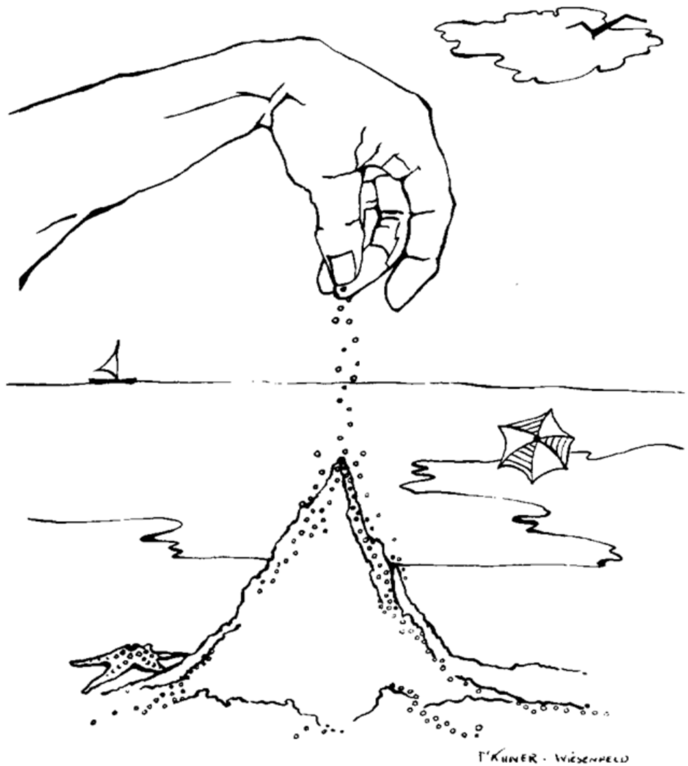
How far do activated random walkers spread from a single source?

Vittoria Silvestri

University of Rome La Sapienza

Banff, 5 July 2022

Self–Organized Criticality (SOC)



Bak, P. (2013). How nature works: the science of self-organized criticality. Springer Science & Business Media.

Activated Random Walks

- Particle configuration $\eta_t : \mathbb{Z}^d \to \mathbb{N} \cup \{s\}$
- Initial configuration η_0 made of i.i.d. Bernoulli(μ) active particles
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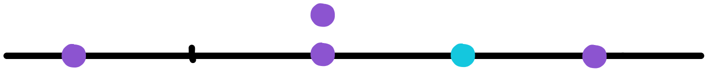

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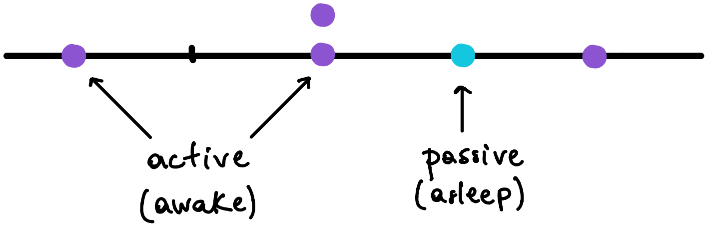

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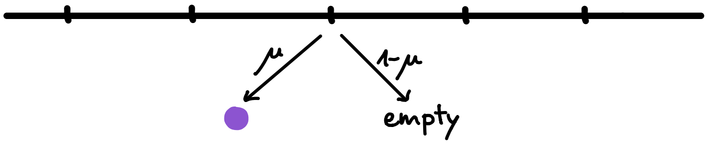

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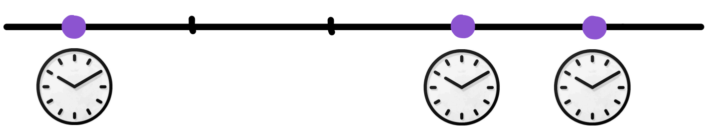

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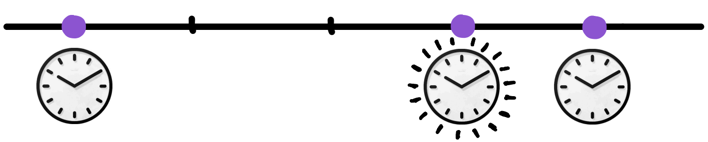

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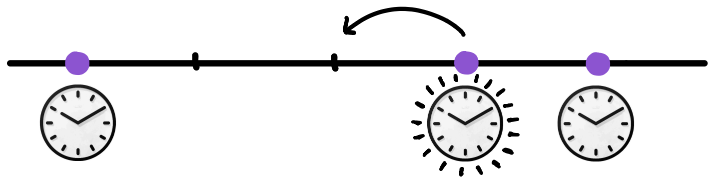

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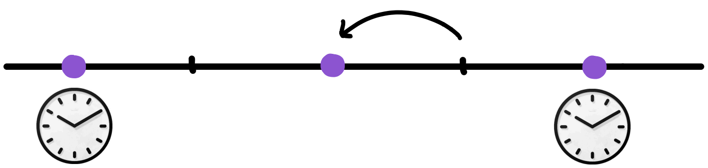

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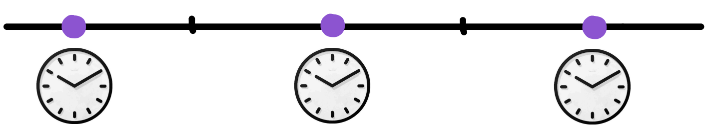

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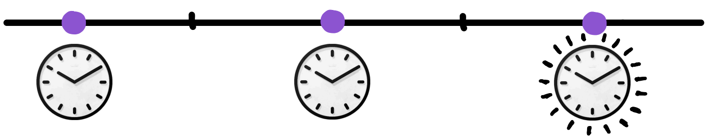

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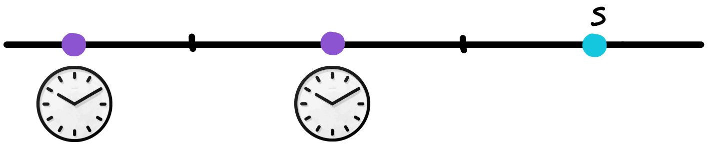

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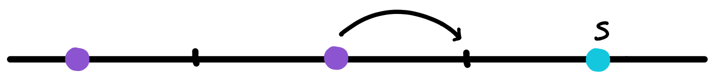

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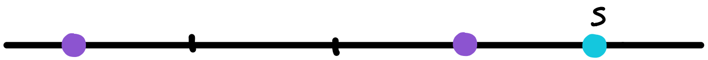

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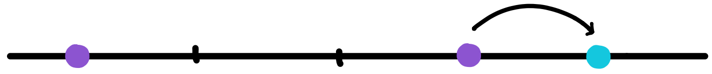

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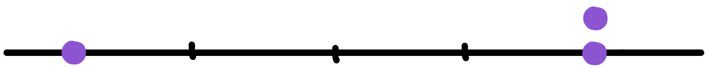

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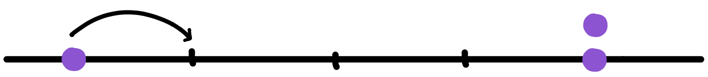

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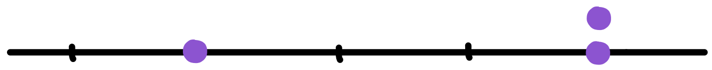

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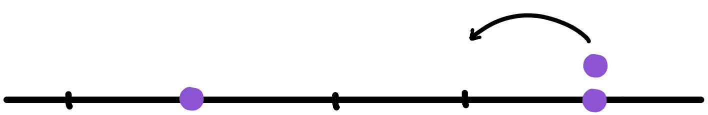

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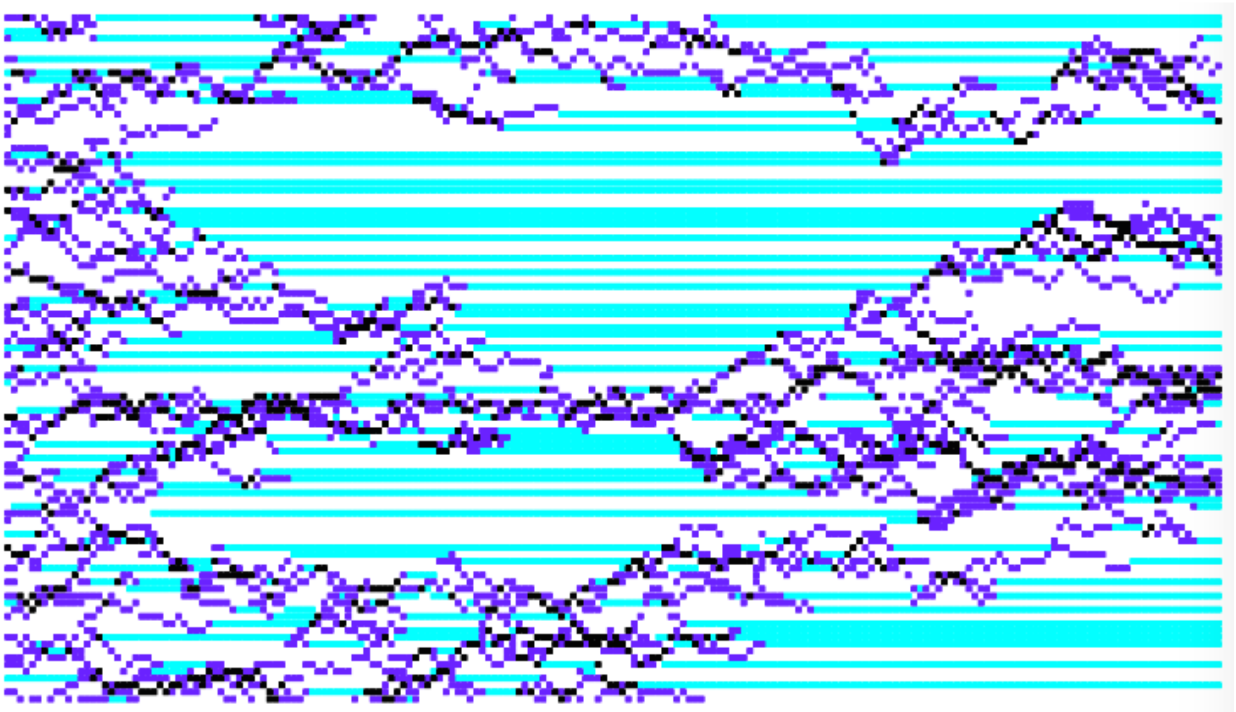
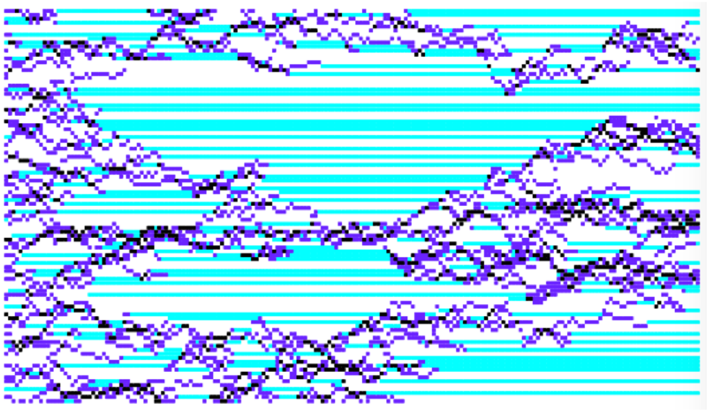


Simulation by Leonardo Rolla (University of Warwick).

Activated Random Walks

Q: Does activity persist forever?



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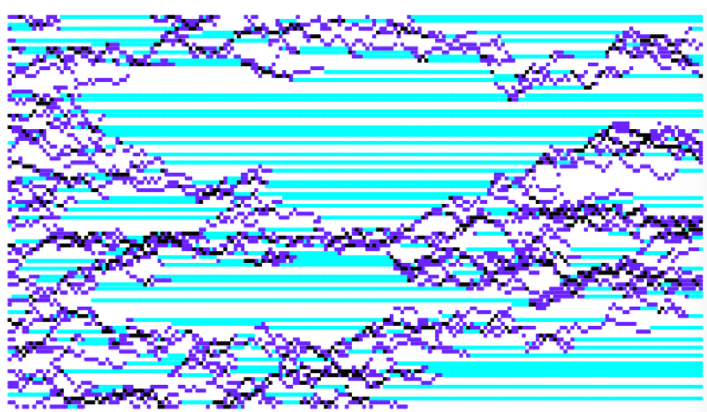
μ small → Fixation

μ large → Explosion



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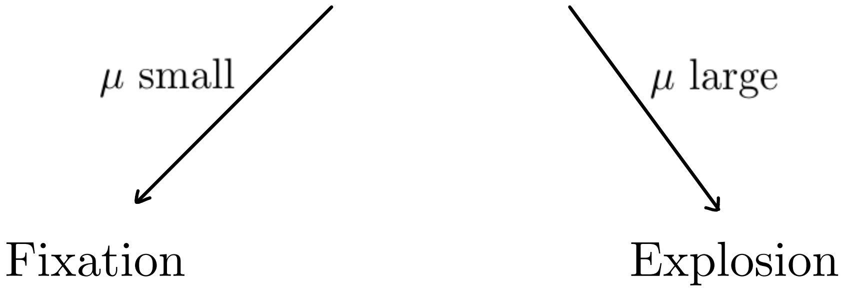

Theorem (Rolla, Sidoravicius, Invent. Math. (2012))

For any sleep rate $\lambda \in (0, \infty]$ *there exists a critical density* $\mu_c(\lambda)$ *such that*

$$\mathbb{P}_\lambda^\mu(\mathit{Fixation}) = \begin{cases} 1, & \text{if } \mu < \mu_c(\lambda) \\ 0, & \text{if } \mu > \mu_c(\lambda). \end{cases}$$

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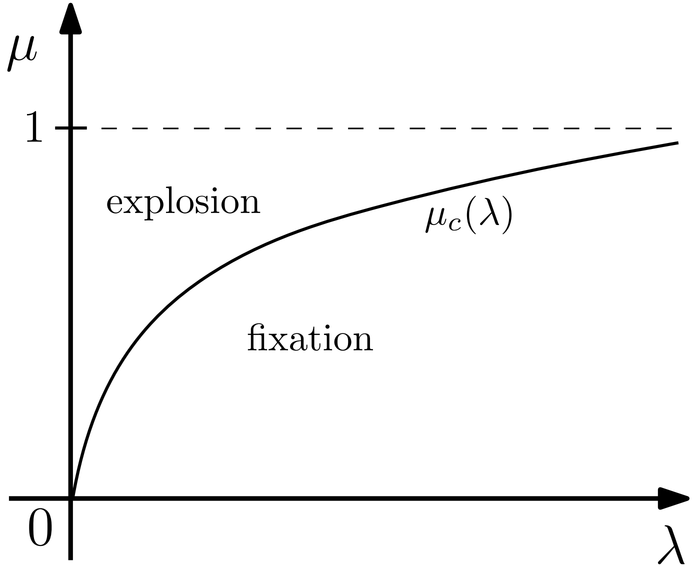

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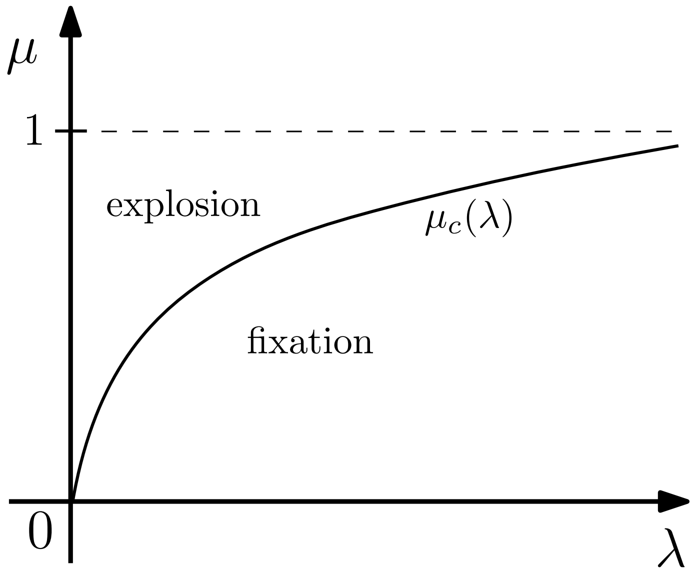
L. Rolla
V. Sidoravicius
O. Zindy

Universality



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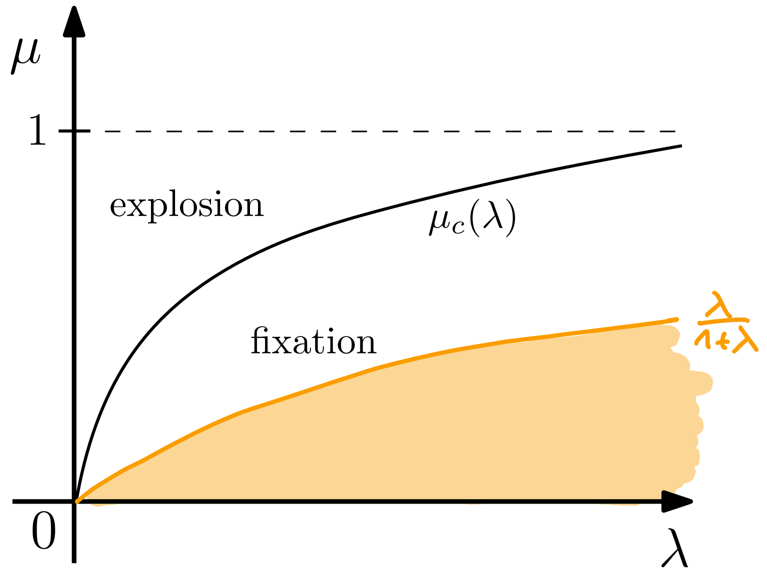

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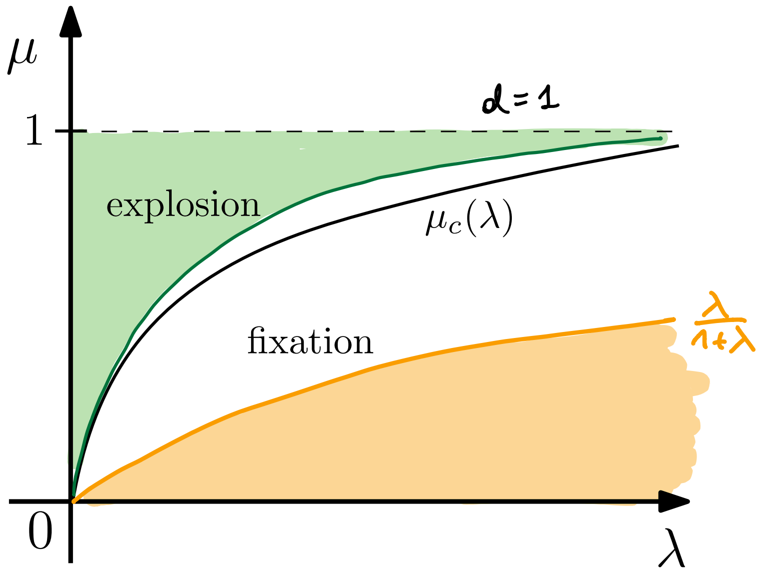

C. Hoffman
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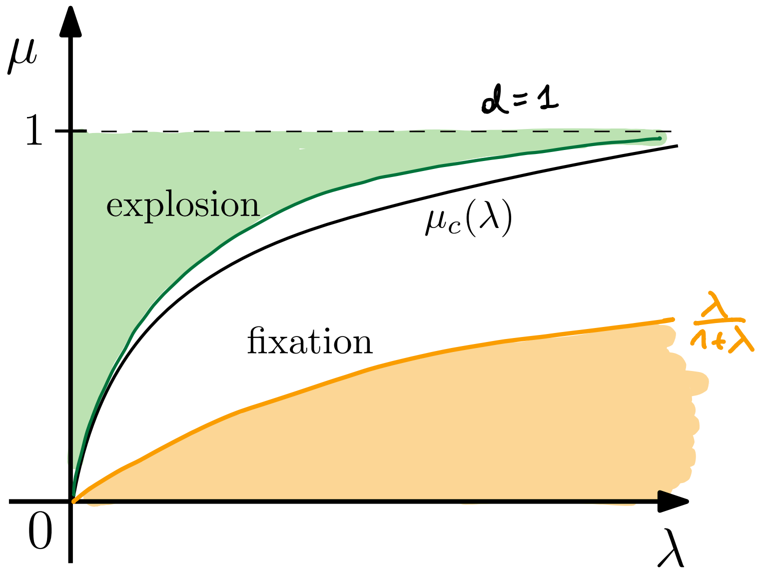
L. Rolla
B. Schapira



