZ**, Phys. Rev. Lett. **109**, 116404 (2012);

Fibonacci pumping







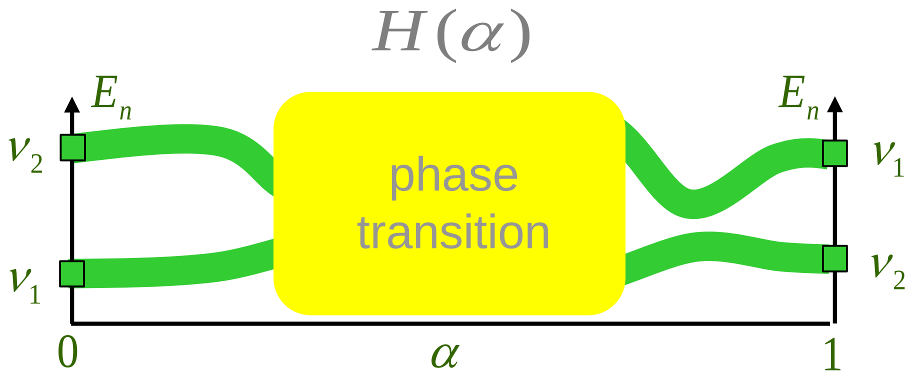


In atoms
M.. Lohse, Nat. Phys. **12**, 350 (2016).
S. Nakajima et al. Nat. Phys. 12, 296 (2016)

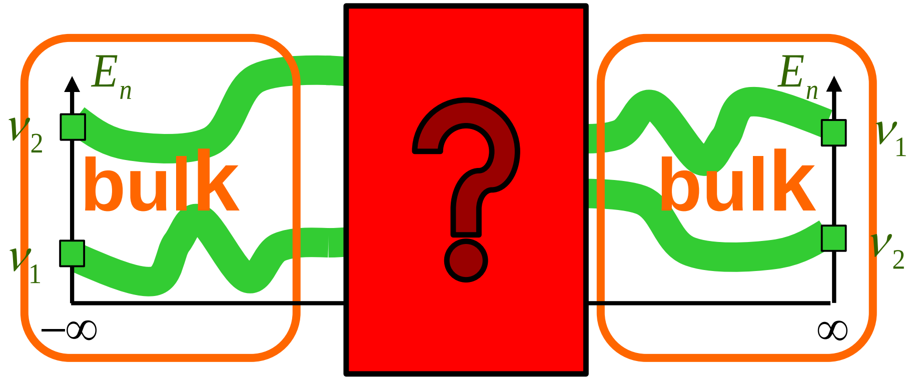
M. Verbin , OZ, Y. Lahini, Y. E. Kraus, and Y. Silberberg, PRB 91, 064201 (2015)

Further implications

- Topological phase transition:



- Topological phase transition in space:



A sharp edge breaks long-range order and the boundary modes may not appear!!

Bulk transitions

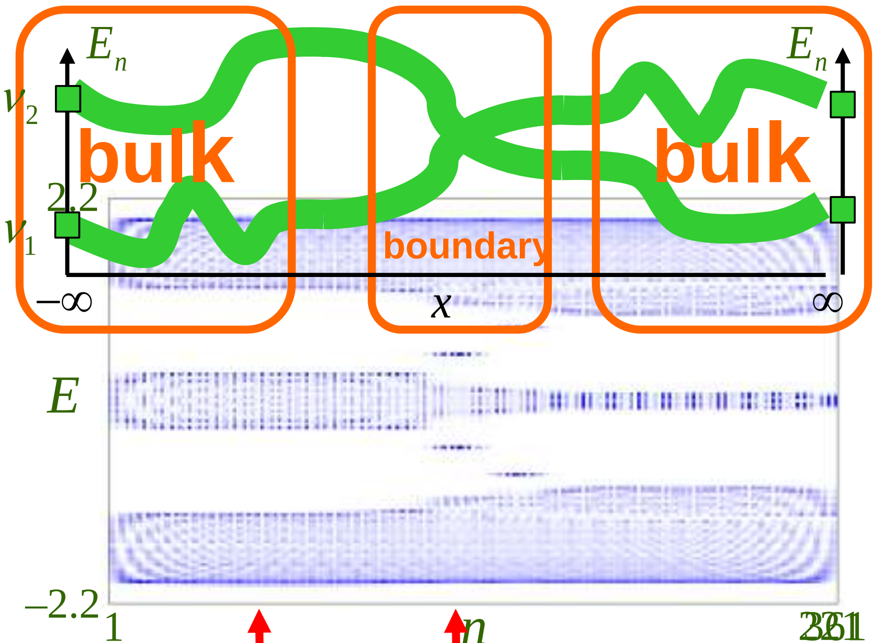
- Adiabatic deformation:

Harper

$$b_{\mathrm{I}} = \frac{2}{1+\sqrt{5}}$$

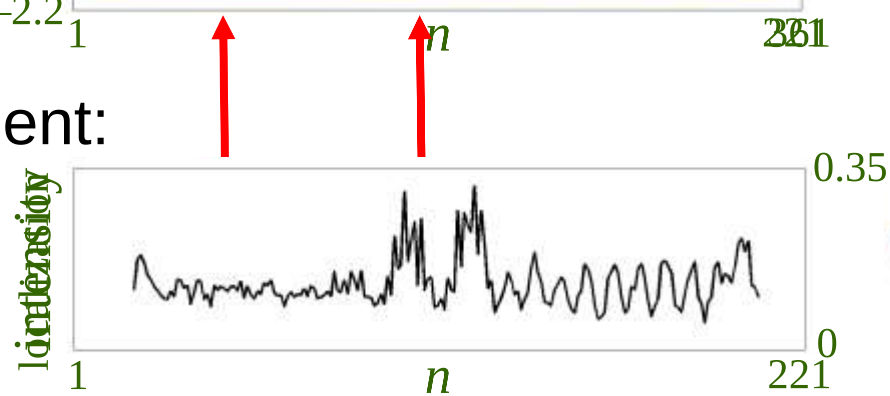
Harper

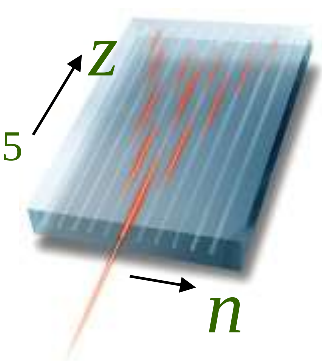
$$b_{\mathrm{II}} = \frac{2}{1+\sqrt{6.5}}$$



- LDOS:

- Measurement:





M. Verbin, **OZ**, Y. E. Kraus, Y. Lahini, and Y. Silberberg, Phys. Rev. Lett. **110**, 076403 (2013)

Bulk transitions

- Adiabatic deformation:

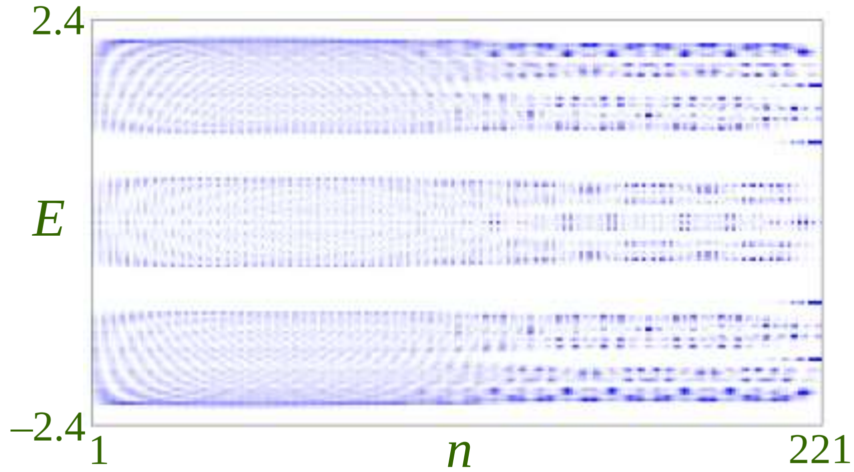
Harper

$$b_{\mathrm{I}} = \frac{2}{1+\sqrt{5}}$$

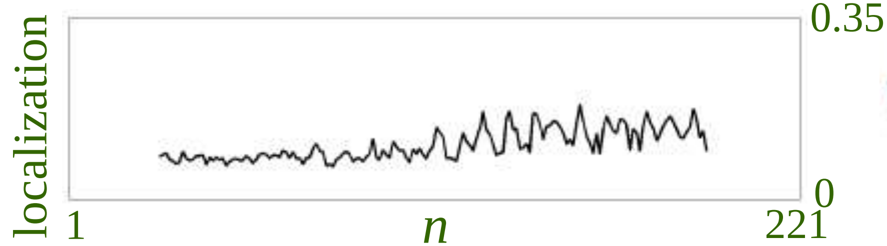
Fibonacci

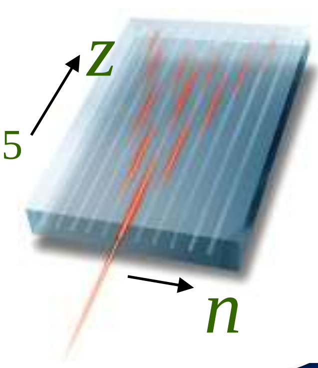
$$b_{\mathrm{II}} = \frac{2}{1+\sqrt{5}}$$