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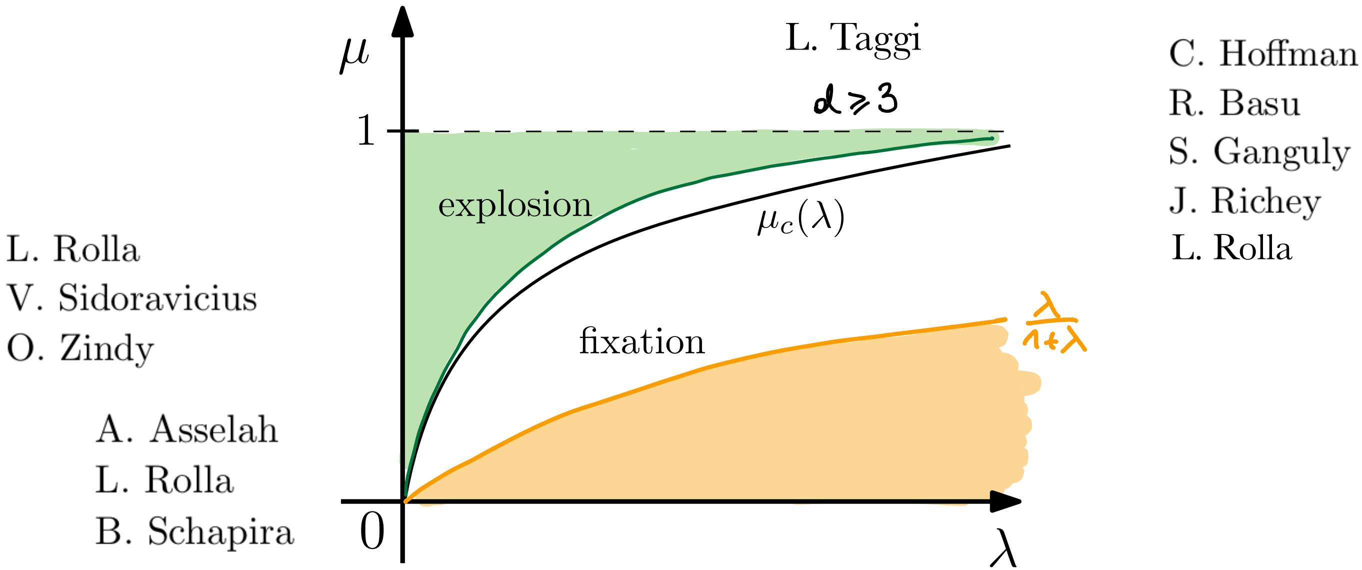

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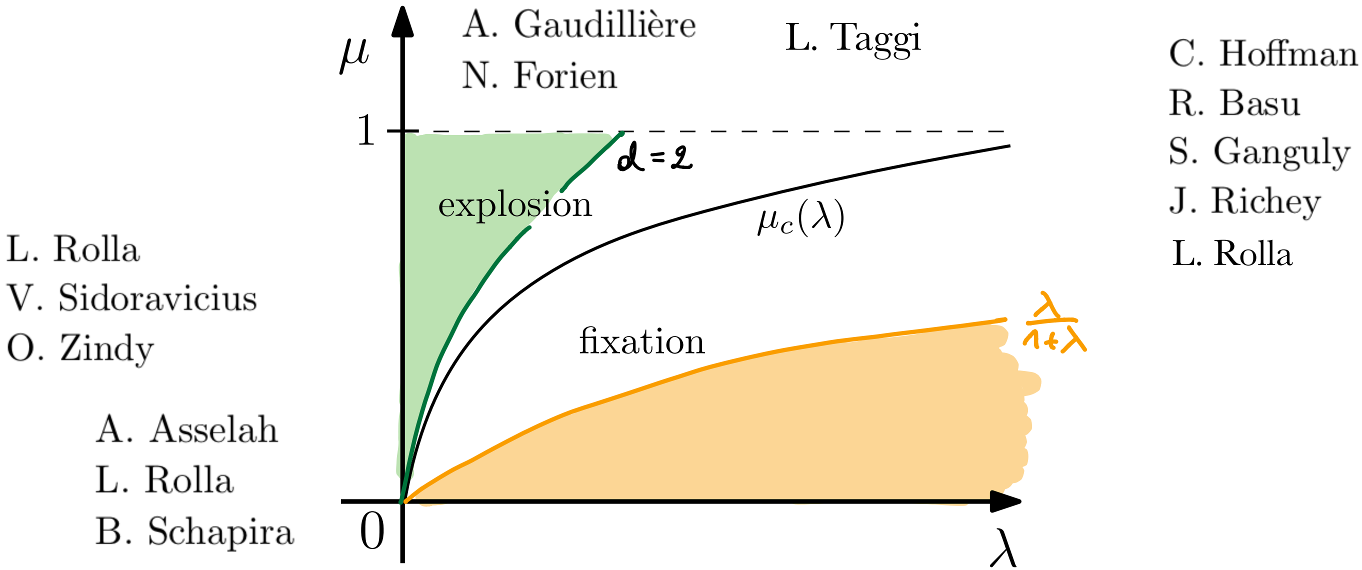

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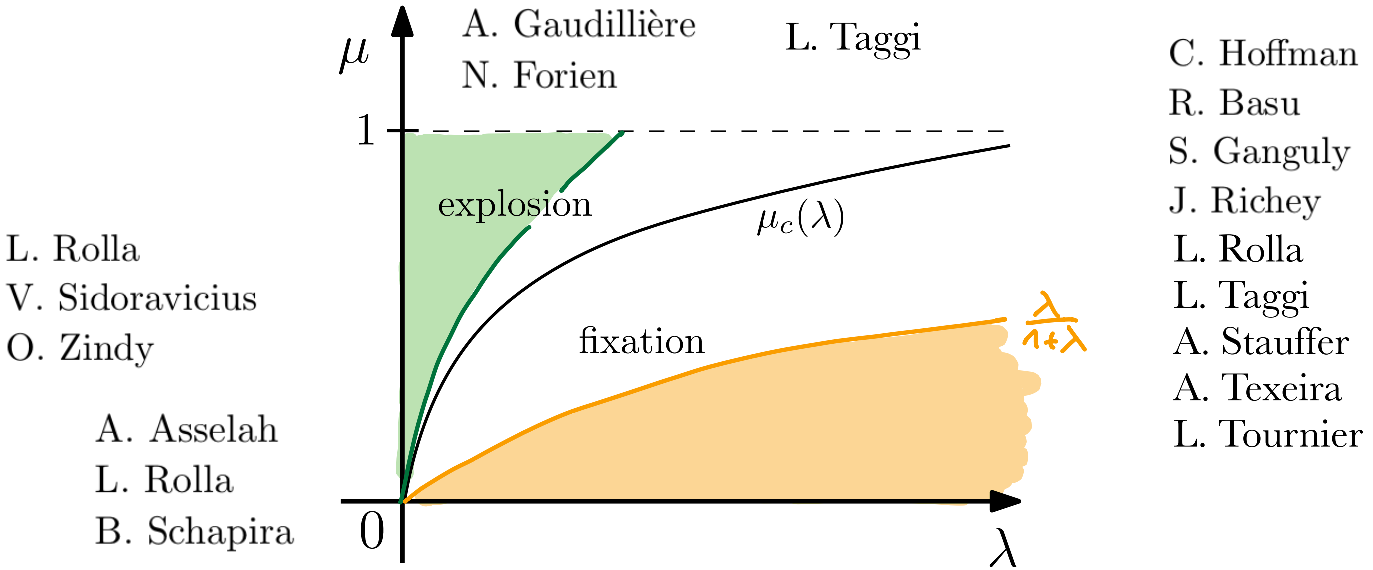
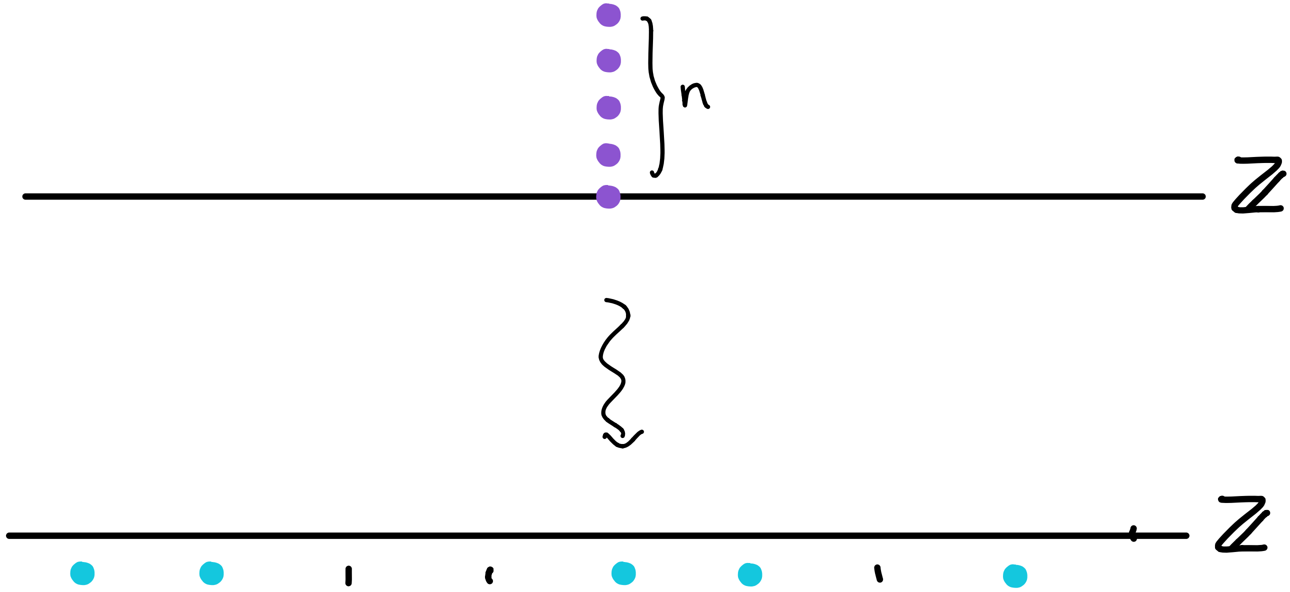


ARWs as a growth model

Q: Start with n walkers at the origin and let the system stabilize. At which density do the particles spread?



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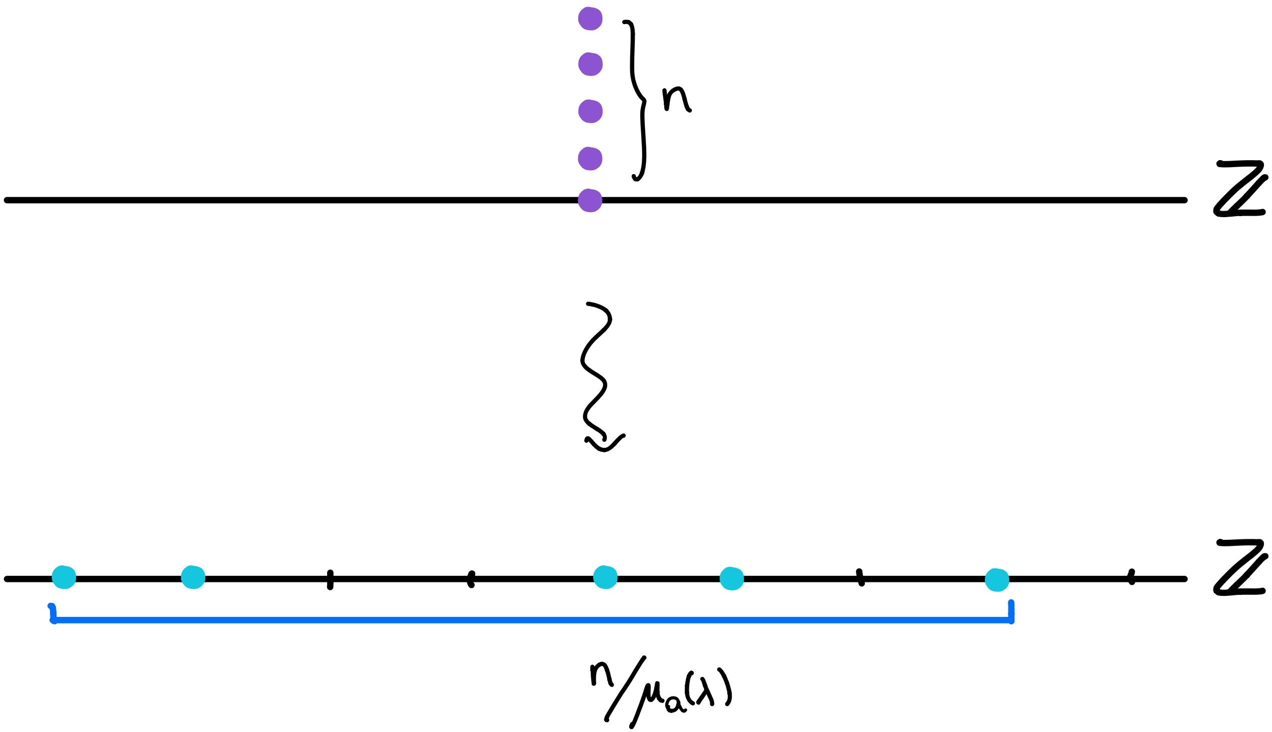

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Video

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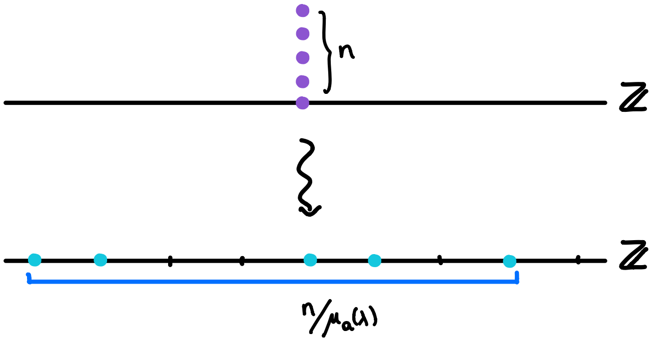


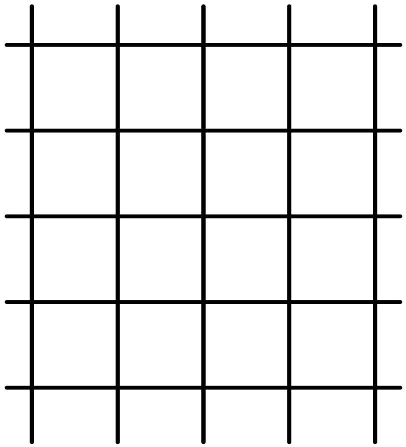
Conjecture

For any sleep rate $\lambda \in (0, \infty]$ *there exists a critical density* $\mu_a(\lambda)$ *such that for any* $\varepsilon > 0$

$$\mathbb{P}_\lambda\left(B_{\frac{n}{\mu_a(\lambda)}(1-\varepsilon)} \subseteq A_n \subseteq B_{\frac{n}{\mu_a(\lambda)}(1+\varepsilon)} \textit{ eventually in } n\right) = 1.$$

ARWs in finite volume

- Start with $\mathbf{1}_{I_N}$, for $I_N \subseteq \mathbb{Z}^d$
- Stabilize with killing on ∂I_N
- Denote by $\mathcal{S}_\lambda(\mathbf{1}_{I_N})$ the final configuration.



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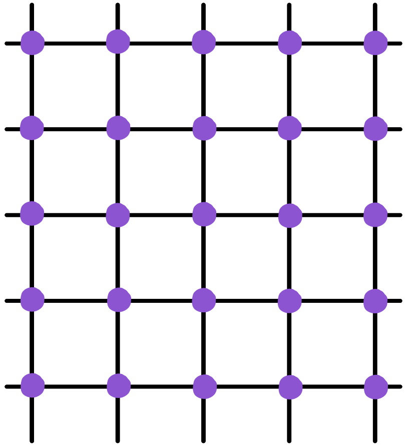

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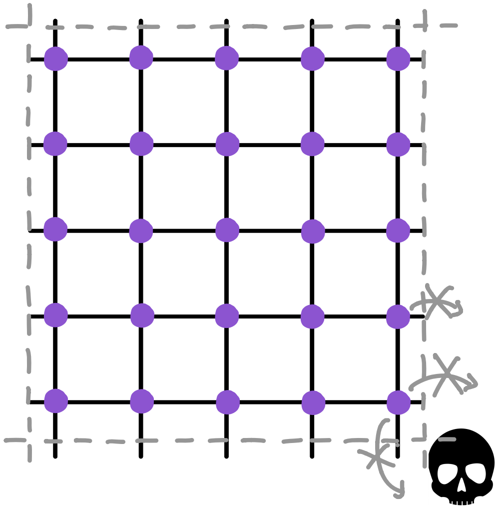

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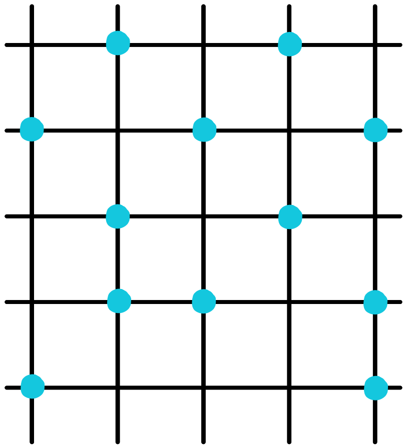
- Stabilize with killing on ∂I_N
- Denote by $\mathcal{S}_\lambda(\mathbf{1}_{I_N})$ the final configuration.



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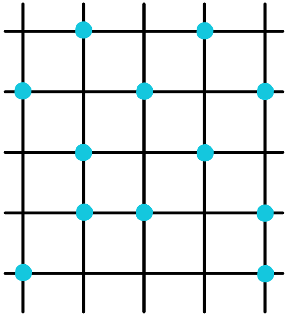


$\mathcal{S}_\lambda(\mathbf{1}_{I_N})$

Conjecture

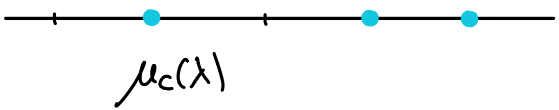
For any sleep rate $\lambda \in (0, \infty]$ *there exists a critical density* $\mu_s(\lambda)$ *such that*

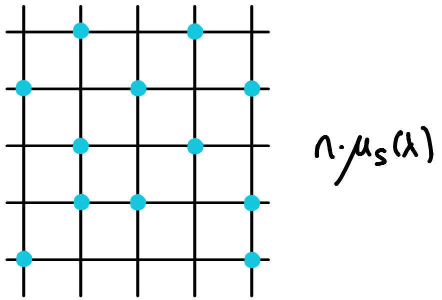
$$\lim_{I_N \nearrow \mathbb{Z}^d} \frac{|\mathcal{S}_\lambda(\mathbf{1}_{I_N})|}{\|I_N\|} = \mu_s(\lambda).$$

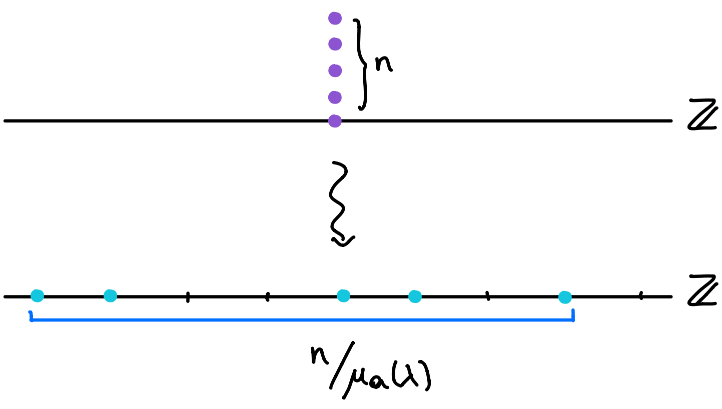
Density conjecture

Conjecture (Universality)

$\mu_c(\lambda) = \mu_a(\lambda) = \mu_s(\lambda)$.







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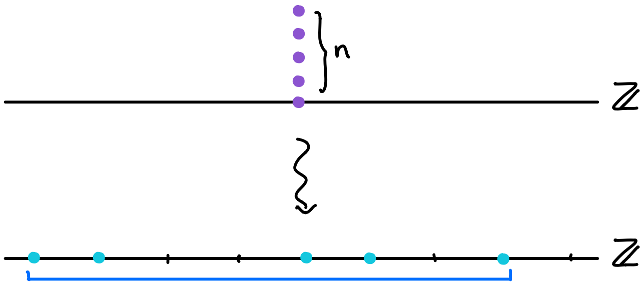

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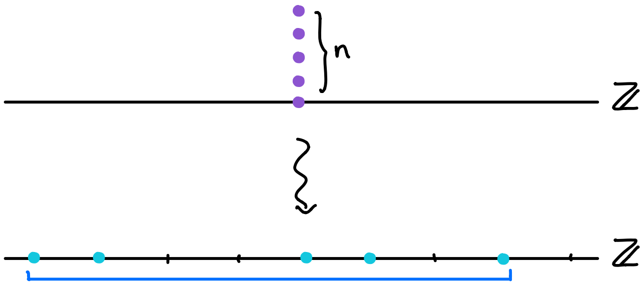
$\mu_c(\lambda) = \mu_a(\lambda) = \mu_s(\lambda)$.

Theorem (Levine, S., JSP (2021))

ARWs on $\mathbb{Z}$. Let A_n denote the set of visited sites until stabilization when starting with n particles at the origin. Then for any sleep rate λ there exist critical densities $\mu_{in}(\lambda)$ and $\mu_{out}(\lambda)$ such that, assuming they are both positive, for any $\varepsilon > 0$ it holds

$$\mathbb{P}_\lambda\Big(A_n \subseteq B_{\frac{n}{\mu_{out}(\lambda)}(1+\varepsilon)},\ \|A_n\| \geq \frac{n}{\mu_{in}(\lambda)}(1-\varepsilon) \text{ eventually in } n\Big) = 1.$$

Density conjecture



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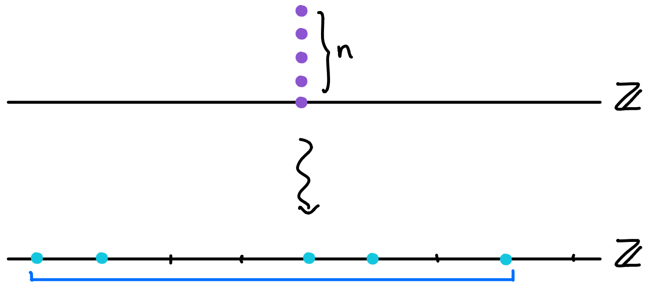

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Definition ($\mu_{out}(\lambda)$)

Start with an i.i.d. Bernoulli configuration on $\mathbb{Z}$. *Let* $w : \mathbb{Z} \to \mathbb{N}$ *denote the number of clock rings at each site of* $\mathbb{Z}$ *until stabilization. Then*

$$\mu_{out}(\lambda) := \sup\{\mu : \mathbb{E}^{\mu}_{\lambda}(w(0)^3) < \infty\}.$$

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$\mu_{out}(\lambda) = \mu_c(\lambda)$.

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For $I \subseteq \mathbb{Z}$ define

$$\mu_{in,I}(\lambda) := \inf\Big\{\mu : \mathbb{P}_\lambda\Big(\frac{|\mathcal{S}(\mathbf{1}_I)|}{\|I\|} > \mu\Big) \leq \|I\|^{-20}\Big\},$$

and set

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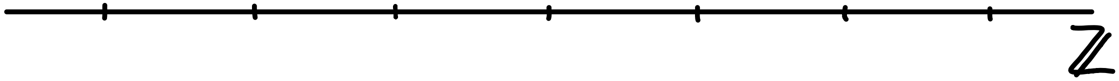
Conjecture

$\mu_{in}(\lambda) = \mu_s(\lambda)$.

Ideas of proof

- Outer bound: Abelian property + coupling with Internal Diffusion Limited Aggregation on Bernoulli vertex percolation.
- Inner bound: Build the stable configuration on progressively larger intervals, using that it only depends on the values of the odometer function on the boundary.

The Abelian Property



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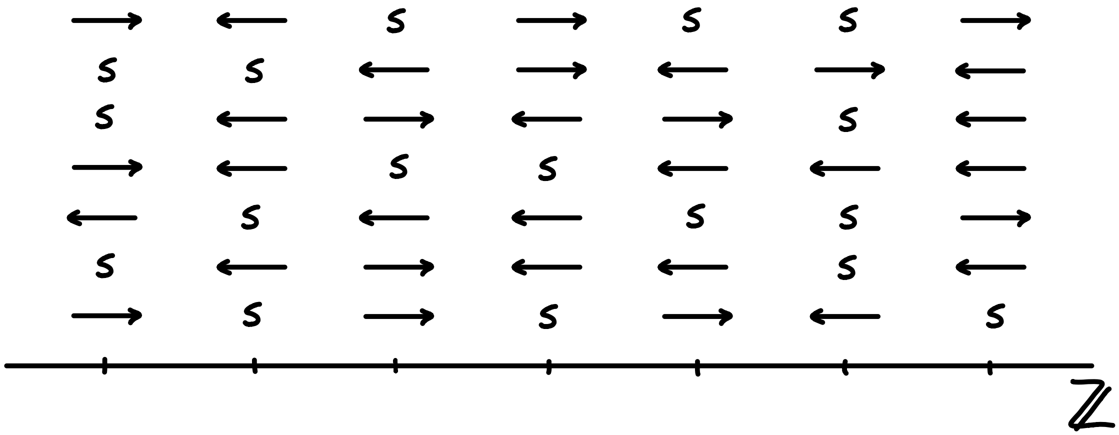

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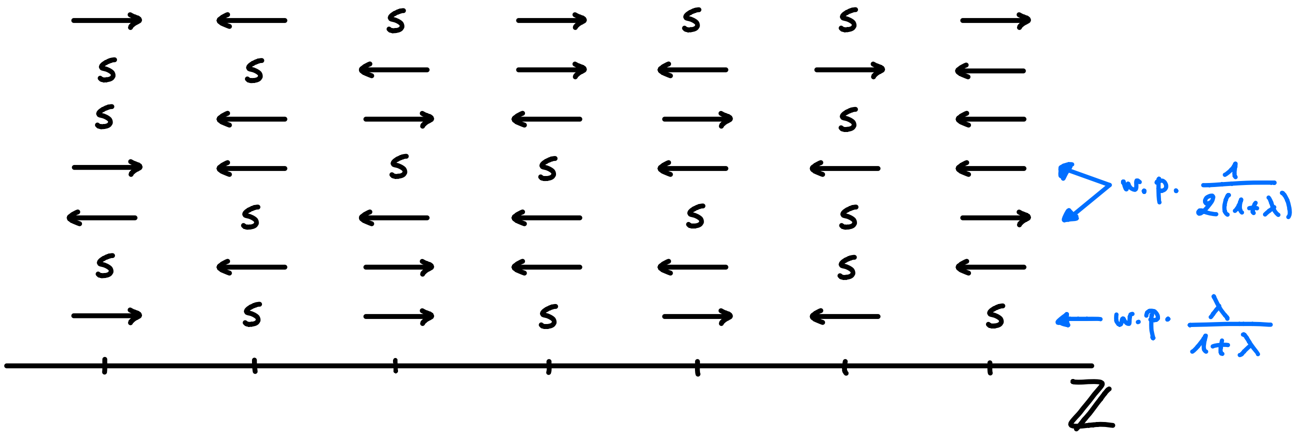

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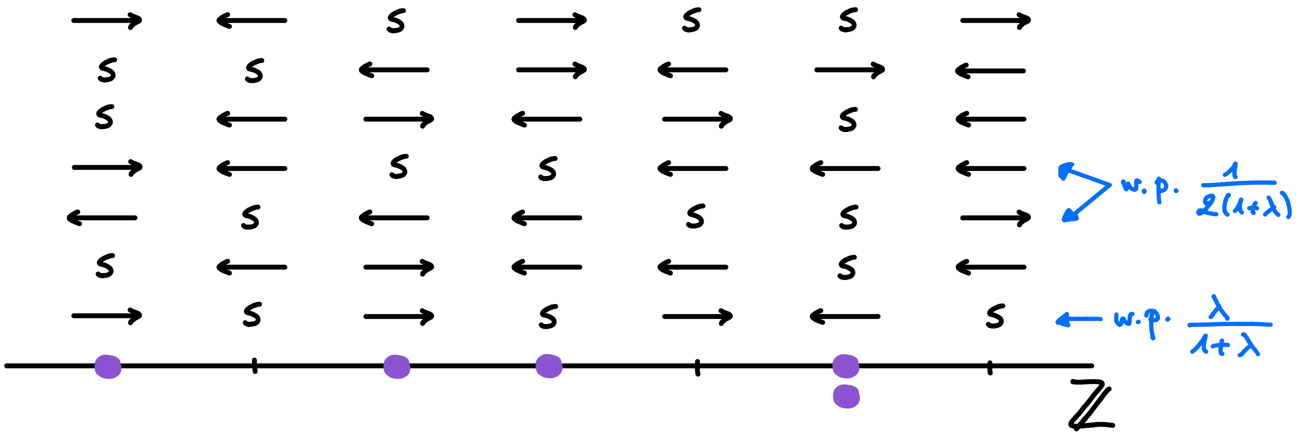

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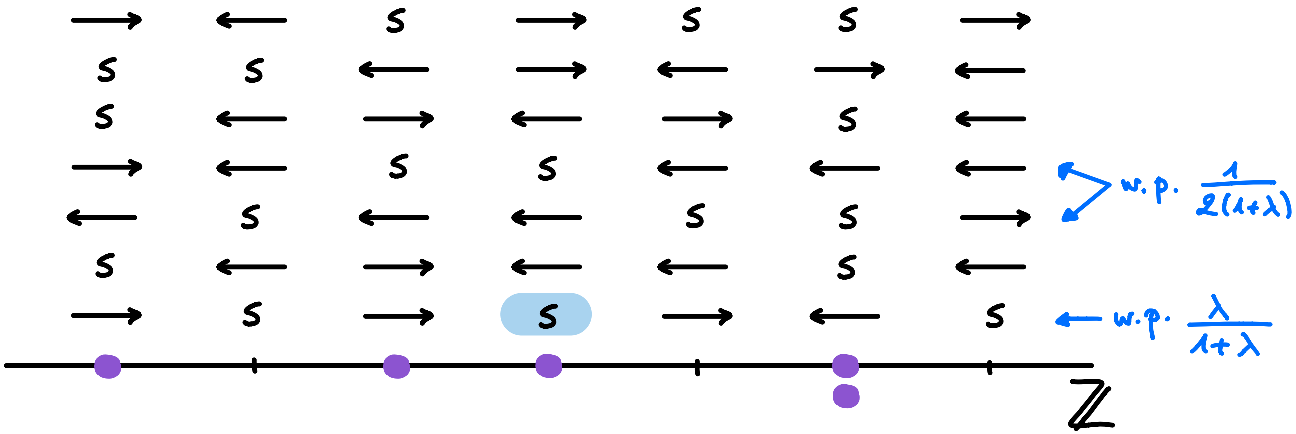

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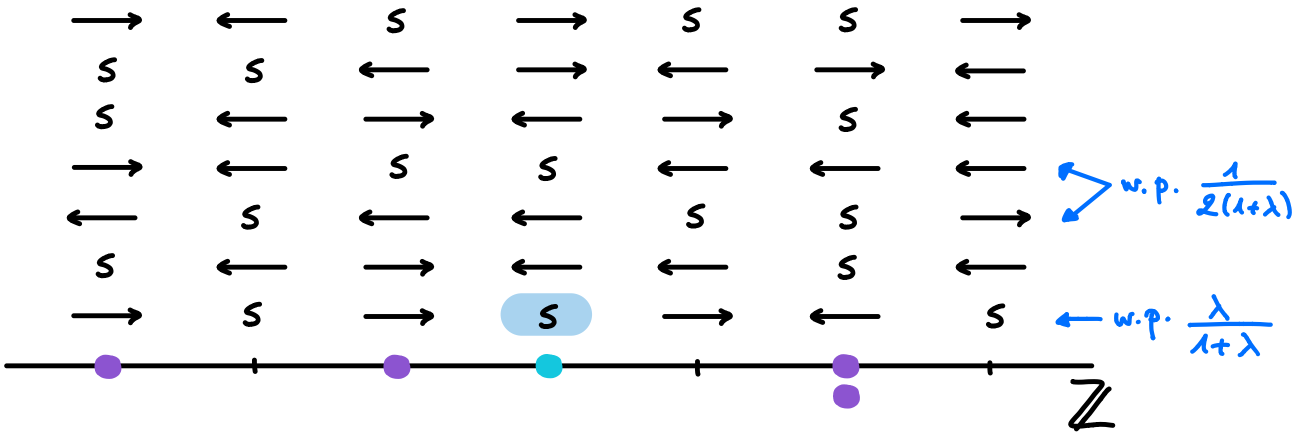

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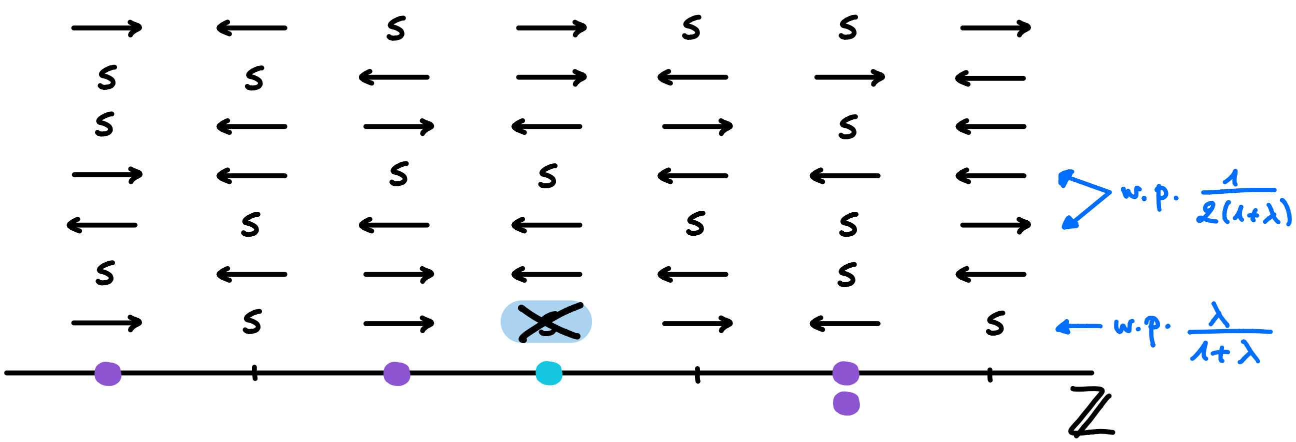

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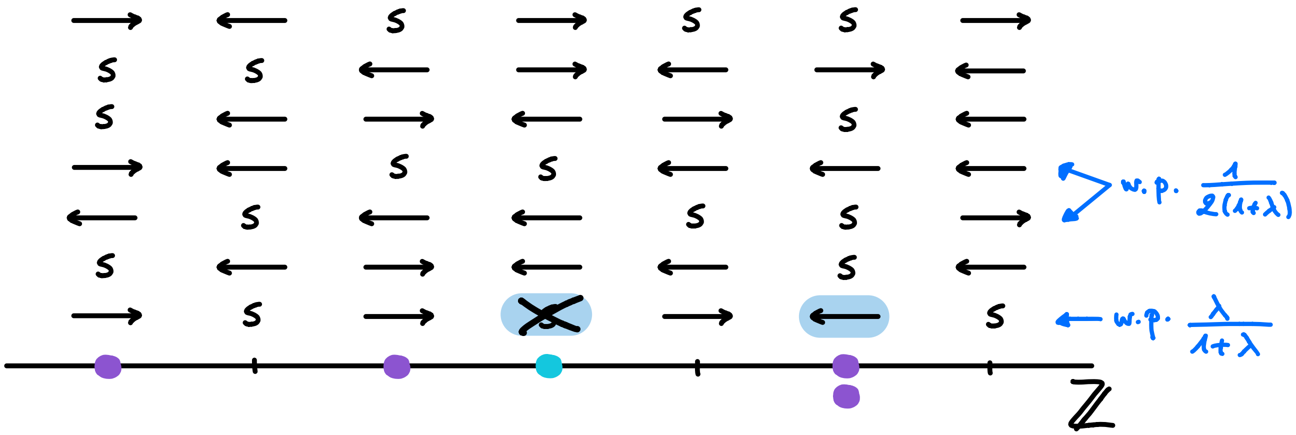

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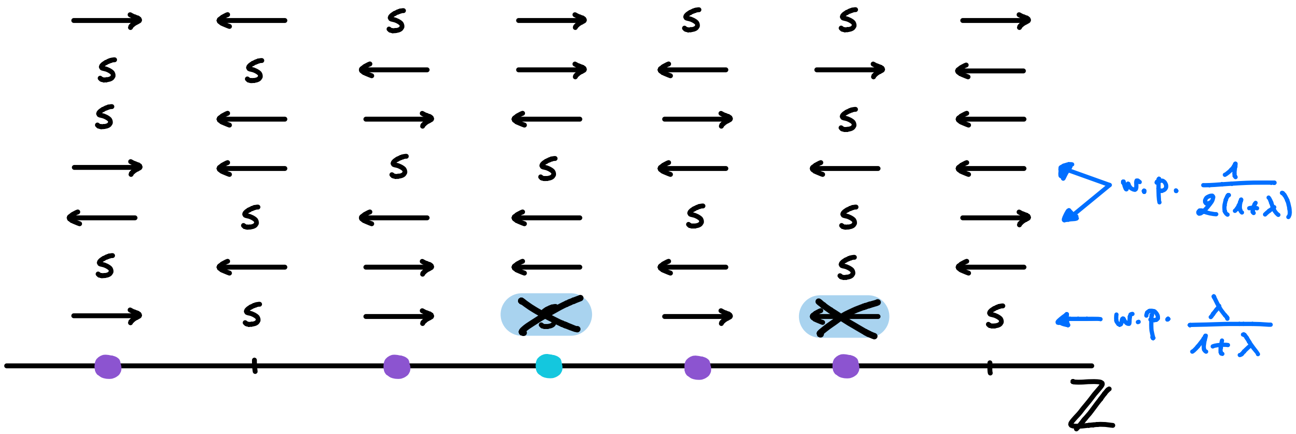

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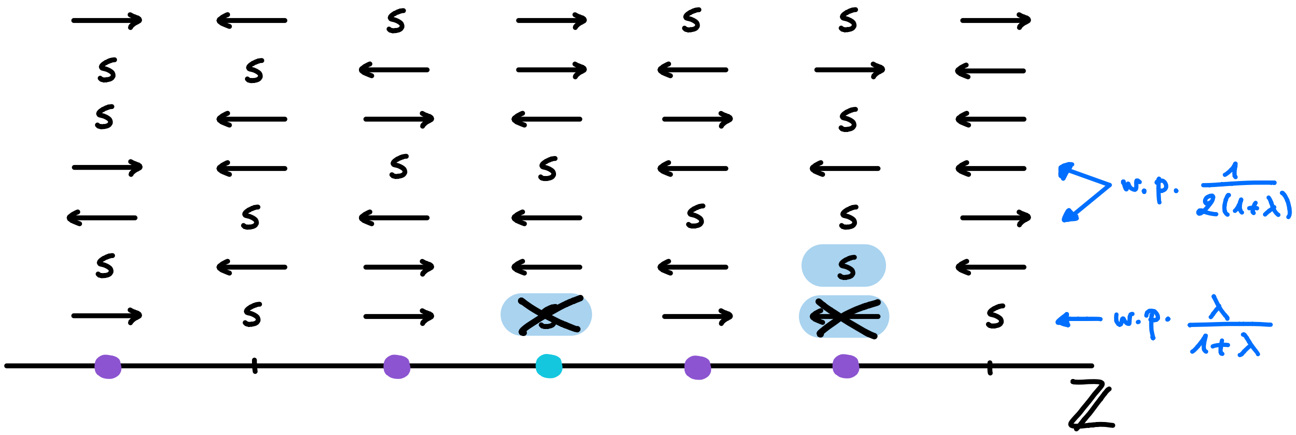