- LDOS:



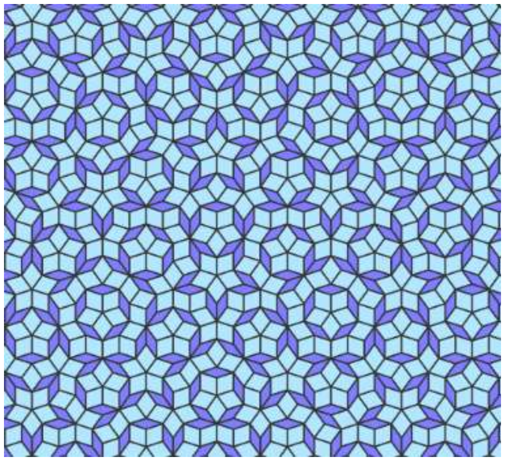
- Measurement:

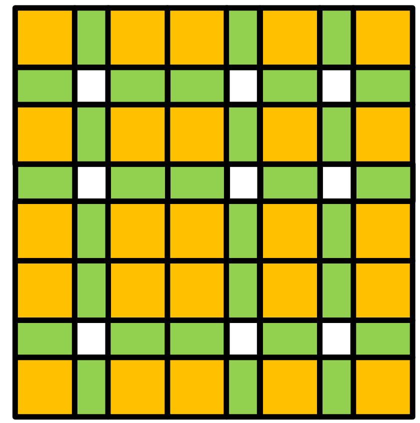


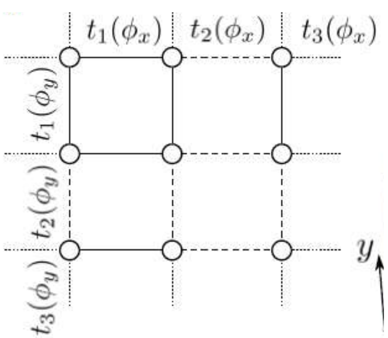


M. Verbin, **OZ**, Y. E. Kraus, Y. Lahini, and Y. Silberberg, Phys. Rev. Lett. **110**, 076403 (2013)

What about 2D quasicrystals?

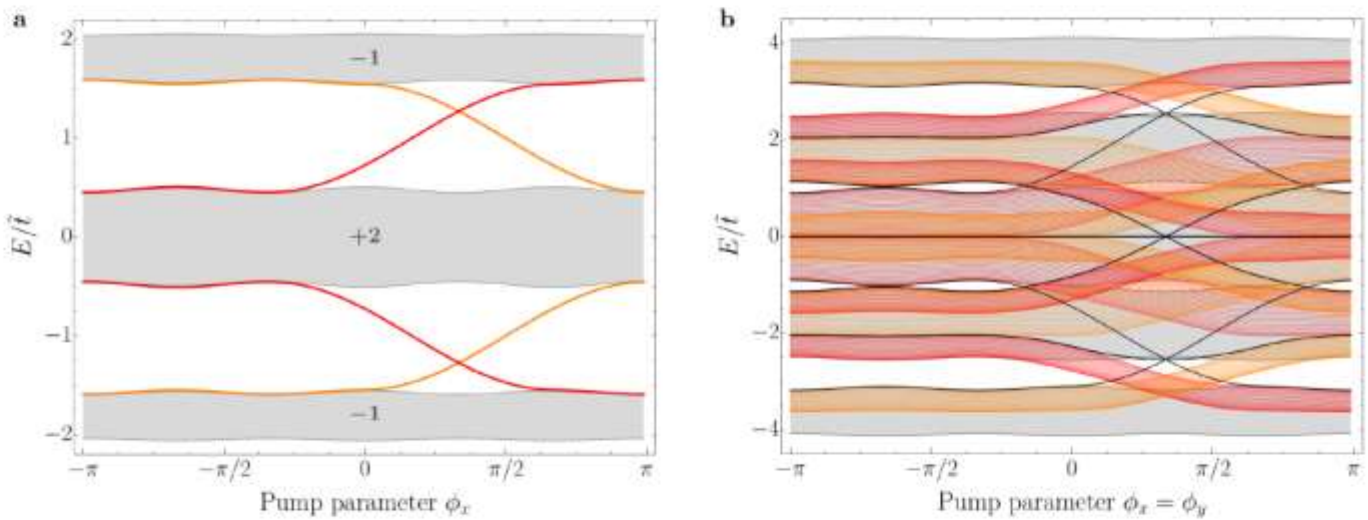


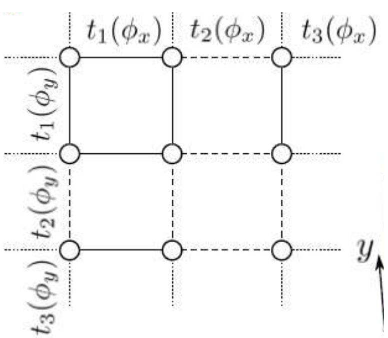




$$H(\phi_x,\phi_y) = \sum_{x,y} \left[t_x + 2t_w \cos(2\pi b_x x + \phi_x)\right] c^{\dagger}_{x,y} c_{x+1,y} + \left[t_y + 2t_z \cos(2\pi b_y y + \phi_y)\right] c^{\dagger}_{x,y} c_{x,y+1} + h.c.$$

What about 2D quasicrystals?





$$H(\phi_x,\phi_y) = \sum_{x,y} \left[t_x + 2t_w \cos(2\pi b_x x + \phi_x)\right] c^\dagger_{x,y} c_{x+1,y} + \left[t_y + 2t_z \cos(2\pi b_y y + \phi_y)\right] c^\dagger_{x,y} c_{x,y+1} + h.c.$$

Outline

Quantum Hall effect ⇔ Topological pumps

Photonic topological pump (1D)

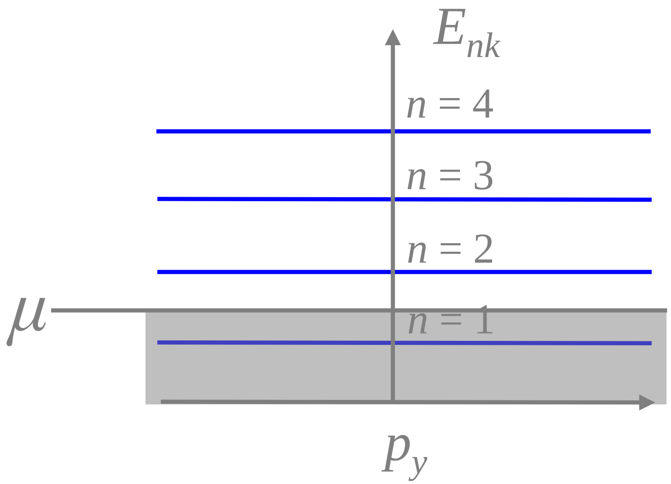
Atomic pumps

Quasiperiodic systems

Four-dimensional QHE

4D quantum Hall effect

S^4



$$I_\alpha = \chi \frac{e^2}{h} \varepsilon_{\alpha\beta\gamma\delta} \frac{B_{\beta\gamma}}{\Phi_0} E_\delta;$$

$$\chi = \frac{1}{(2\pi)^2} \iint\iint_{BZ} \mathrm{d}k_x \mathrm{d}k_y \mathrm{d}k_z \mathrm{d}k_w \Omega \wedge \Omega$$

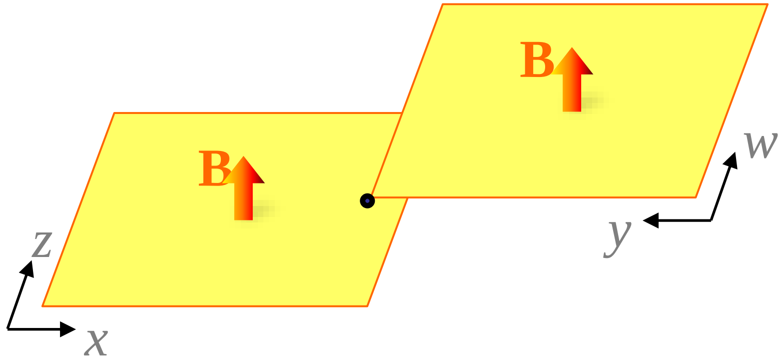
$$B_{\beta\gamma} = \partial_\beta A - \partial_\gamma A$$

First derivations:

J. E. Avron *et al.*, Comm. Math. Phys. 124, 595 (1989).
J. Fröhlich and B. Perdini, in Mathematical Physics 2000 (Imperial College Press, London, United Kingdom)..
S.-C. Zhang and J. Hu, Science Vol. 294, 823 (2001):
X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).

4D quantum Hall effect

K. Kraus, Z. Ringel, and OZ, PRL **111**, 226401 (2013)



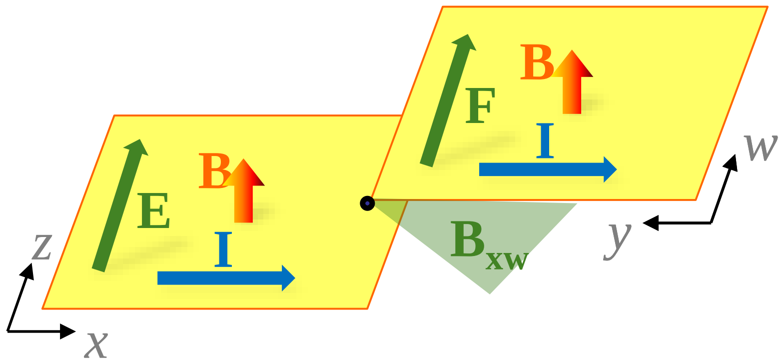
$$I_\alpha = \chi \frac{e^2}{h} \varepsilon_{\alpha\beta\gamma\delta} \frac{B_{\beta\gamma}}{\Phi_0} E_\delta;$$

$$\chi = \frac{1}{(2\pi)^2} \iint\iint_{BZ} \mathrm{d}k_x \mathrm{d}k_y \mathrm{d}k_z \mathrm{d}k_w \Omega_{xz} \Omega_{yw}$$

See also PRL **115**, 195303 (2015), PRA **93**, 043827 (2016), PRB **93** 245113 (2016)

4D quantum Hall effect

K. Kraus, Z. Ringel, and OZ, PRL **111**, 226401 (2013)



$$I_y = \chi \frac{e^2}{h} \varepsilon_{x\beta\gamma\delta} \frac{B_{\beta\gamma}}{\Phi_0} E_\delta;$$

Lorentz-type response

$$I_v = I_w = 0$$

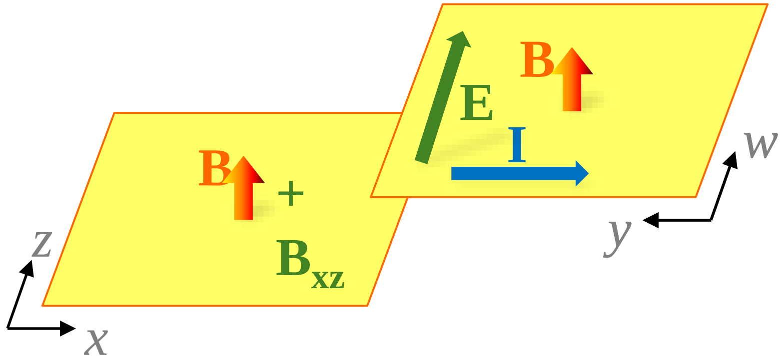
$$I_x = \frac{e^2}{h} n_{yw} \nu_{xz} E_z;$$

$$I_y = \chi \frac{e^2}{h} \frac{B_{xw}}{\Phi_0} E_z$$

See also PRL **115**, 195303 (2015), PRA **93**, 043827 (2016), PRB **93** 245113 (2016)

4D quantum Hall effect

K. Kraus, Z. Ringel, and OZ, PRL **111**, 226401 (2013)



$$I_y = \chi \frac{e^2}{h} \varepsilon_{x\beta\gamma\delta} \frac{B_{\beta\gamma}}{\Phi_0} E_\delta ;$$

Lorentz-type response

$$I_v = I_w = 0$$

$$I_x = \frac{e^2}{h} n_{yw} \nu_{xz} E_z ;$$

$$I_y = \chi \frac{e^2}{h} \frac{B_{xw}}{\Phi_0} E_z$$

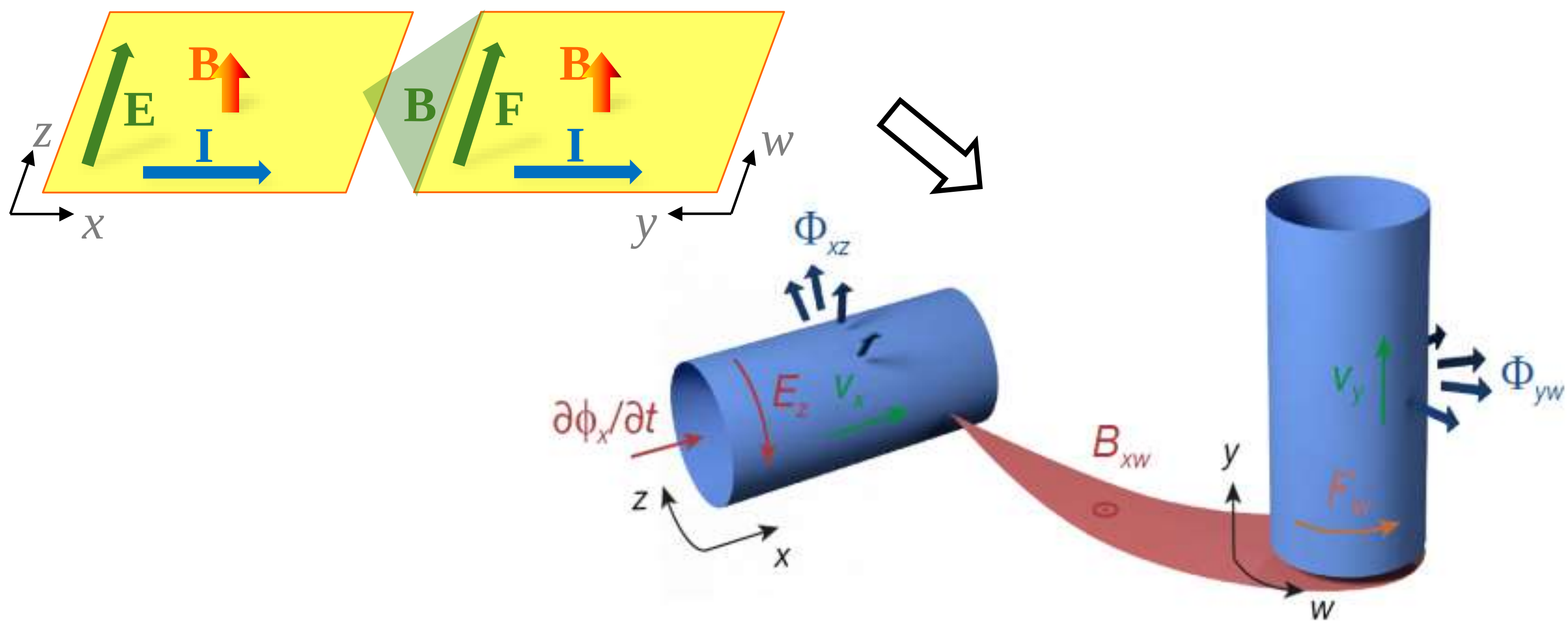
Density-type response

$$I_z = I_x = I_w = 0$$

$$I_y = \chi \frac{e^2}{h} \frac{B_{xz}}{\Phi_0} E_w + \frac{e^2}{h} \nu_{yw} n_{xz} E_w ;$$

See also PRL **115**, 195303 (2015), PRA **93**, 043827 (2016), PRB **93** 245113 (2016)

4D QHE => 2D topological pumps



$$H(\phi_x,\phi_y)=\sum_{\mathrm{x,y}}[t_x\left(c^{\dagger}_{\mathrm{x,y}}c_{x+1,\mathrm{y}}+h.c.\right)+2t_z\cos(2\pi(\Phi_{xz}x+B_{yz}y)+\phi_x)c^{\dagger}_{\mathrm{x,y}}c_{\mathrm{x,y}}$$