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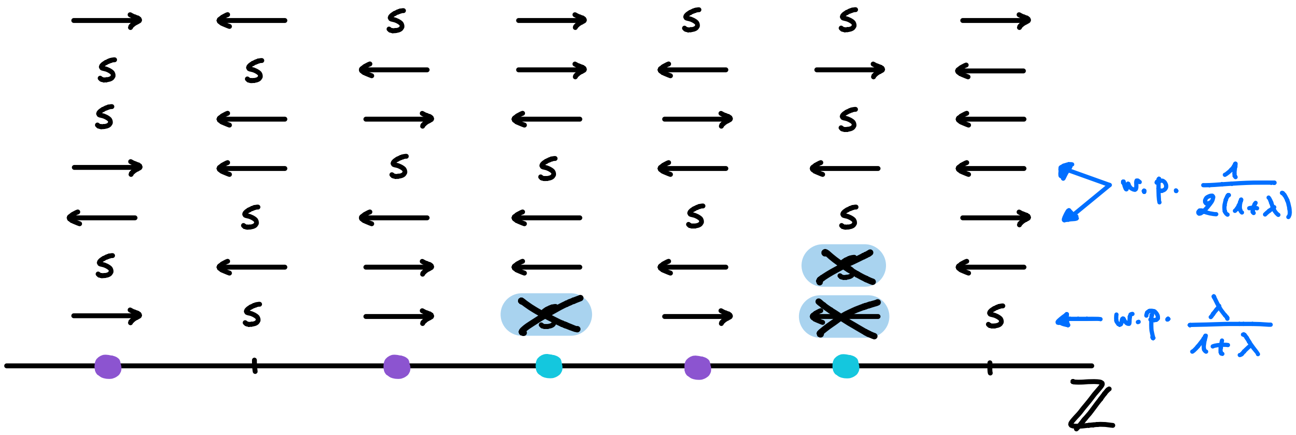

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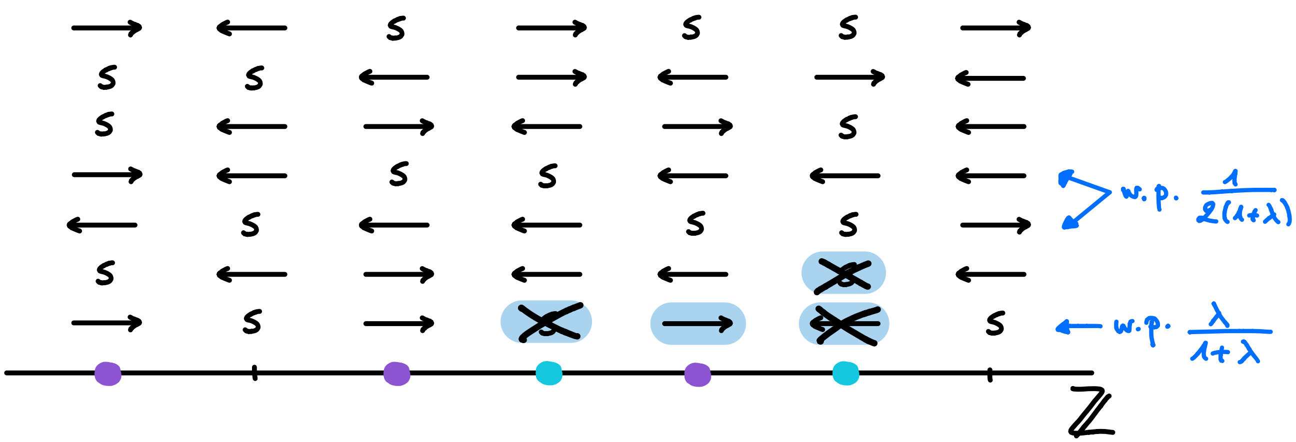

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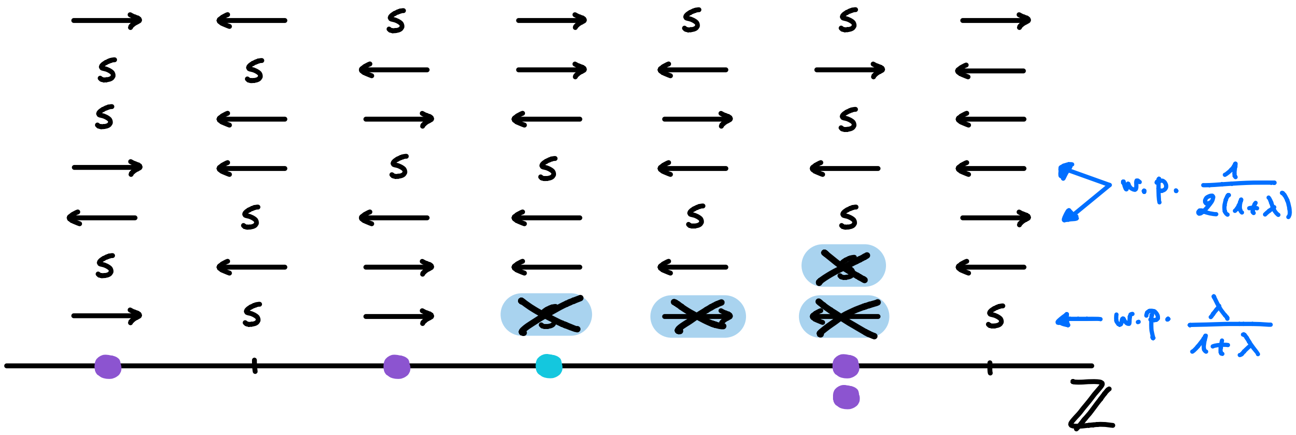

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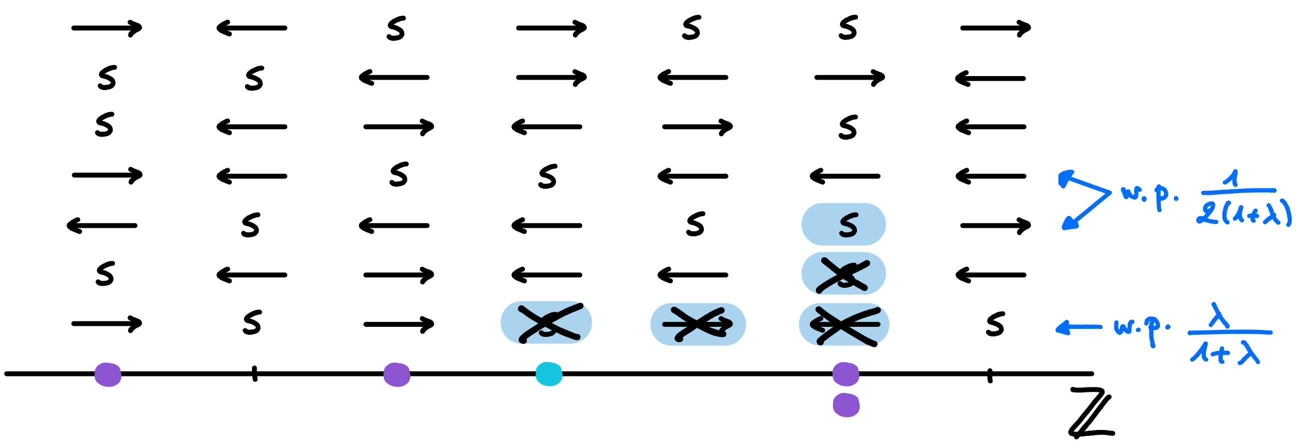

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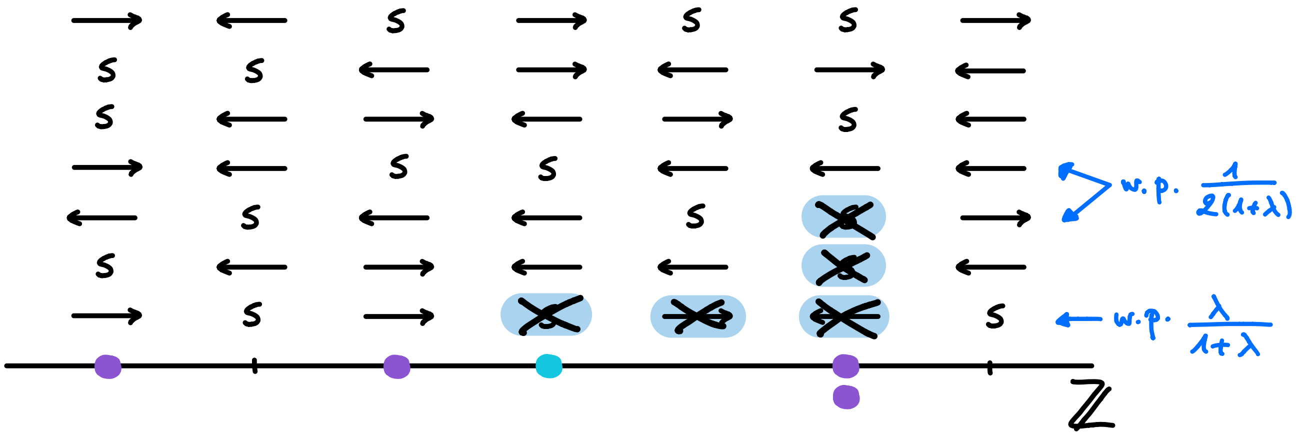

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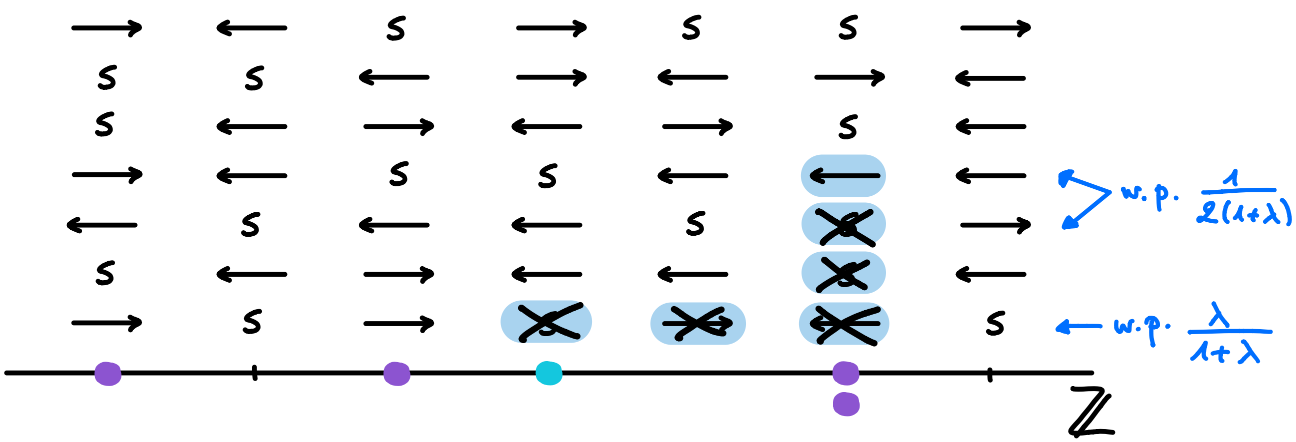

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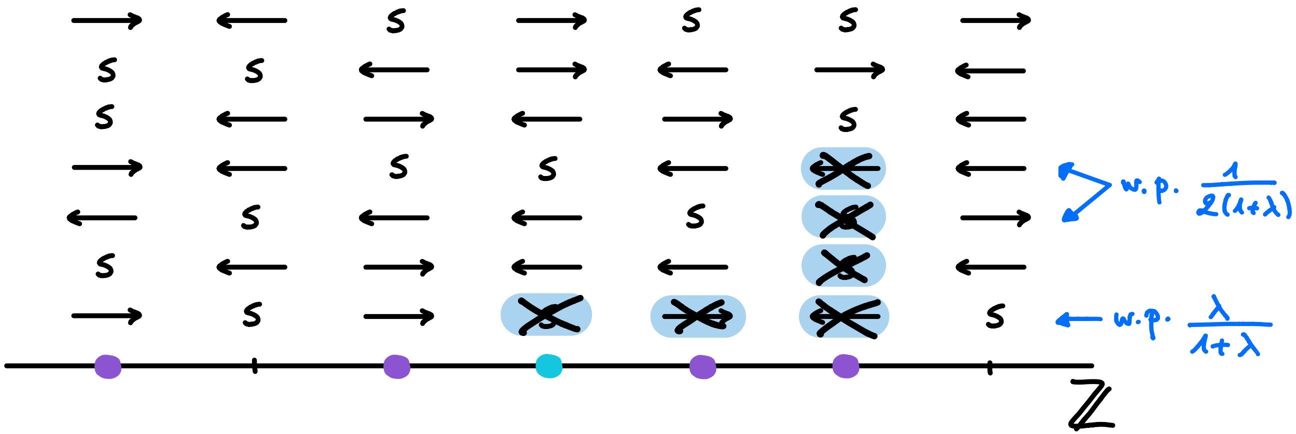

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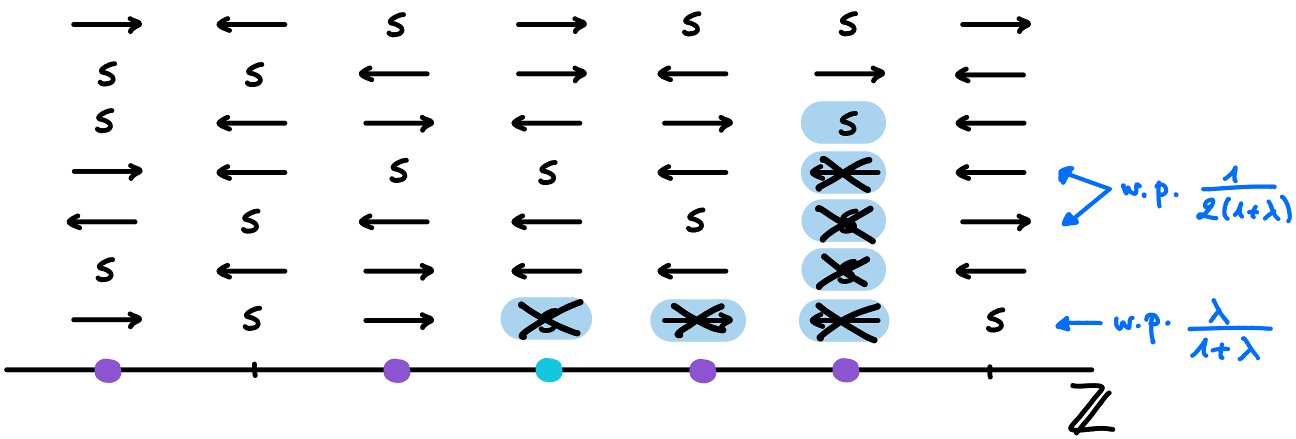

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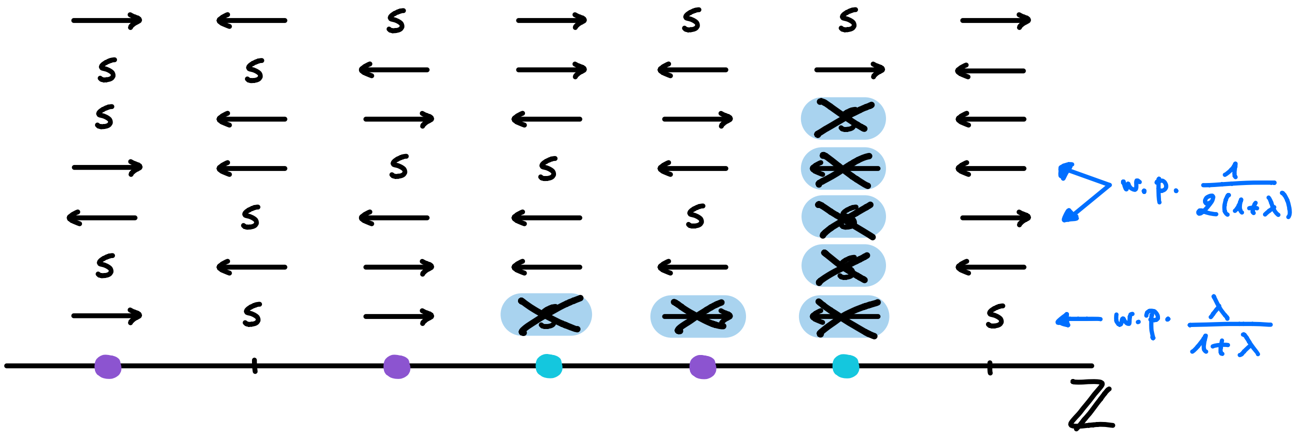

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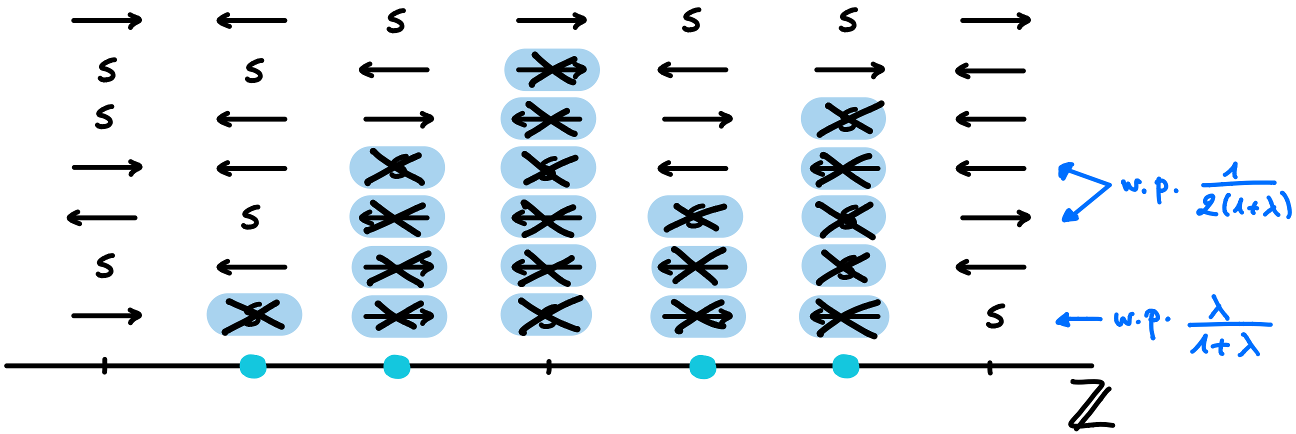

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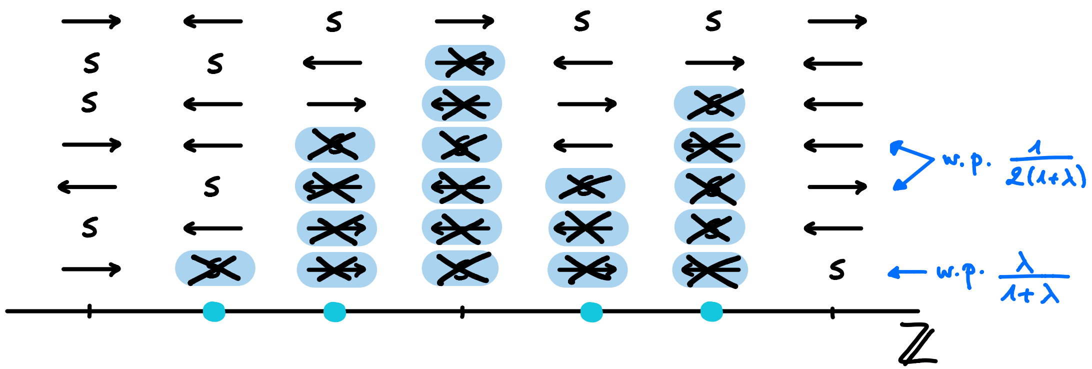
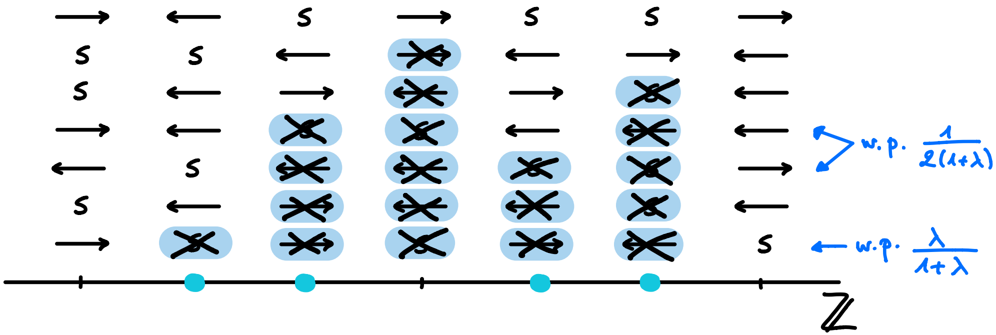


Theorem (P. Diaconis, W. Fulton (1991))

- *The final configuration does not depend on the order of topplings.*
- *The number of instructions used per site does not depend on the order of topplings.*

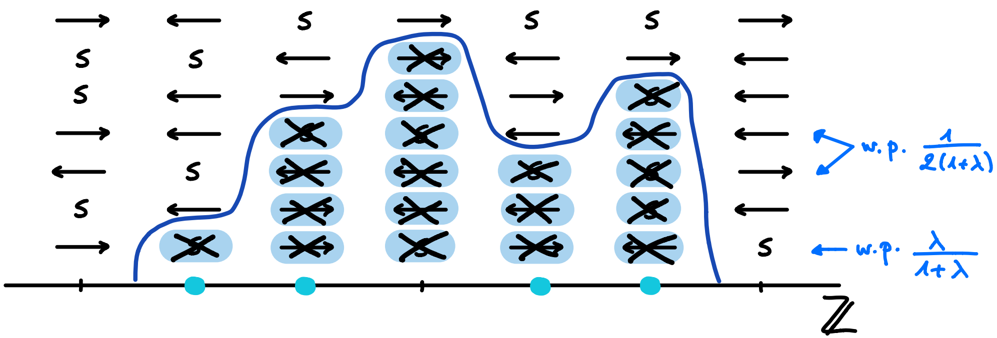
The Abelian Property



Definition (Odometer function)

For each $x \in \mathbb{Z}$ the odometer function $w : \mathbb{Z} \to \mathbb{Z}_+$ is given by $w(x) =$ number of instruction used at x until stabilization.