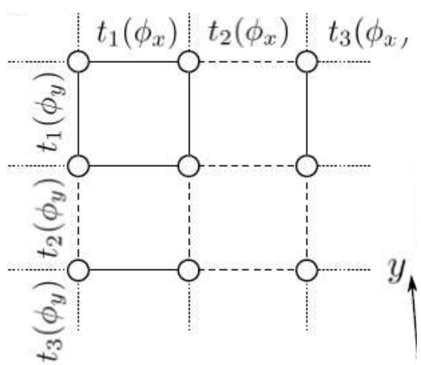
$$+t_y\left(c^{\dagger}_{\mathrm{x,y}}c_{\mathrm{x,y+1}}+h.c.\right)+2t_w\cos(2\pi(\Phi_{yw}+B_{yw})y+\phi_y)c^{\dagger}_{\mathrm{x,y}}c_{\mathrm{x,y}}]$$

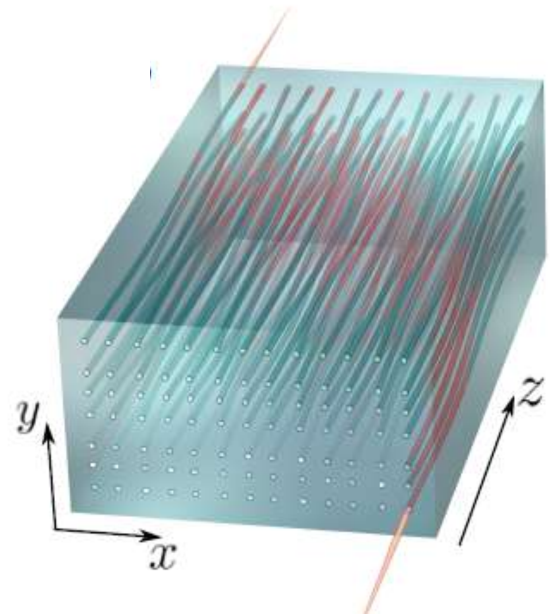
K. Kraus, Z. Ringel, and OZ, PRL **111**, 226401 (2013)

2D photonic topological pump (2D + 2)

- Topological charge pump in each direction:

$$H(\phi_x,\phi_y)=\sum_{\mathrm{x,y}}\left[t_x+2t_w\cos(2\pi b_x x+\phi_x)\right]c^{\dagger}_{\mathrm{x,y}}c_{x+1,\mathrm{y}}$$
$$+\left[t_y+2t_z\cos(2\pi b_y y+\phi_y)\right]c^{\dagger}_{\mathrm{x,y}}c_{\mathrm{x,y+1}}+h.c.$$

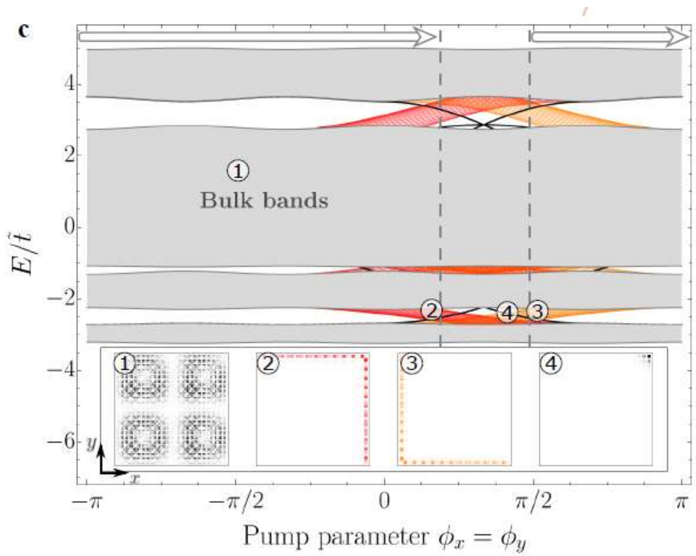


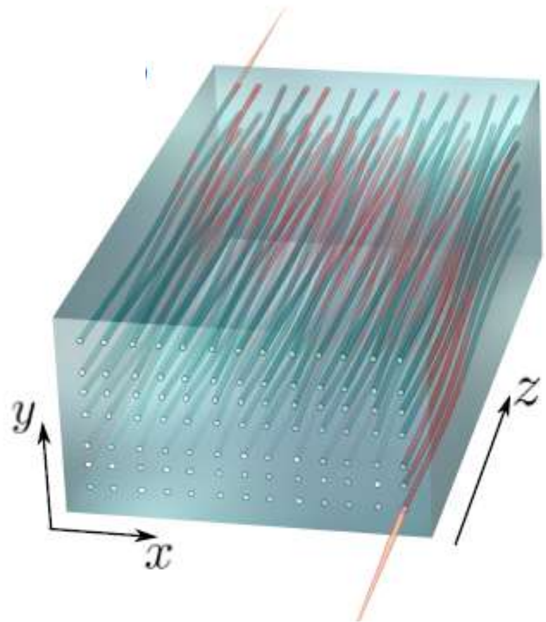


Nature **553**, 59 (2018)

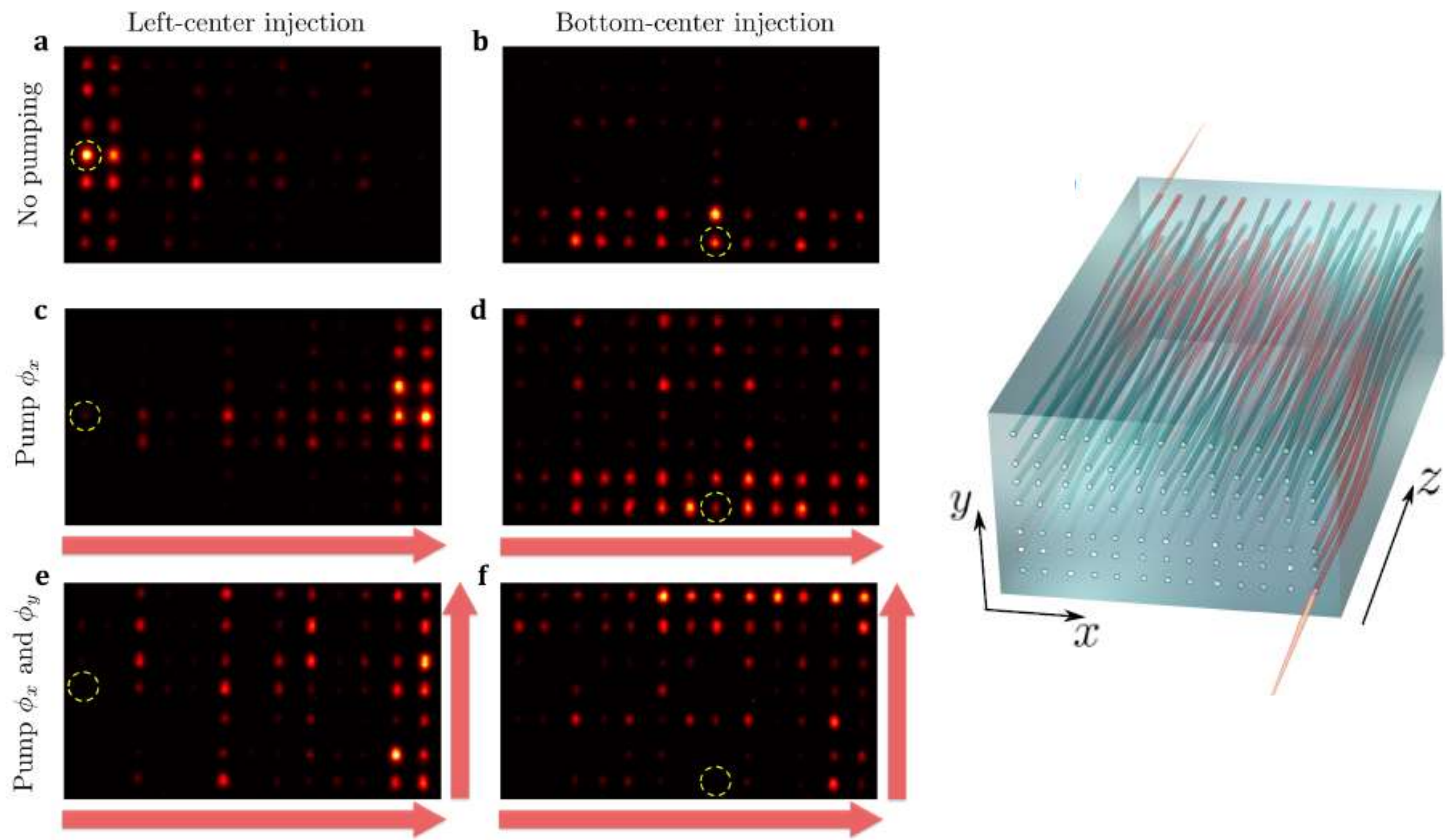
2D photonic topological pump (2D + 2)

- Topological charge pump in each direction:



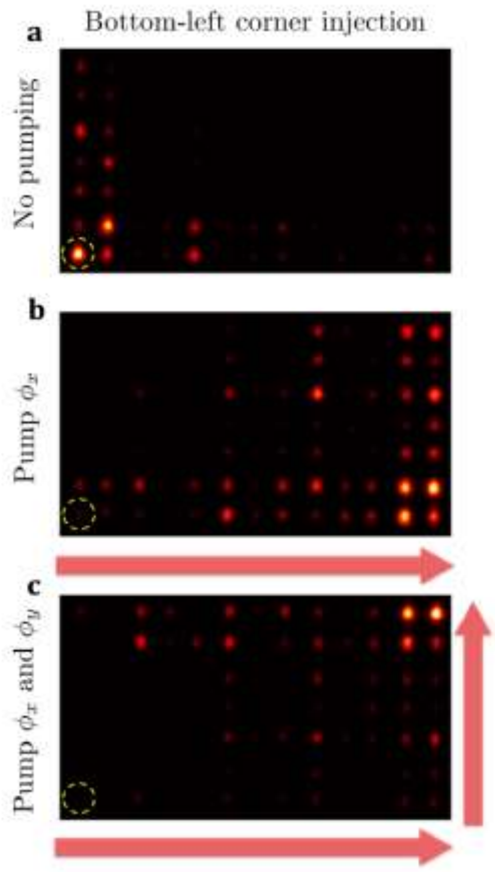


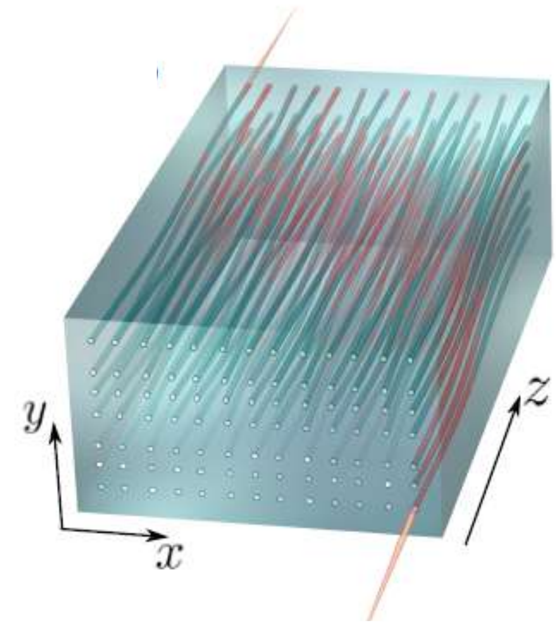
2D photonic topological pump (2D + 2)



Nature **553**, 59 (2018)

2D photonic topological pump (2D + 2)





$$I_y = \chi \frac{e^2}{h} \frac{B_{xw}}{\Phi_0} E_z;$$

Outline

Quantum Hall effect ⇔ Topological pumps

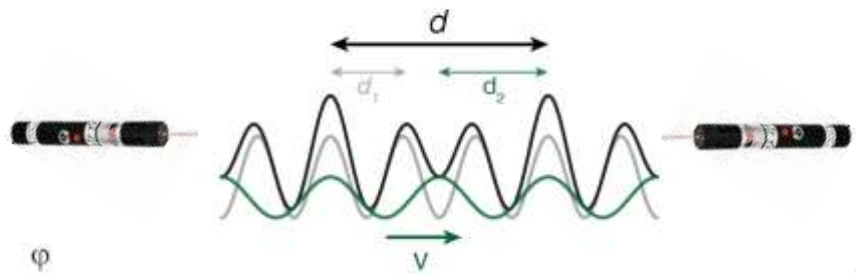
Photonic topological pump (1D)

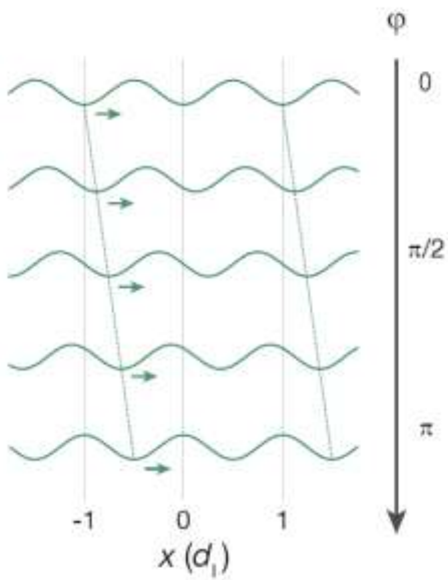
Atomic pumps

Quasiperiodic systems

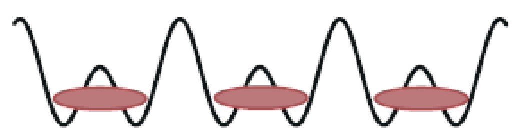
Four-dimensional QHE

Atomic topological pumps

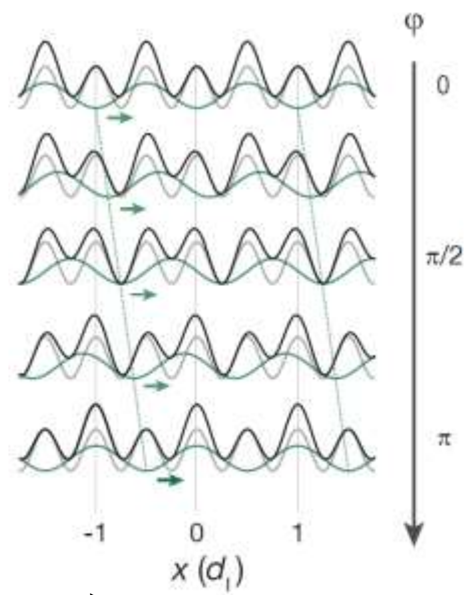


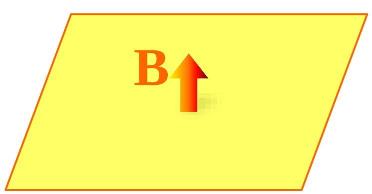


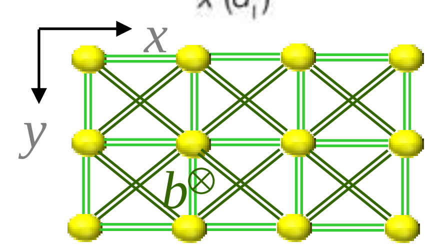
$$V_\mathrm{s} \sin^2\left(\pi x/d_\mathrm{s} + \pi/2\right) + V_\mathrm{l} \sin^2\left(\pi x/d_\mathrm{l} - \varphi/2\right)$$



Cool atoms into a Mott insulator state
Homogeneous delocalization over first Brillouin zone

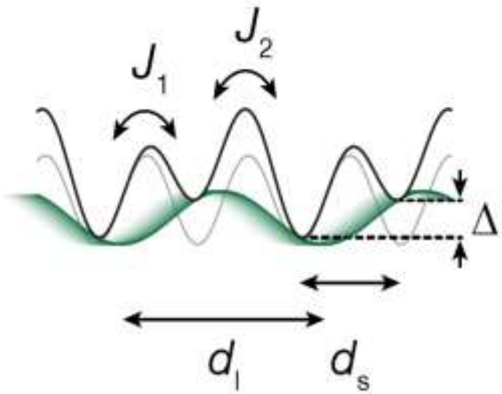




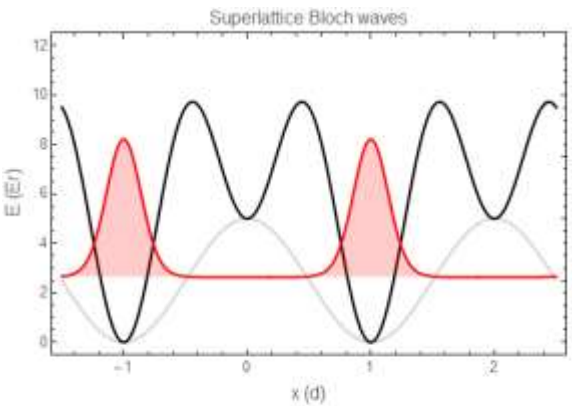


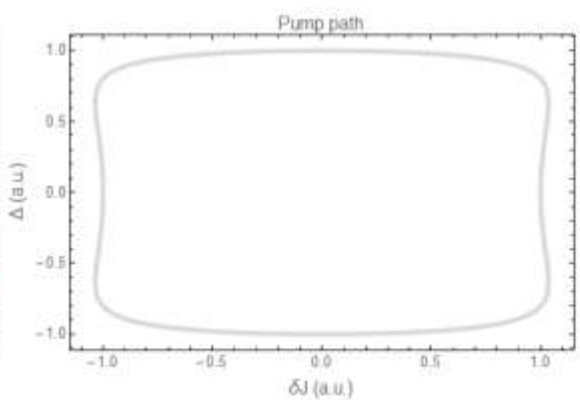
Nat. Phys. **12**, 350 (2016)

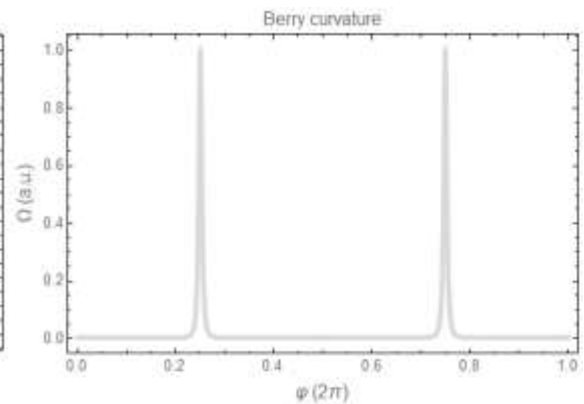
Atomic topological pumps



$$\hat{H}(\varphi) = -\sum_m \left(J_1(\varphi)\, \hat{b}_m^\dagger \hat{a}_m + J_2(\varphi)\, \hat{a}_{m+1}^\dagger \hat{b}_m + \text{h.c.} \right) + \frac{\Delta(\varphi)}{2} \sum_m \left(\hat{a}_m^\dagger \hat{a}_m - \hat{b}_m^\dagger \hat{b}_m \right)$$



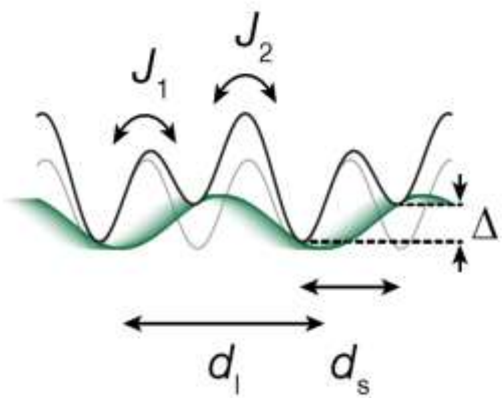




$$\boxed{\langle x \rangle = \iint \langle v_n(k_x, t) \rangle \, dk_x dt = C_n d_1}$$

Nat. Phys. **12**, 350 (2016)

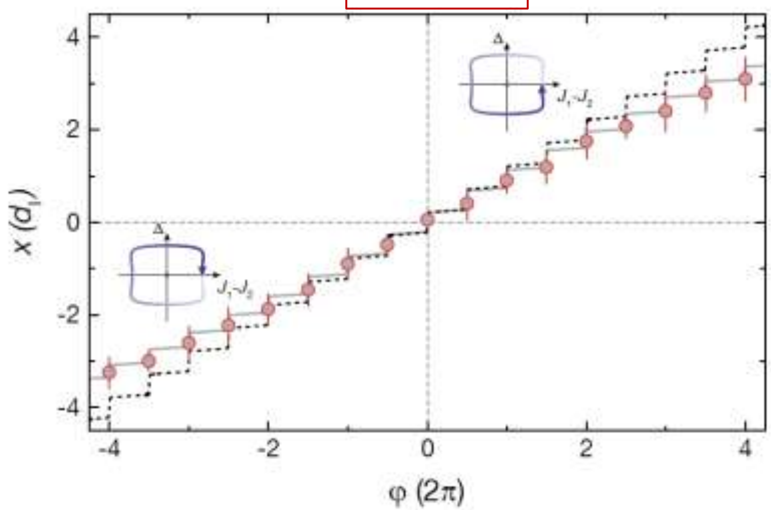
Atomic topological pumps

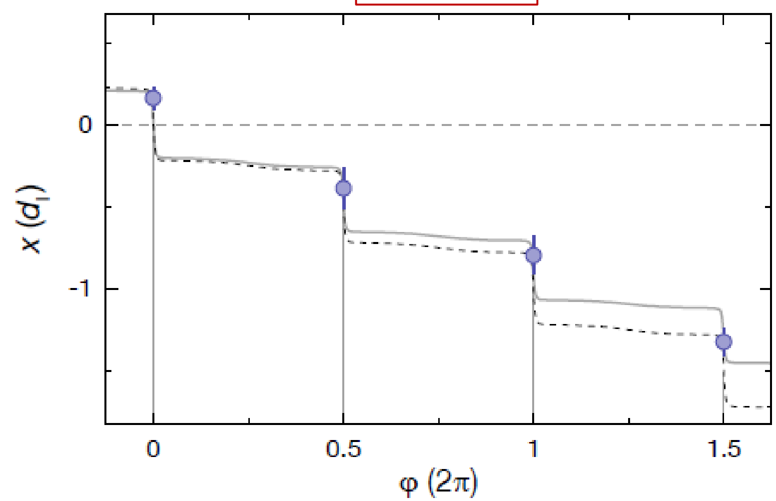


$$\hat{H}(\varphi) = -\sum_m \left(J_1(\varphi)\, \hat{b}^\dagger_m \hat{a}_m + J_2(\varphi)\, \hat{a}^\dagger_{m+1} \hat{b}_m + \mathrm{h.c.} \right) + \frac{\Delta(\varphi)}{2} \sum_m \left(\hat{a}^\dagger_m \hat{a}_m - \hat{b}^\dagger_m \hat{b}_m \right)$$

$C_1 = +1$

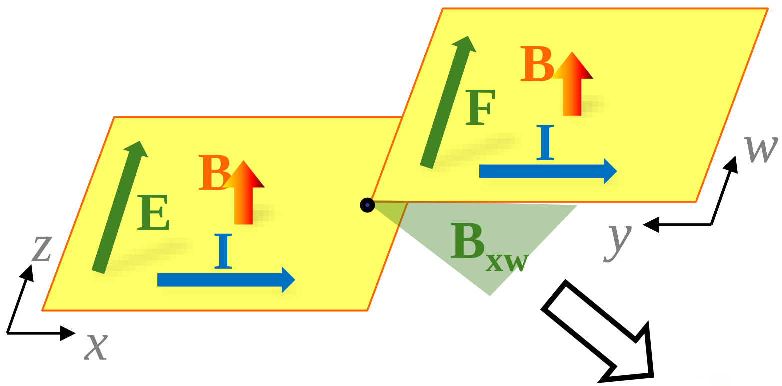
$C_1 = -1$



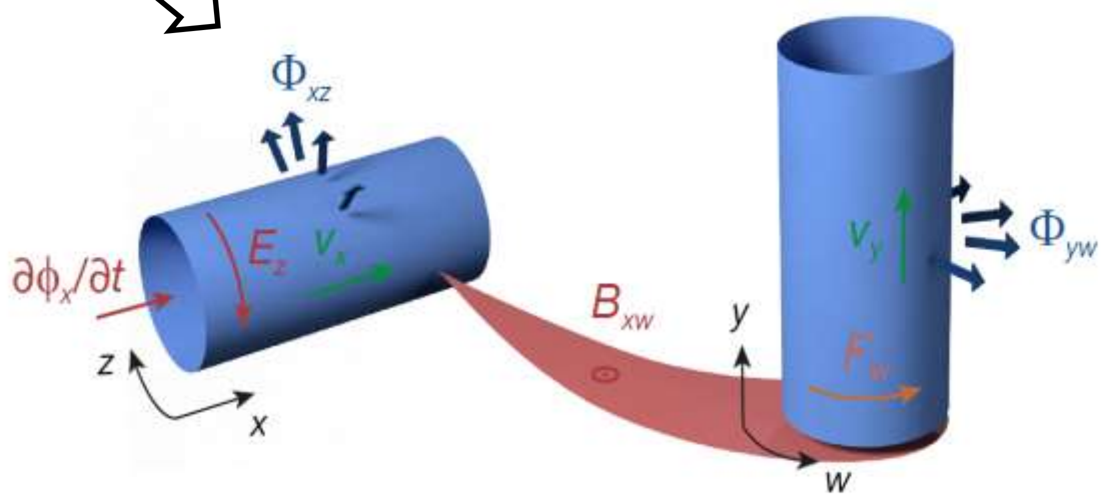


Nat. Phys. **12**, 350 (2016)