The Abelian Property



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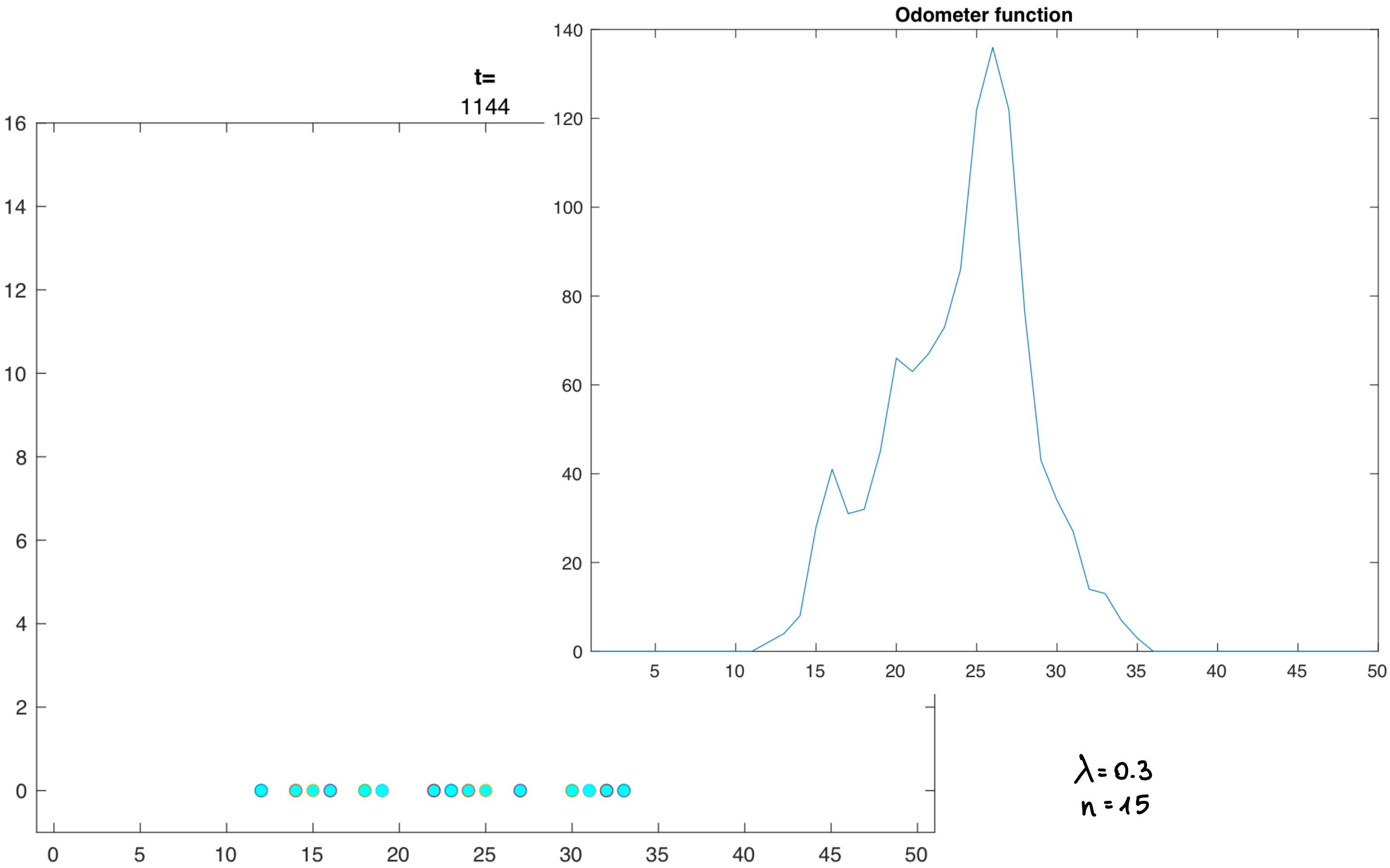
- The odometer function does not depend on the order of topplings [Abelian Property].
- Ignoring sleep instructions can only increase the odometer [Least Action Principle].

The Abelian Property

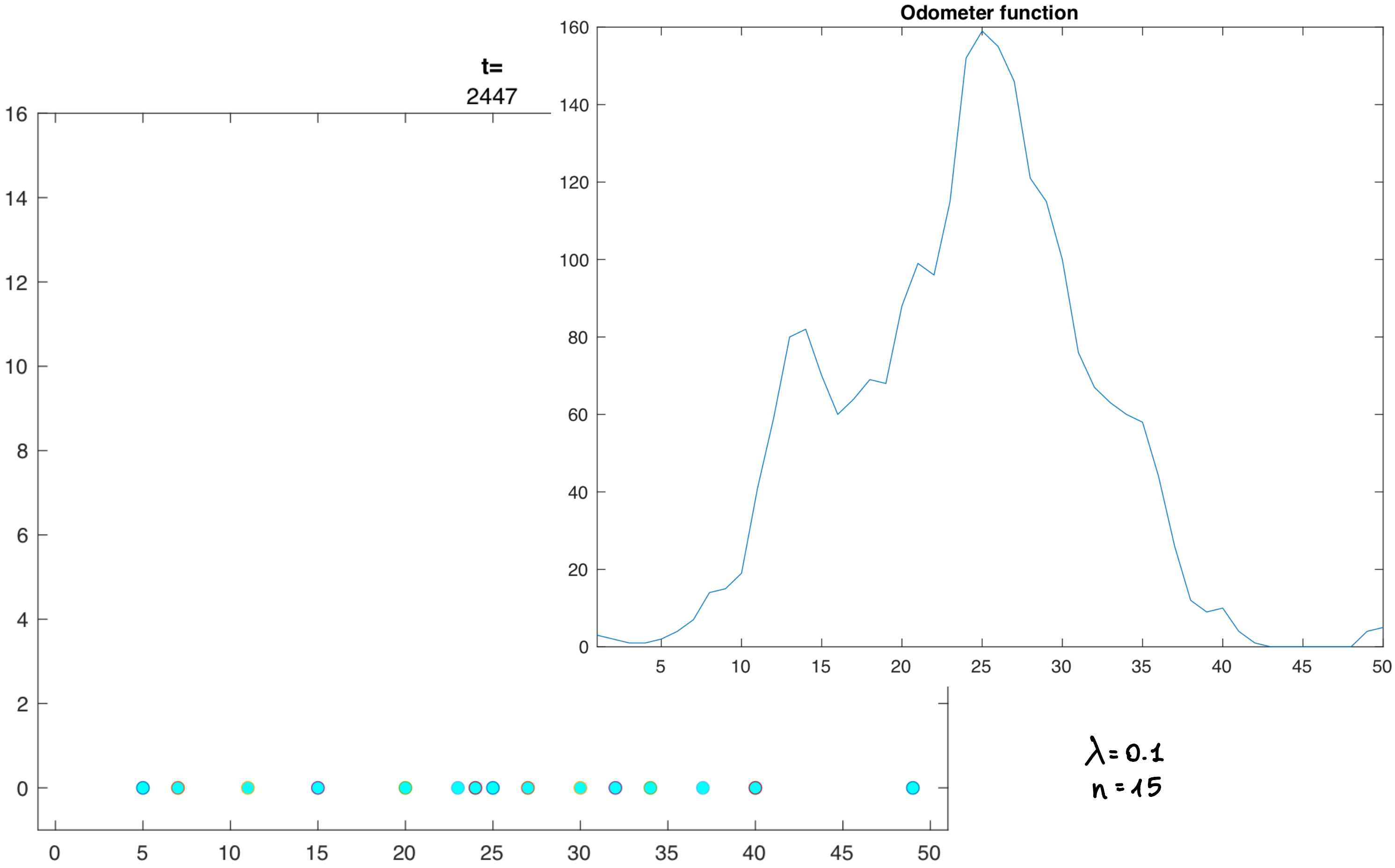
Video

The Abelian Property



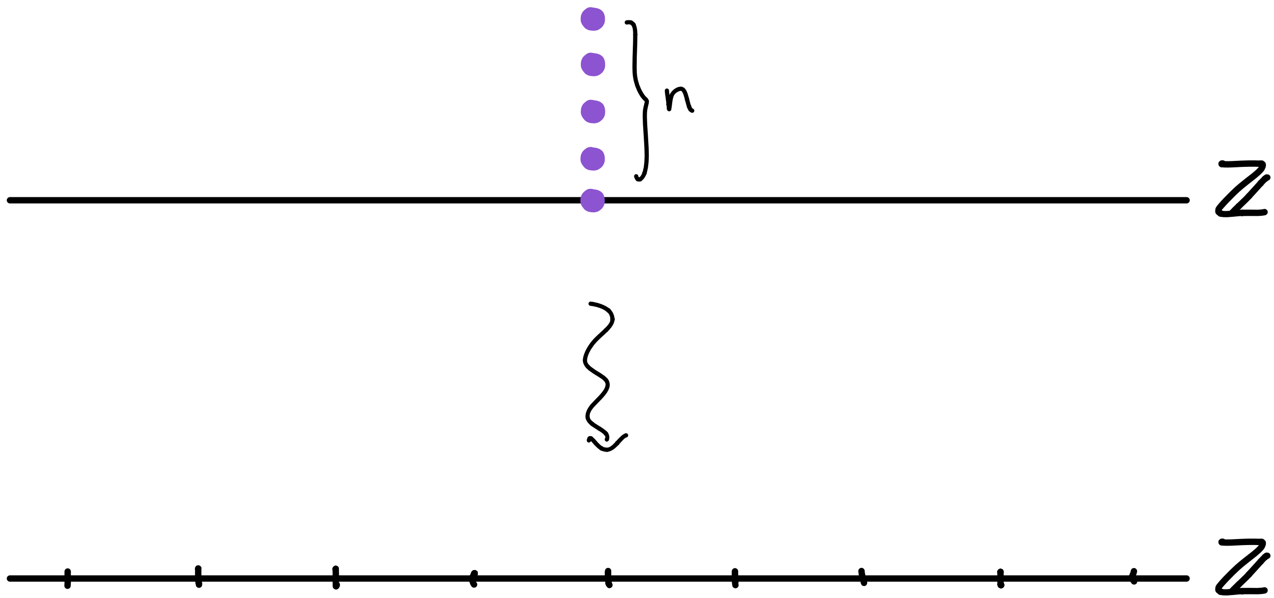
$\lambda = 0.3$
$n = 15$

The Abelian Property



$\lambda = 0.1$
$n = 15$

Ideas of proof: the outer bound



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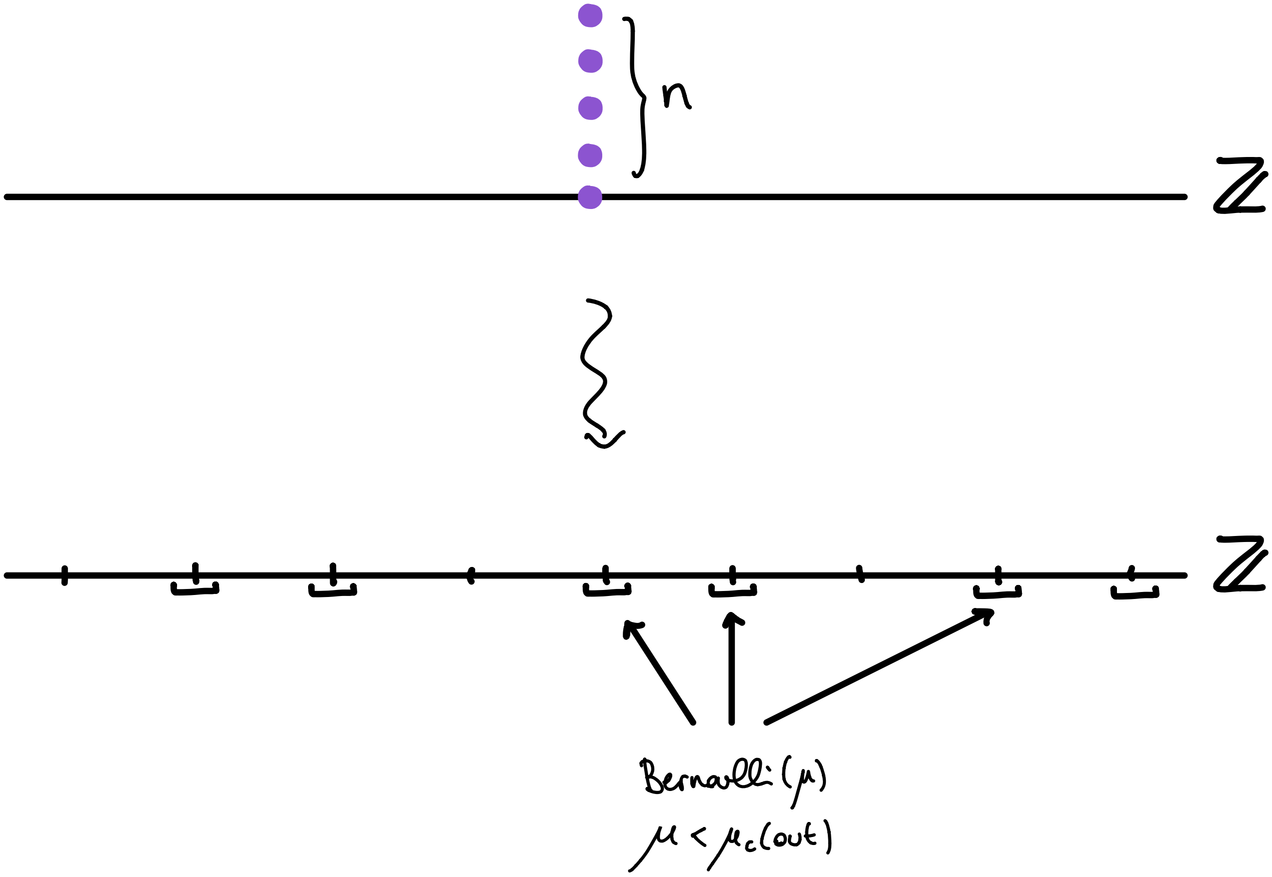

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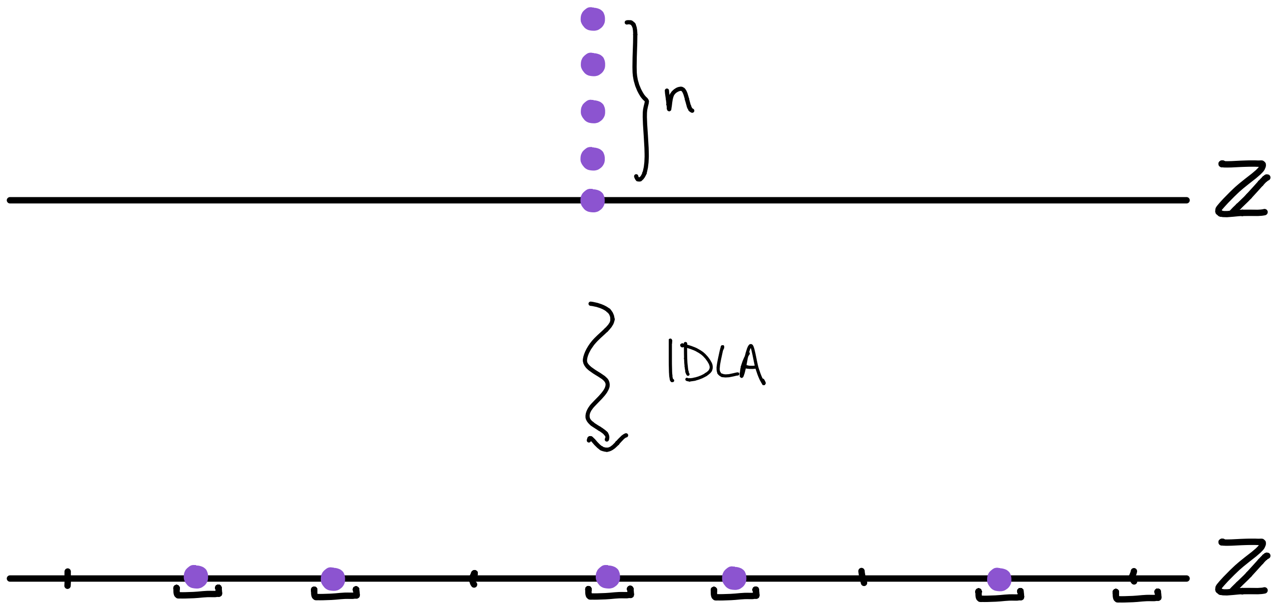

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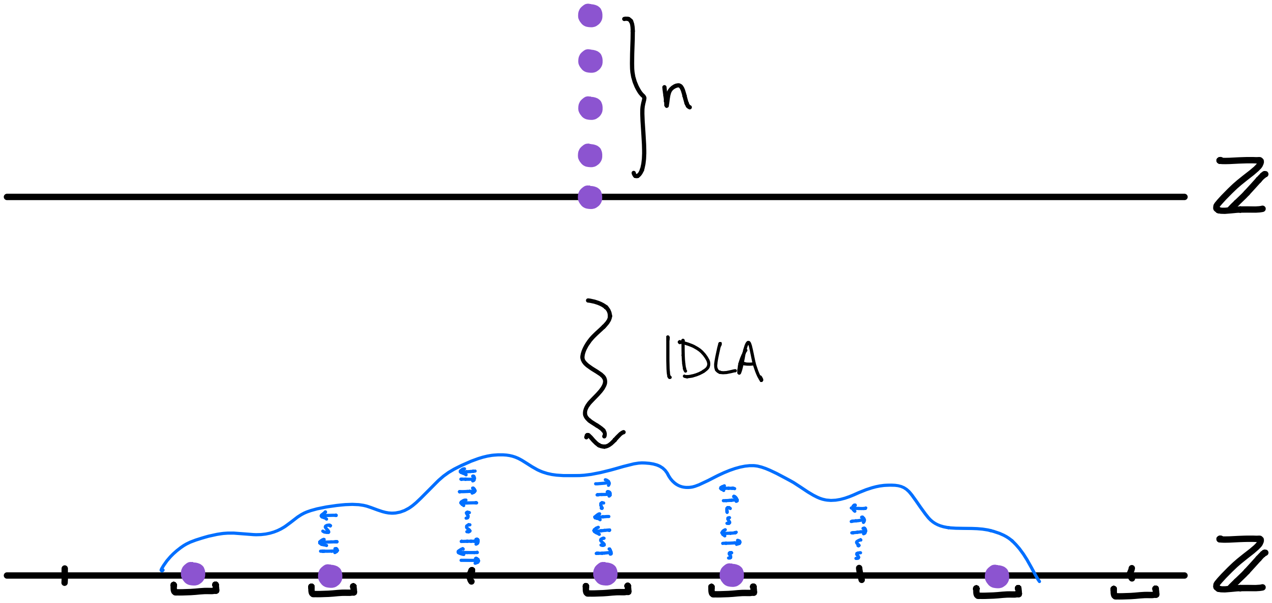

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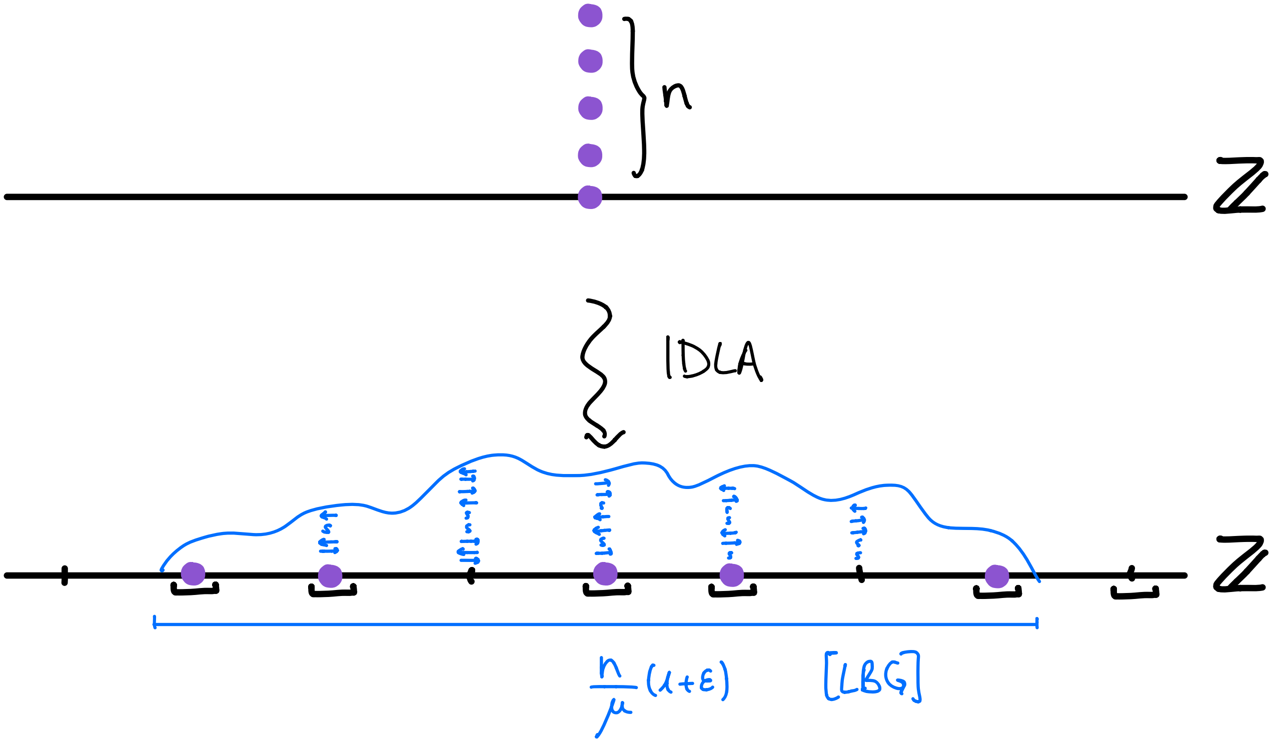

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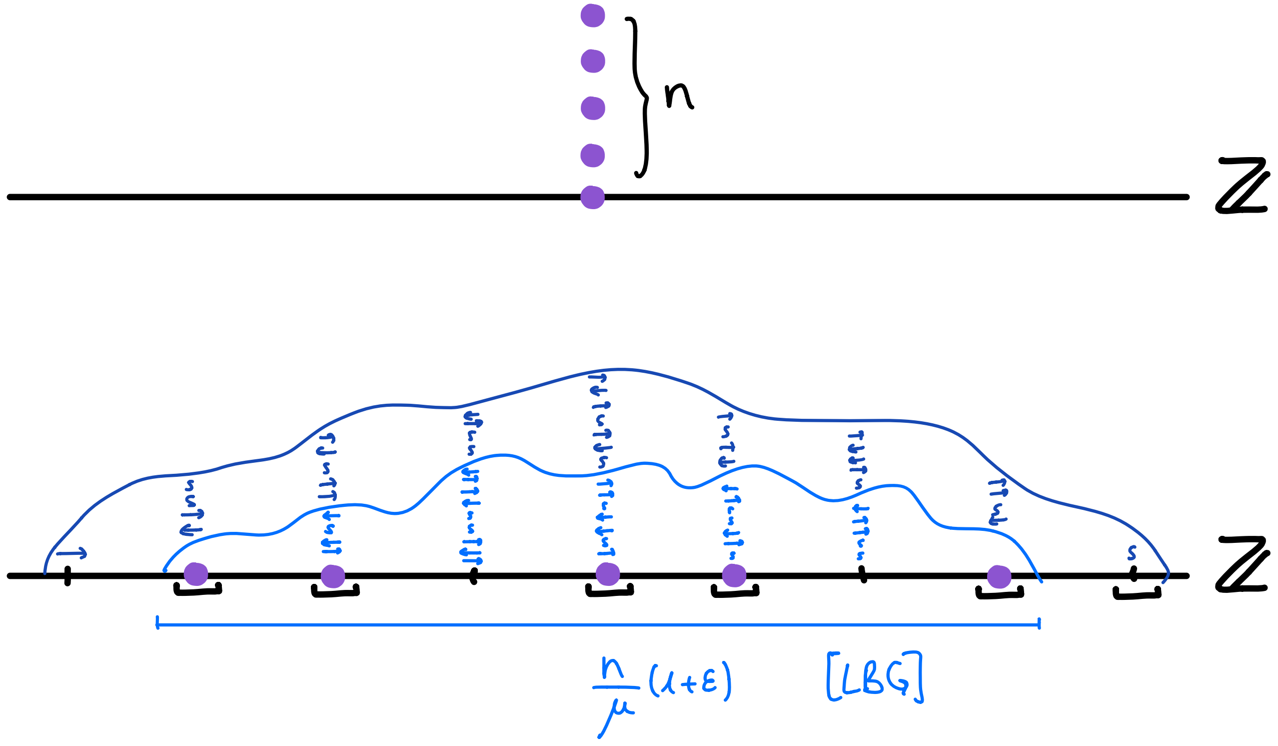

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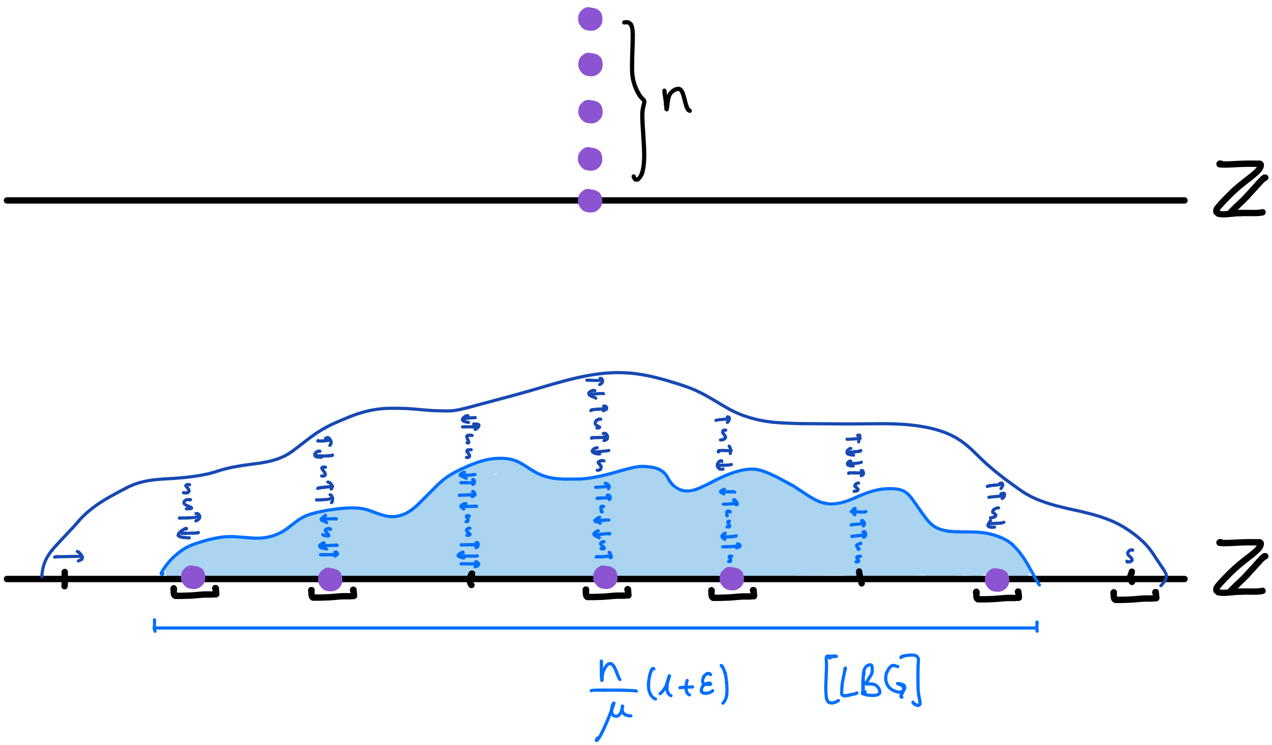

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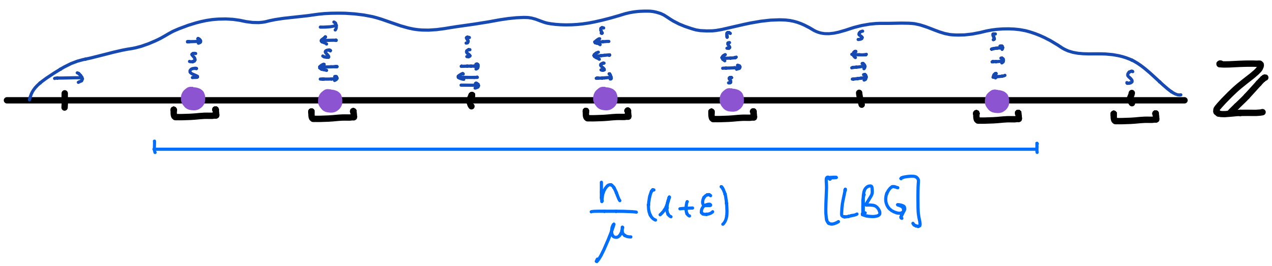

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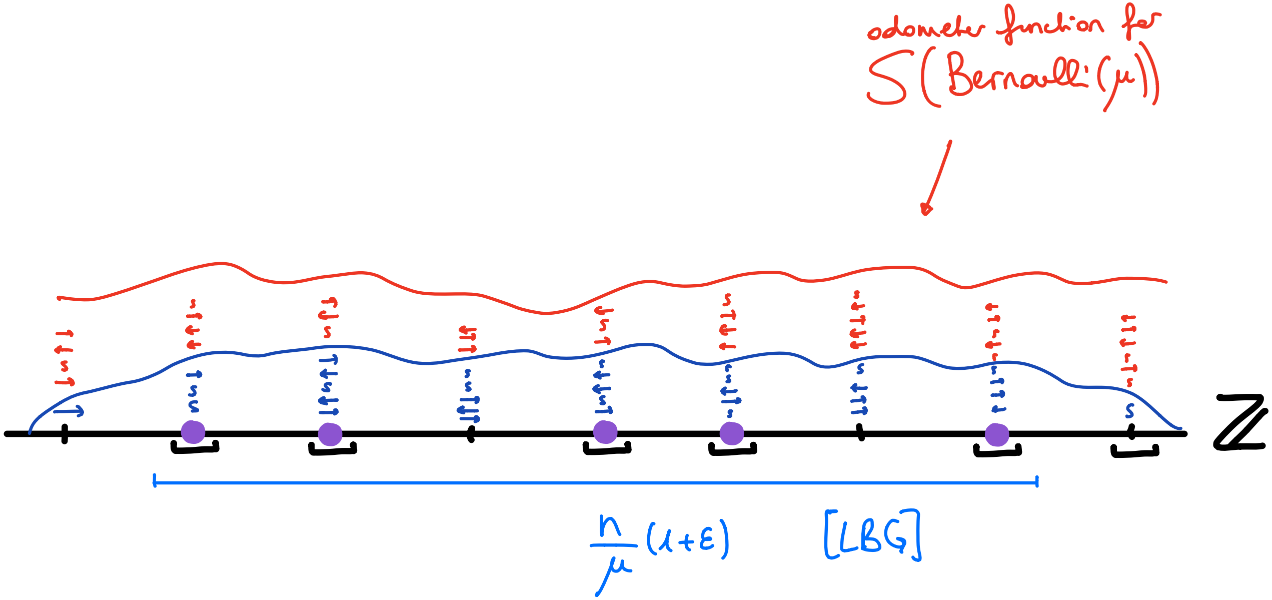

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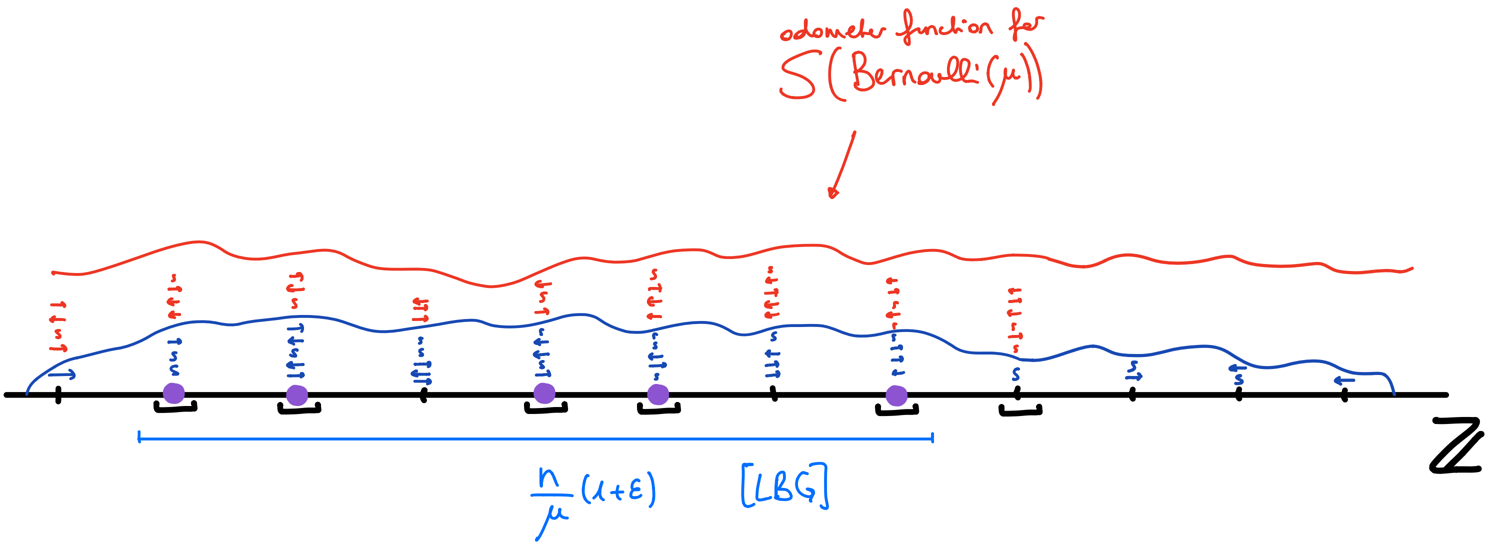

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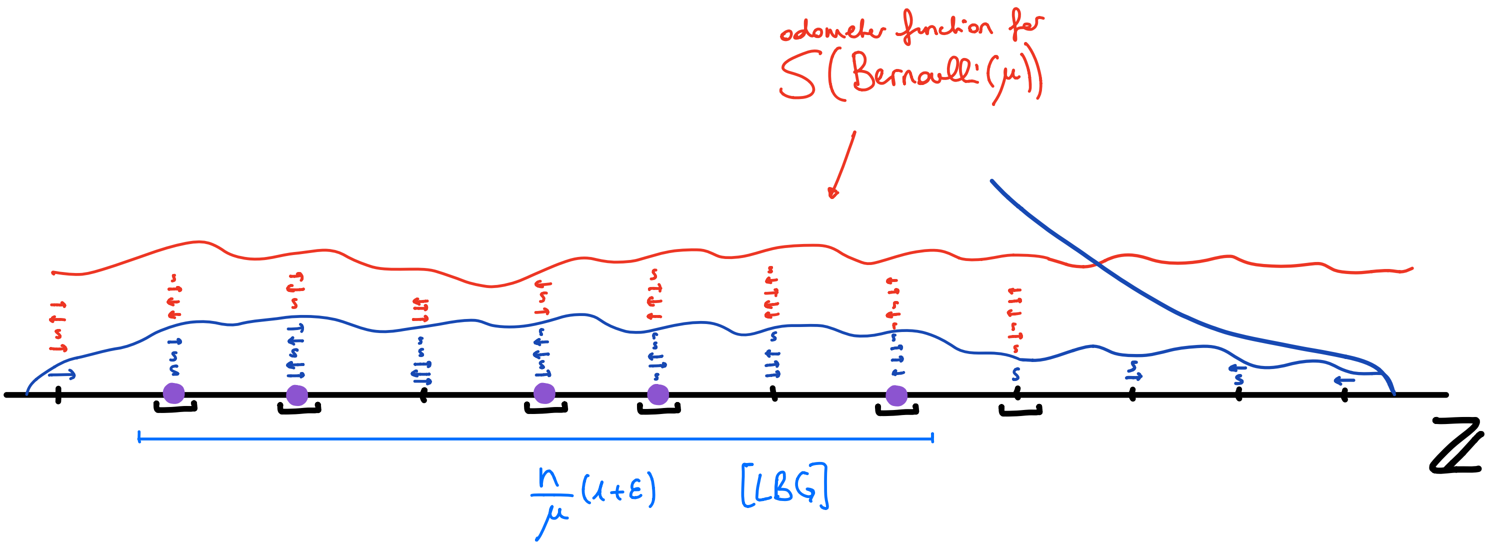

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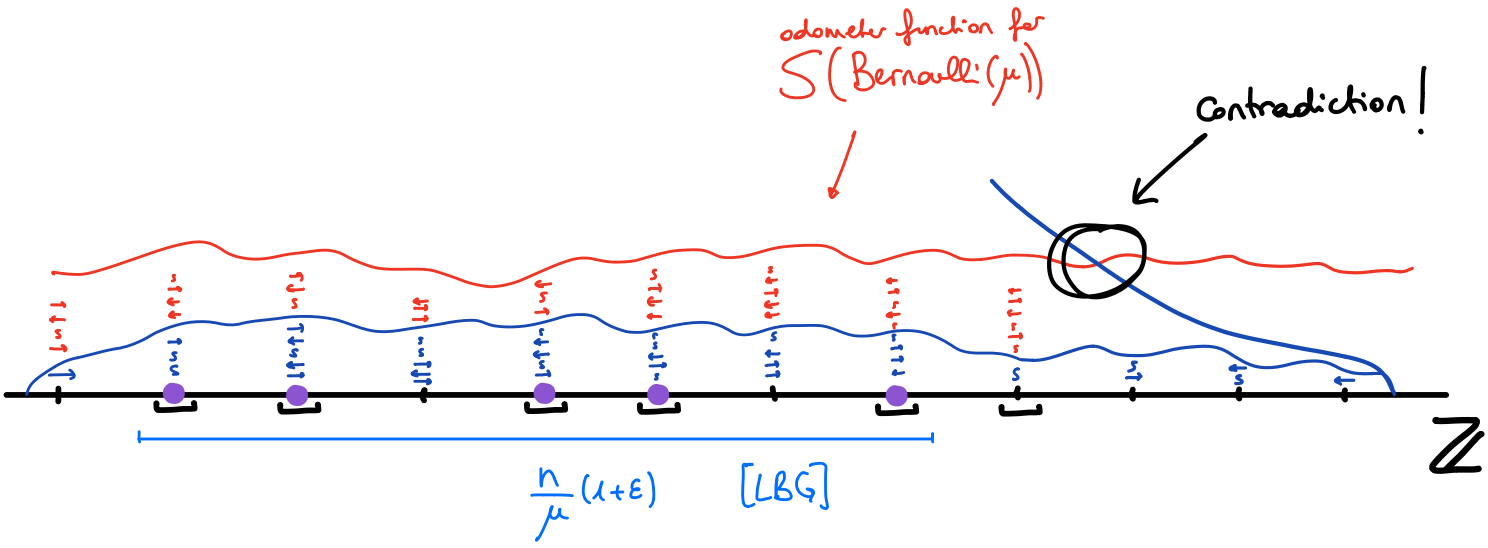