2D topological charge pump

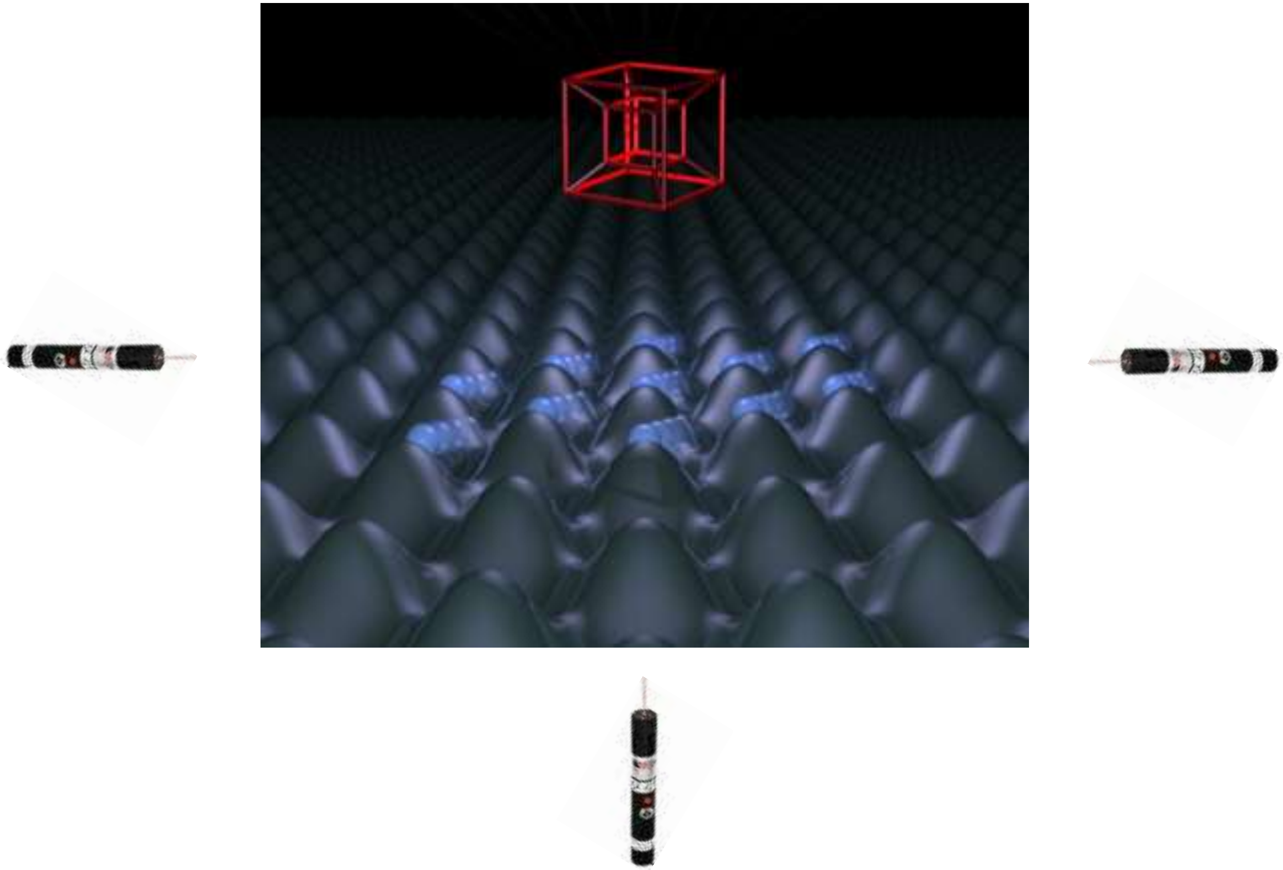


$$I_y = \chi \frac{e^2}{h} \frac{B_{xw}}{\Phi_0} E_z;$$



2D topological charge pump

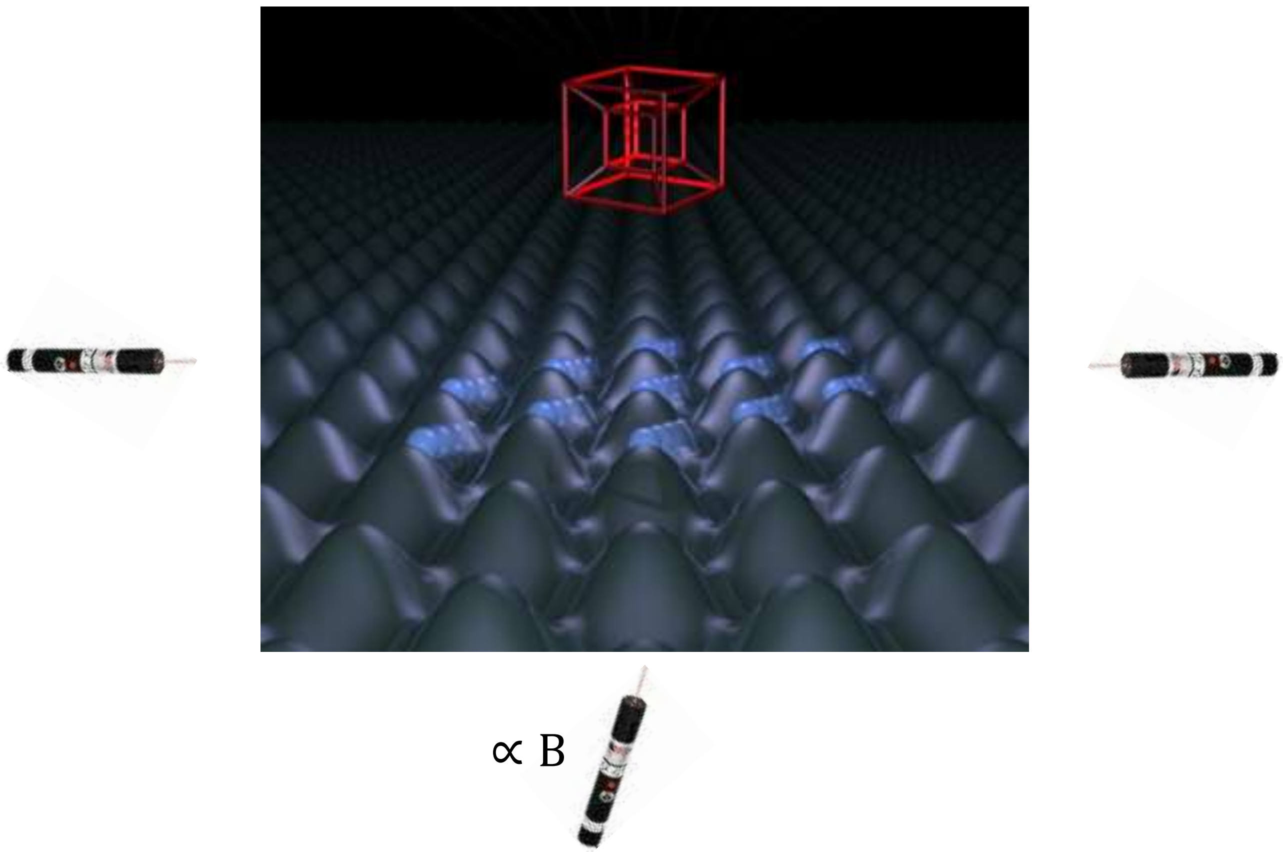
- An optical superlattice



Nature **553**, 55 (2018)