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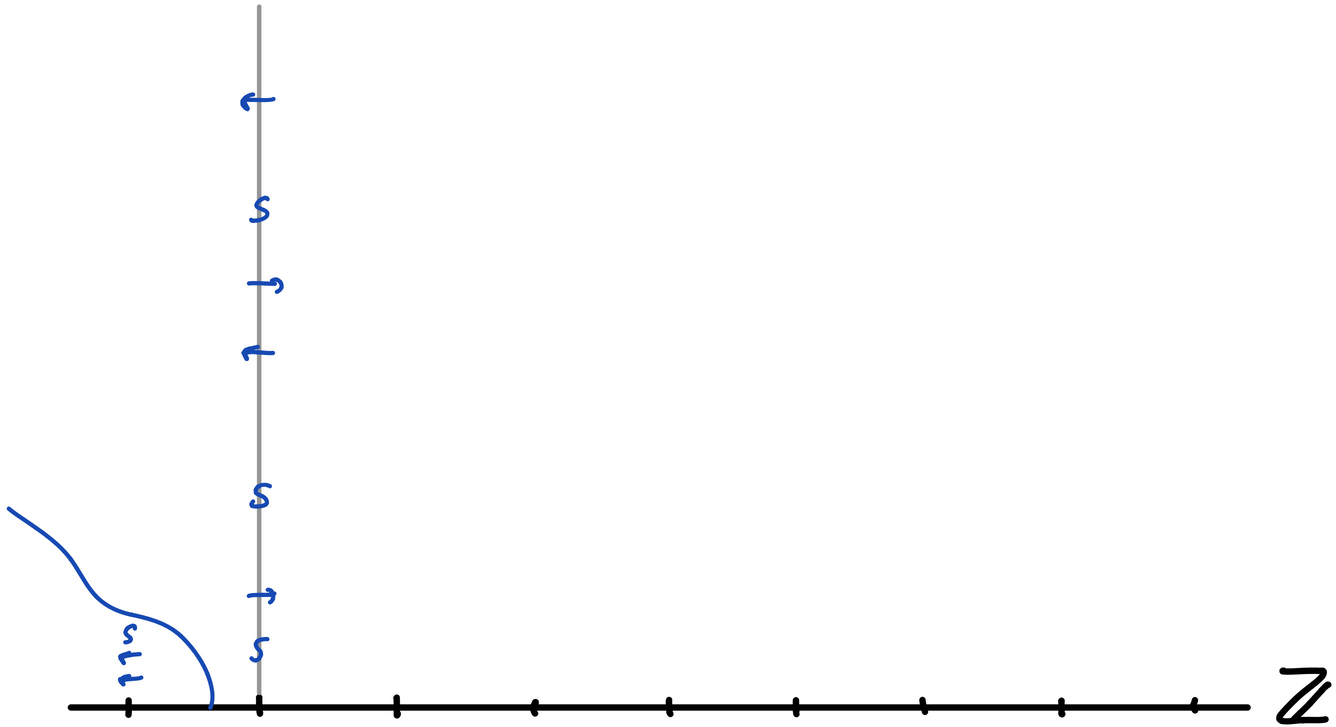

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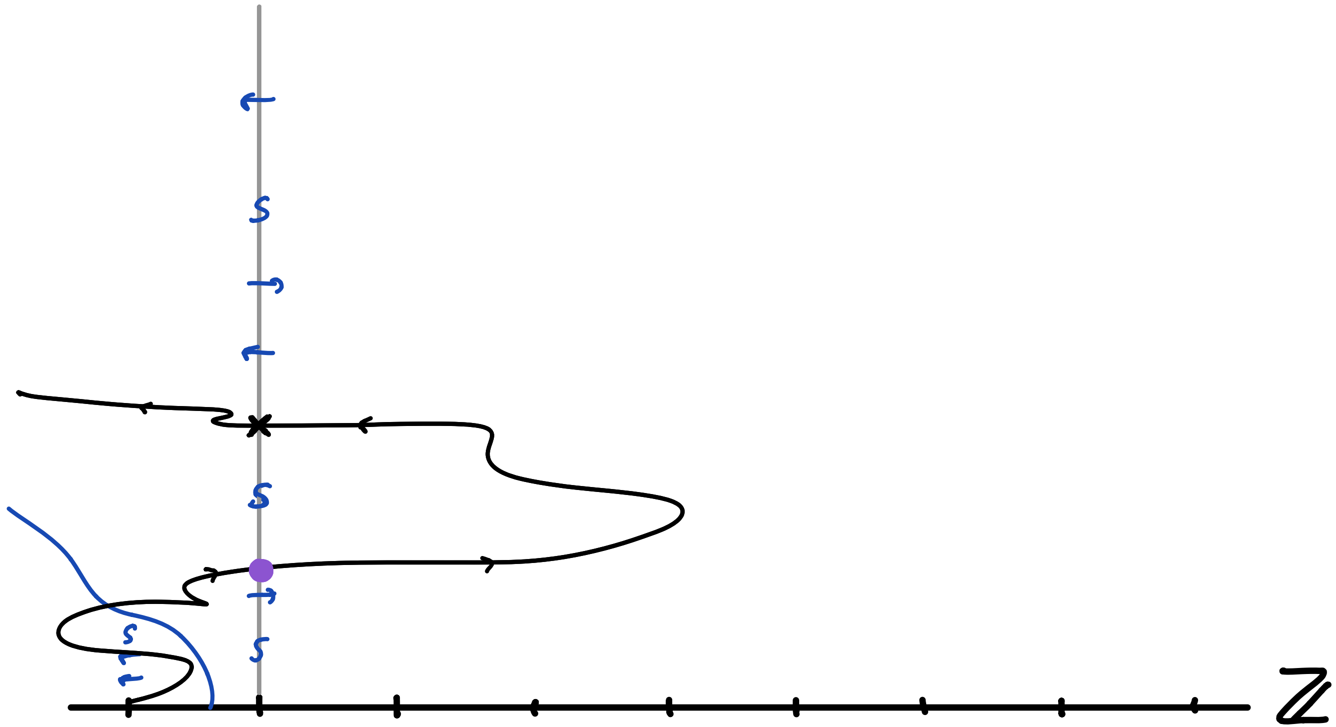

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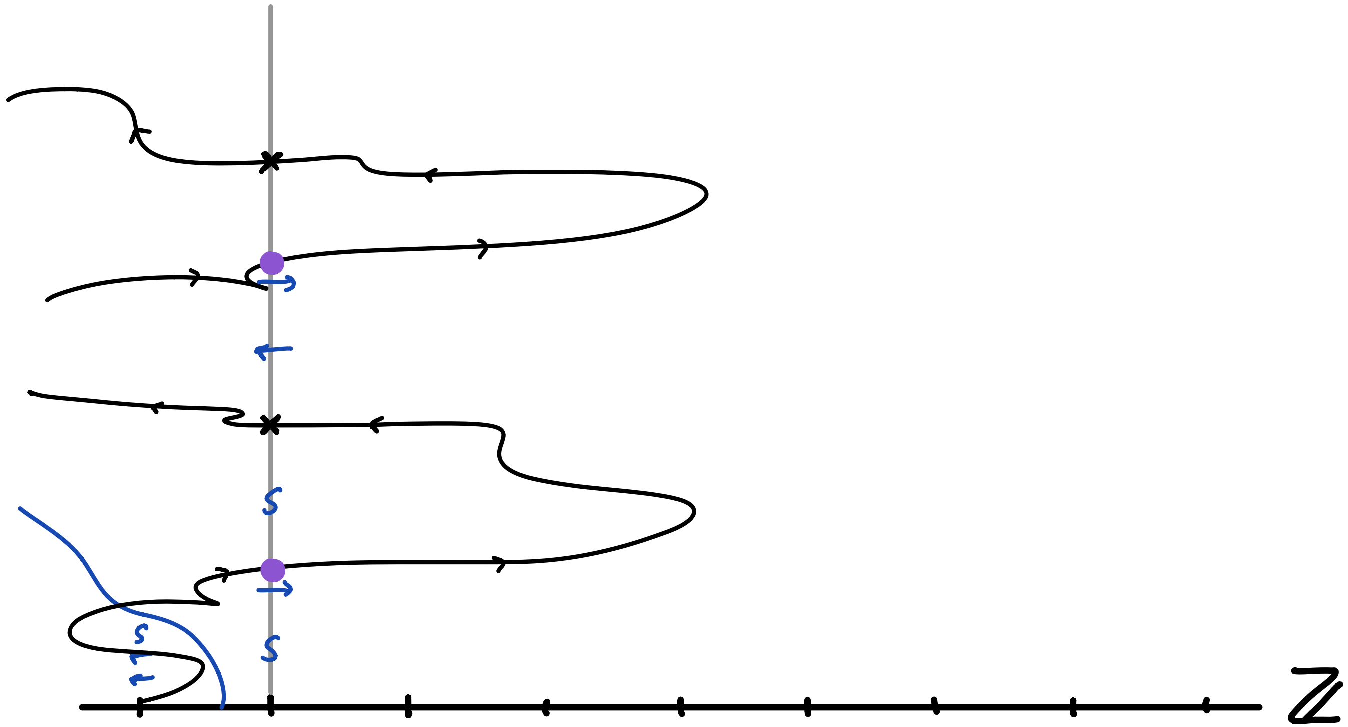

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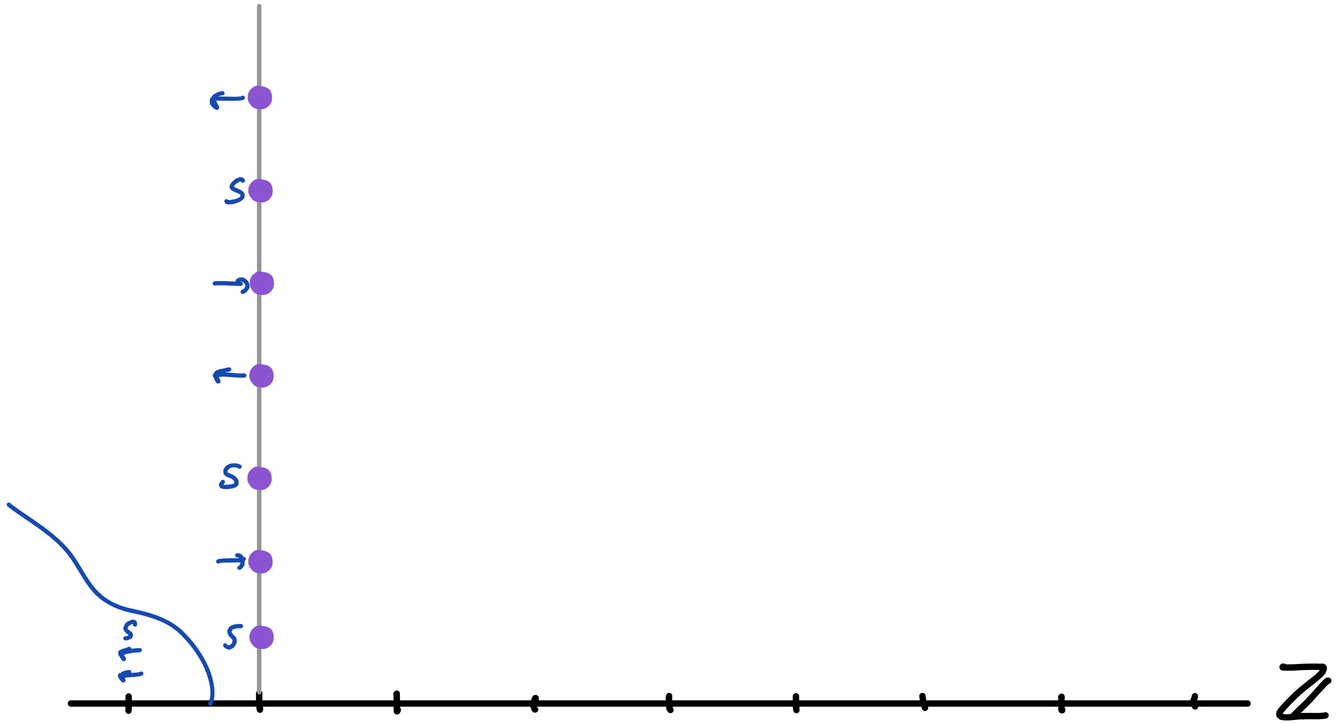

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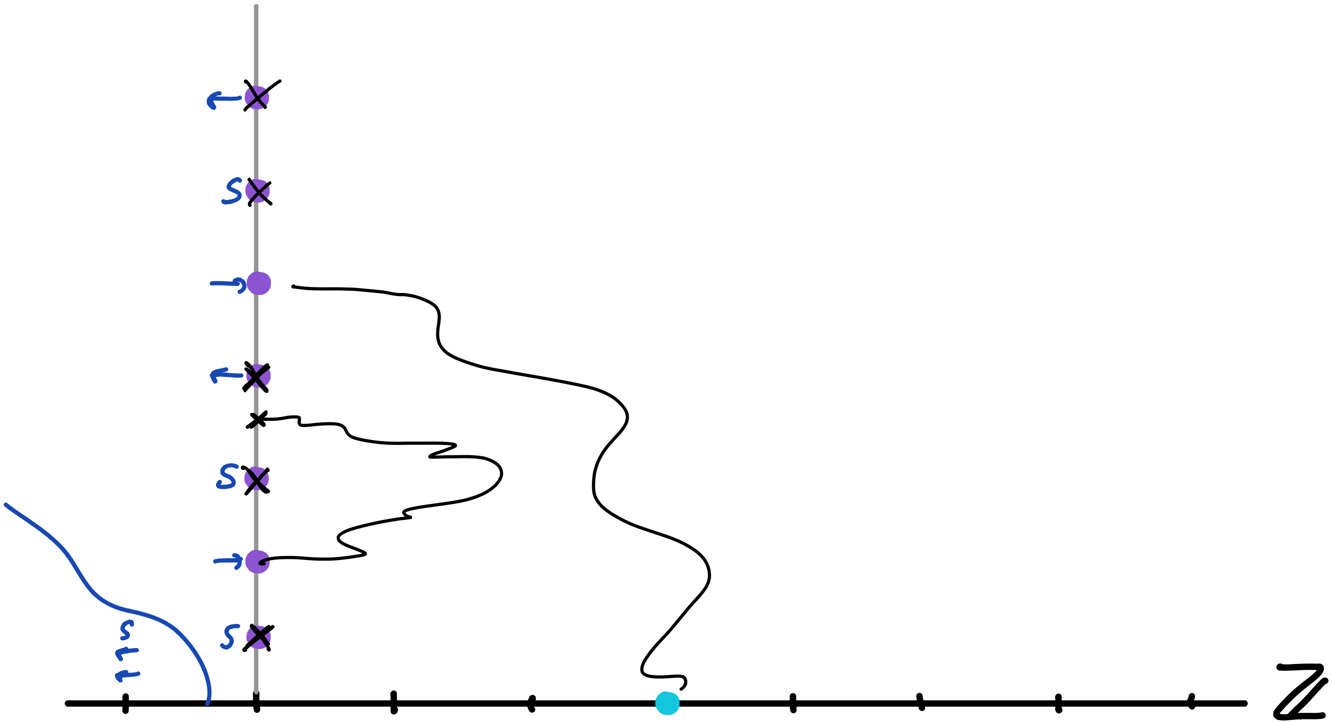

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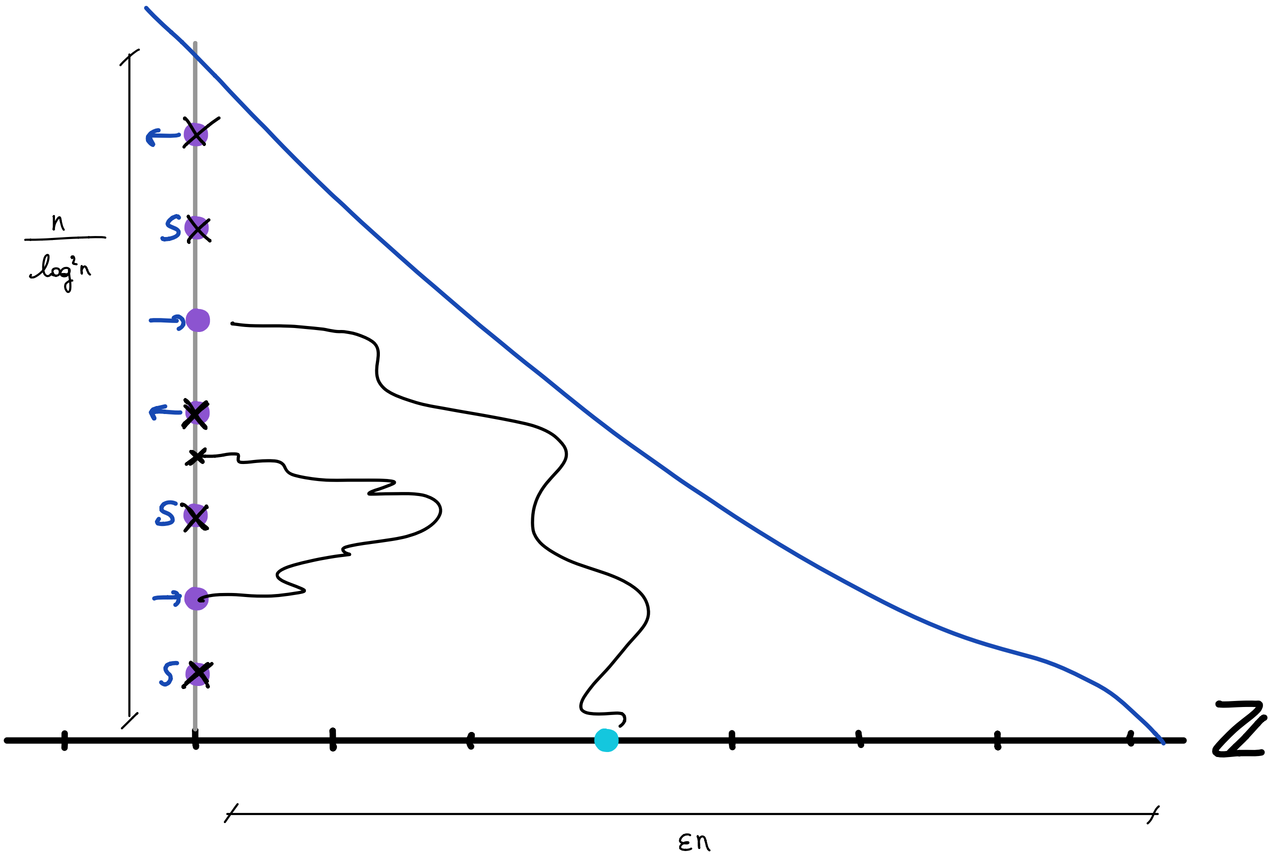
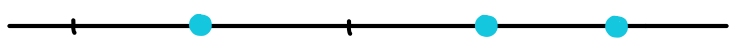


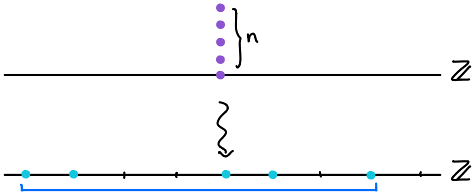
What next?

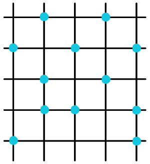
What next?

- Density conjecture: $\mu_c(\lambda) = \mu_a(\lambda) = \mu_s(\lambda)$
- Mixing of ARWs in finite volume
- Description of the critical state
- Stabilization time of ARWs at fixed density
- Sleepers in the initial state on $\mathbb{Z}^d$?

What next?