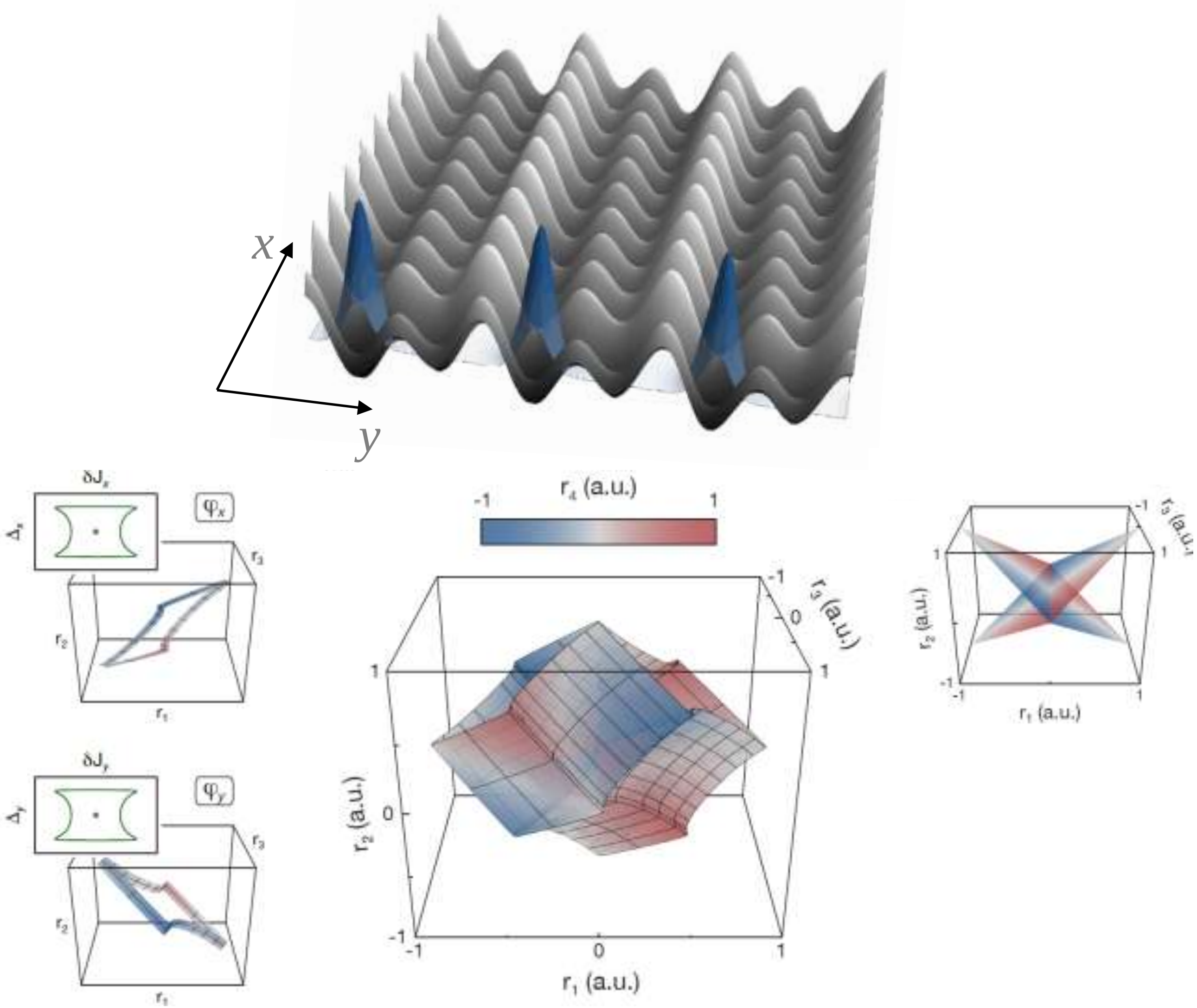
2D topological charge pump

- An optical superlattice



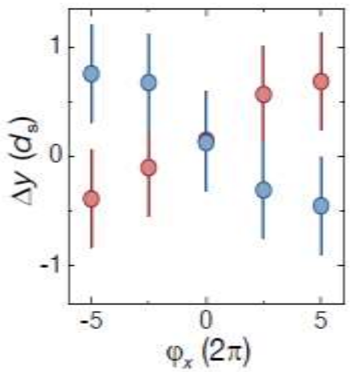
Nature **553**, 55 (2018)

2D topological charge pump



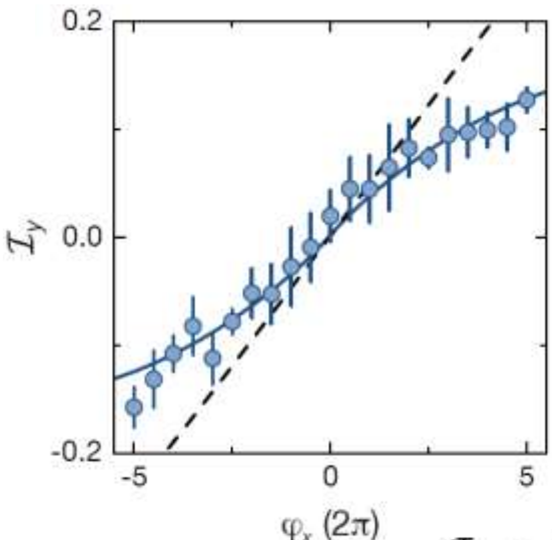
Nature **553**, 55 (2018)

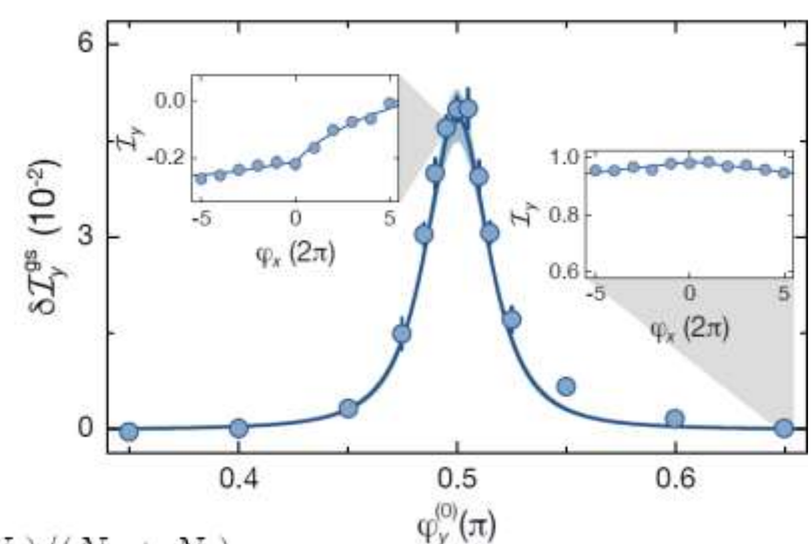
Bulk response



$$I_y = \chi \frac{e^2}{h}\frac{B_{xw}}{\Phi_0}E_z$$

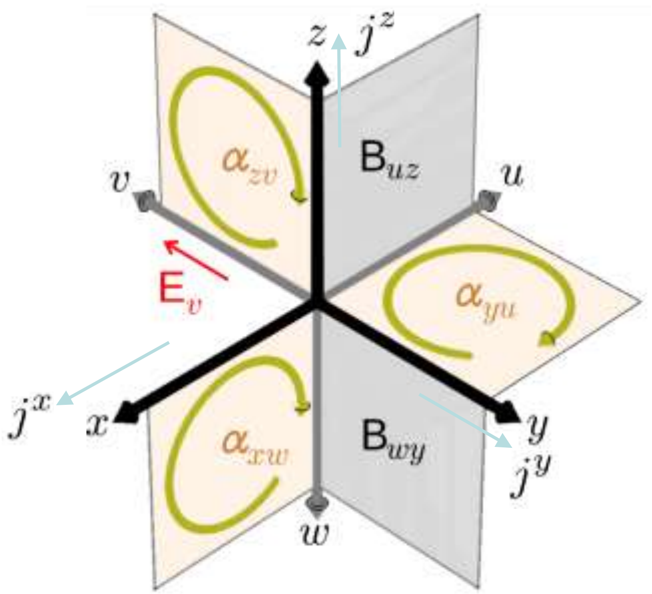
$$\boxed{\chi = 1.07(8)}$$





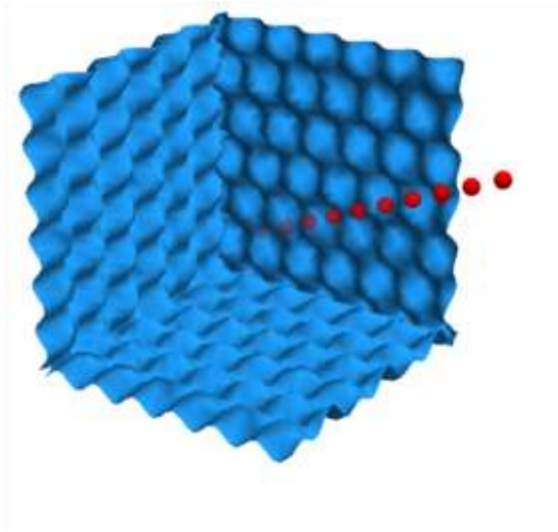
$$\mathcal{I}_y = (N_o - N_e)/(N_o + N_e)$$

6D QHE and 3D topological pumps



$$j^z = \frac{\nu_1^{zv}}{2\pi} E_v \qquad j^y = \frac{\nu_2^{yvuz}}{(2\pi)^2} B_{uz} E_v$$

$$j^x = \frac{\nu_3}{(2\pi)^3} B_{uz} B_{wy} E_v$$



Induced motion after a cycle

$$\delta z_{\mathrm{COM}} = \frac{\nu_1^{z\phi_z}}{\alpha_{zv}} \qquad \delta y_{\mathrm{COM}} = \frac{\nu_2^{yz\phi_y\phi_z} \bar{B}_{uz}}{\alpha_{zv}\alpha_{yu}}$$

$$\delta x_{\mathrm{COM}} = \frac{\nu_3 \bar{B}_{uz} \bar{B}_{wy}}{\alpha_{xw}\alpha_{yu}\alpha_{zv}}$$

I. Petrides, H. M. Price, and **OZ**, arXiv:1804.01871

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Photonic topological pumps

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Atomic topological pump

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