- Density conjecture: $\mu_c(\lambda) = \mu_a(\lambda) = \mu_s(\lambda)$
- Mixing of ARWs in finite volume
- Description of the critical state
- Stabilization time of ARWs at fixed density
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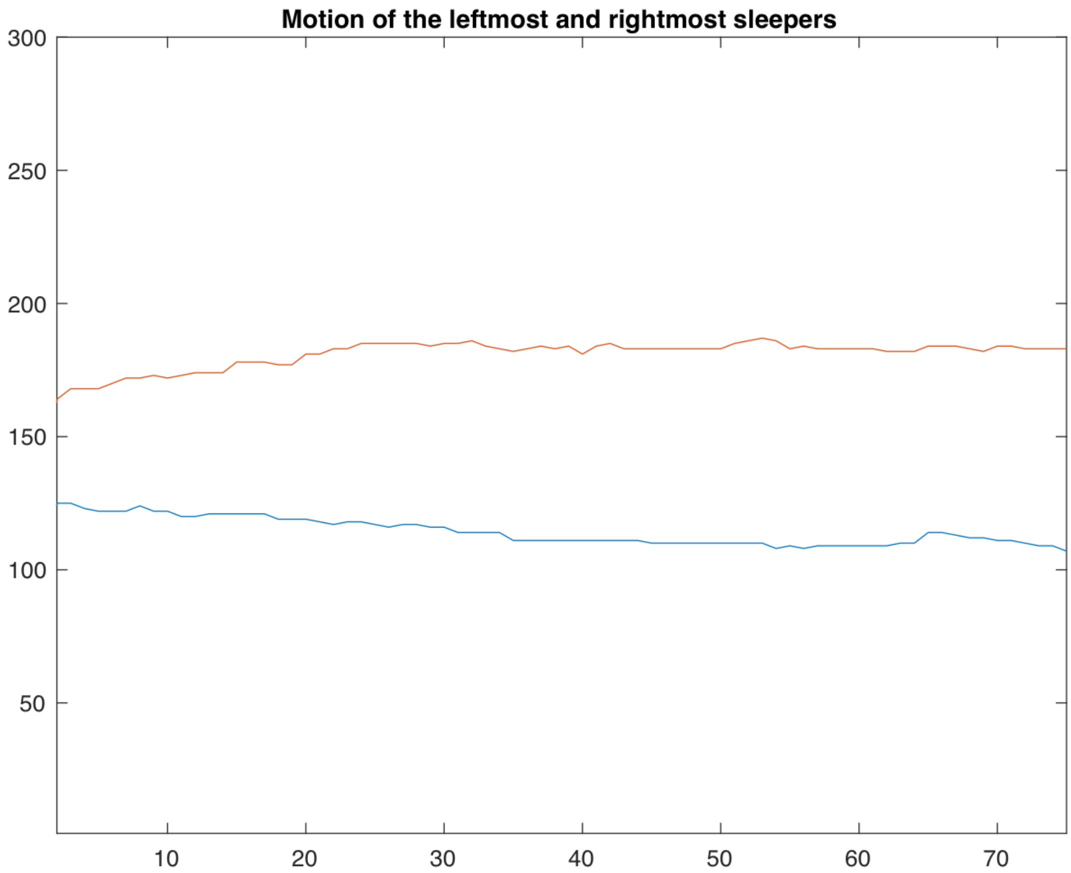
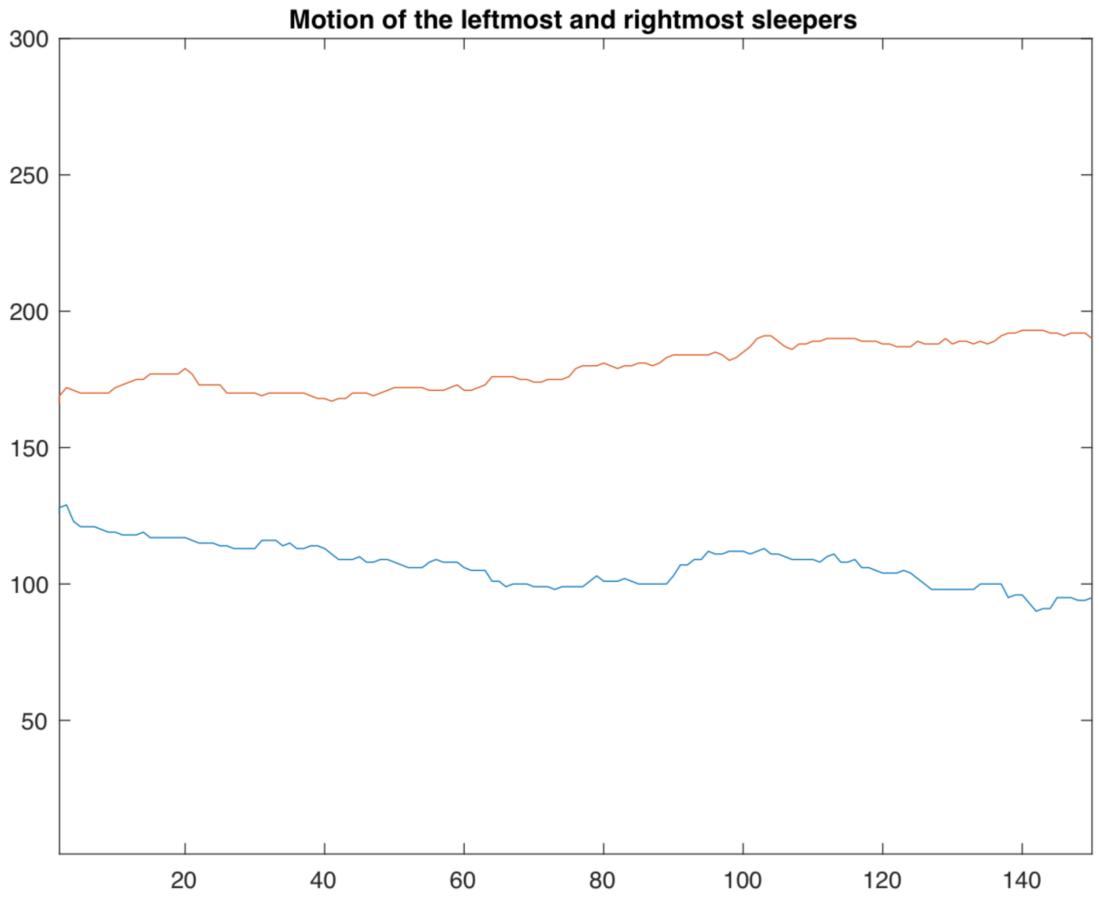
What next?

N=300
t=N/4



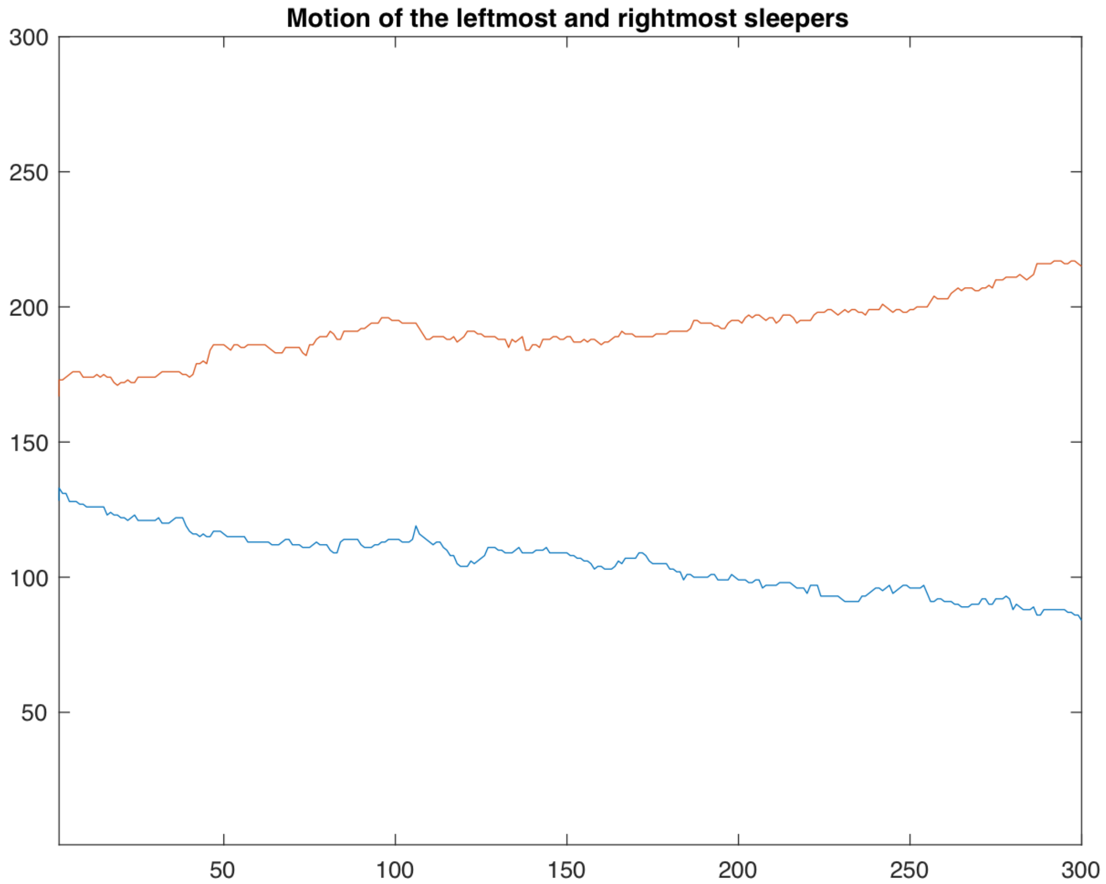
What next?

N=300
t=N/2

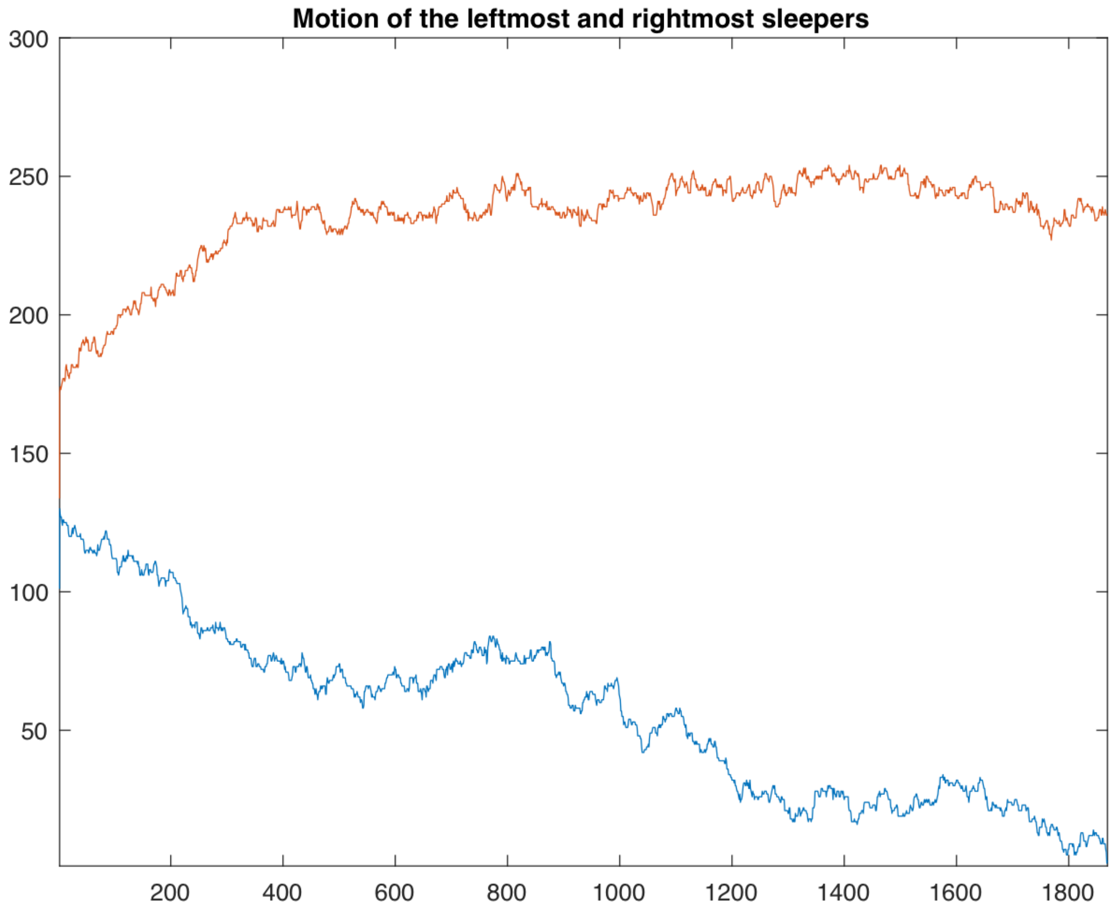


What next?

N=300
t=N



What next?



N = 300
t = 10N

What next?

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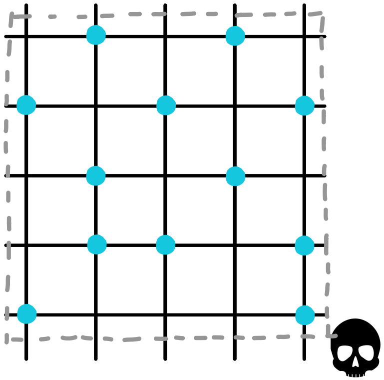

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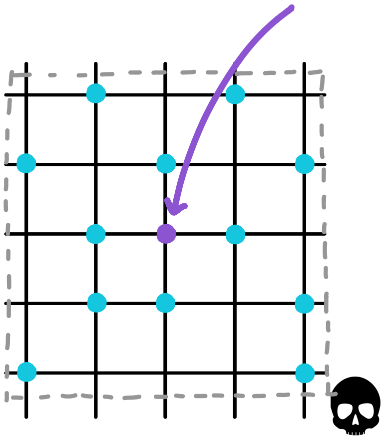

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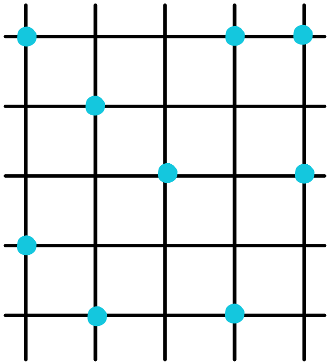

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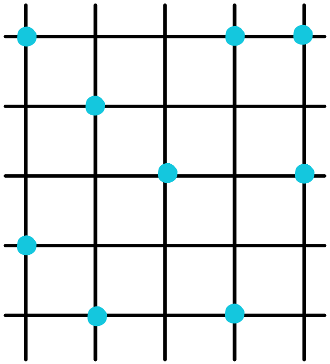

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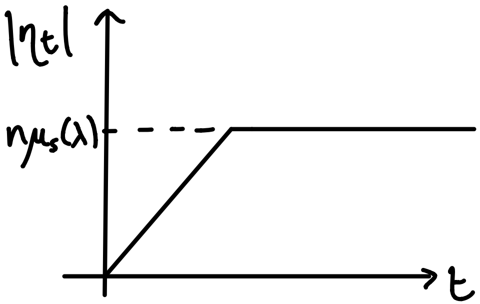
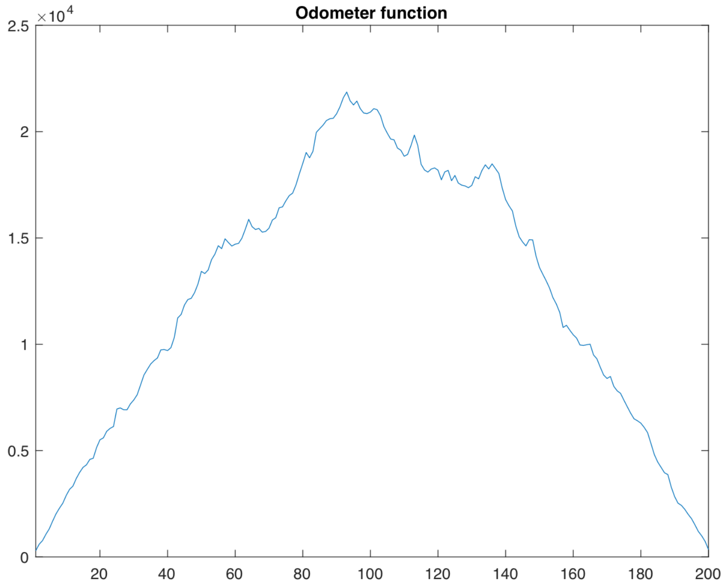
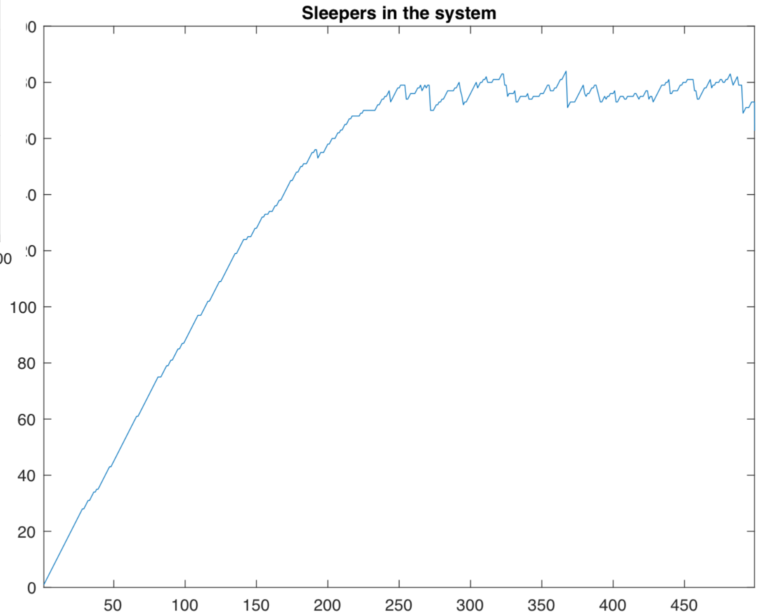


What next?



$\lambda = 0.9$
$N = 200$
$t = 500$



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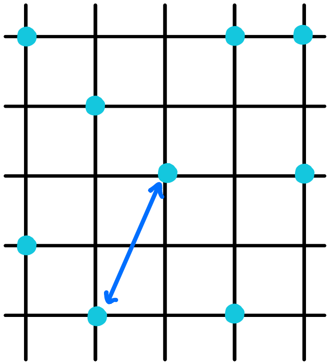

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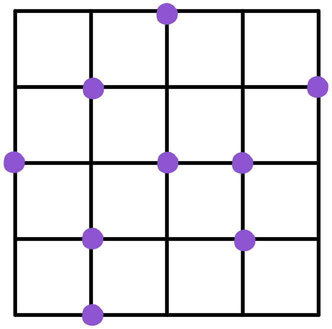

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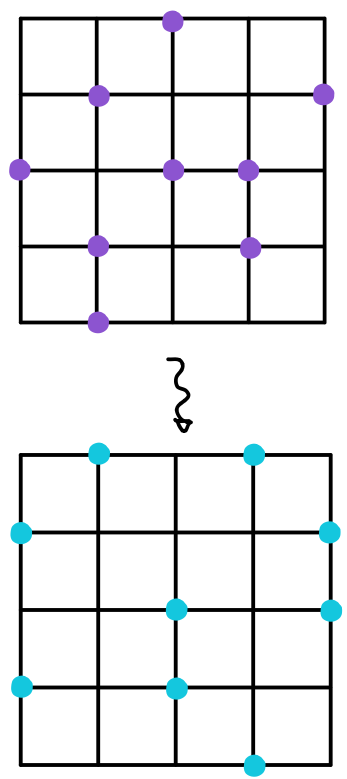

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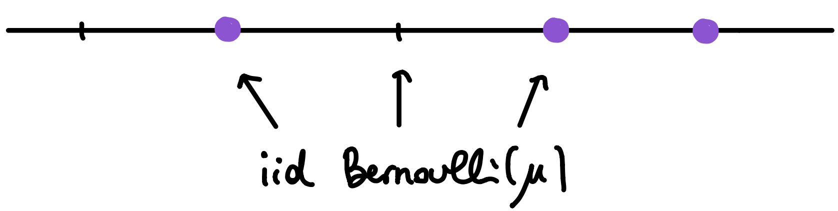

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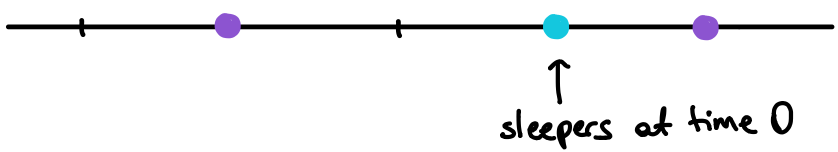

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Thank you!