A Search Game Model of Ambush vs Active Search Predation

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Predators have two modes of searching for prey:

1. Active (cruising, or moving) search

2. Ambush ('sit-and-wait') Search

Originally (Shoener, 1971) species were classified in a binary manner, more recently (Cooper, 2005) on a scale between 1. and 2. (perhaps fraction of time spent in each mode).

Mode switching usually explained in terms of changes in conditions, either external (weather, visibility, prey density) or internal (hunger level).

Our Thesis: *Possibility of ambush deters prey from moving - this makes systematic (exhaustive) search more effective. Similarly, possibility of exhaustive search encourages prey movement, rendering is vulnerable to ambush.*

michael

blomquist

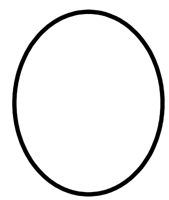
1.jpg

He tried to be calm, think rationally. The choice was to wait or to get the hell out. If the marksman was still there, the latter alternative was assuredly not a good idea. If he waited where he was the marksman would calmly walk up to the Fortress, find him and shoot him at close range.

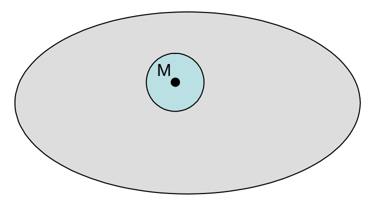
From: Girl With the Green Tatoo, Steig Larsen

Princess and Monster Games

P and M move at unit speed until the first time T that they meet, which is the payoff of a zero sum game (Monster minimizer).



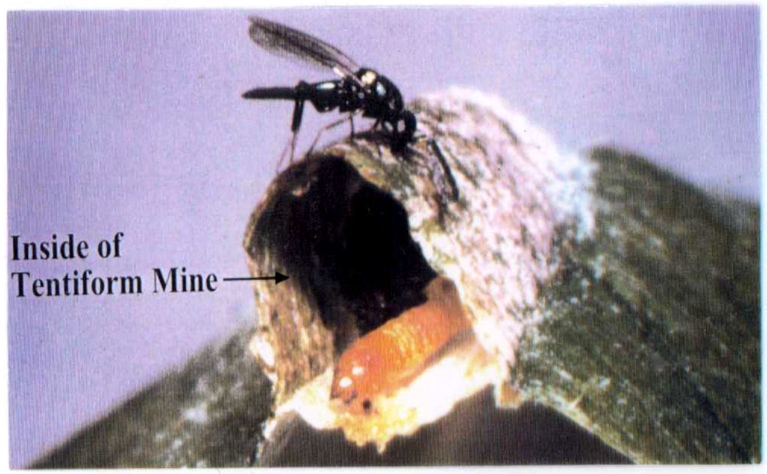
Alpern, Zelikin
‘co-ha-to’



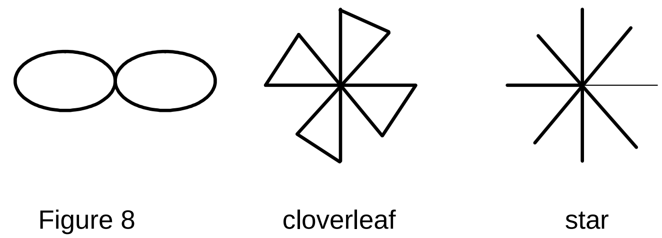
Gal, Lalley and Robbins
local oscillations for M

existence of Value follows from Alpern-Gal minimax theorem

Leaf Miner Larvae moves in
white annular area


Inside of
Tentiform Mine

Gal's 'No-Loitering' Assumption



Optimal play on figure 8 requires ambushing (loitering). Alpern-Asic 80's.

Noisy Game on n-Star

State is number of periods of rays k searched since Princess last moved.

$$V_k = Val \text{ of } \begin{array}{l|l|l|} \text{P}\backslash\text{M} & \text{Stay} & \text{Move} \\ \hline \text{Ambush} & 1 + V_k & 0 \\ \hline \text{Search} & \left(\frac{1}{n-k}\right)\frac{1}{2} + \left(1 - \frac{1}{n-k}\right)(1 + V_{k+1}) & 1 + V_0 \\ \hline \end{array}$$

Note that this cannot be solved recursively due to V_k on both sides.

Our Game Γ

Search region X has size (measure) 1. T capture time payoff, $\hat{T}$ its expected value. Searcher strategy $s(t)$ is fraction of X searched at time t after last (randomizing) move of Hider. So $s'(t) = 0$ when ambushing and $s'(t) = 1$ when searching. Interpret intermediate $s'(t)$ as fraction of time searching. Or simply speed. Thus $s(0) = 0$, $s(\tau) = 1$. Pure strategy of Hider is time interval m between moves.

- For $t \leq m$, $\Pr(T \leq t) = s(t)$.

- If $T \geq m$ Hider caught at time m with probability $1 - s'(m)$; reaches new random position with probability $s'(m)$

Main Equation for Expected Capture Time $\hat{T}$

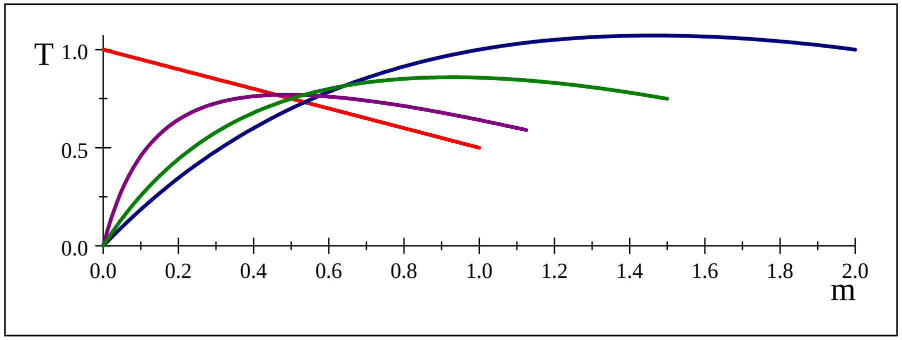
$\hat{T} = \hat{T}(s, m)$ is given by

$$\hat{T} = \int_0^m \overbrace{t\ s'(t)}^{T<m}\ dt + (1 - \overbrace{s(m)}^{T \geq m}) \left(\Big(1 - \overbrace{s'(m)}^{T=m}\Big) m + \overbrace{s'(m)}^{T>m} \left(m + \hat{T}\right) \right)$$

Solution for steady-state predators

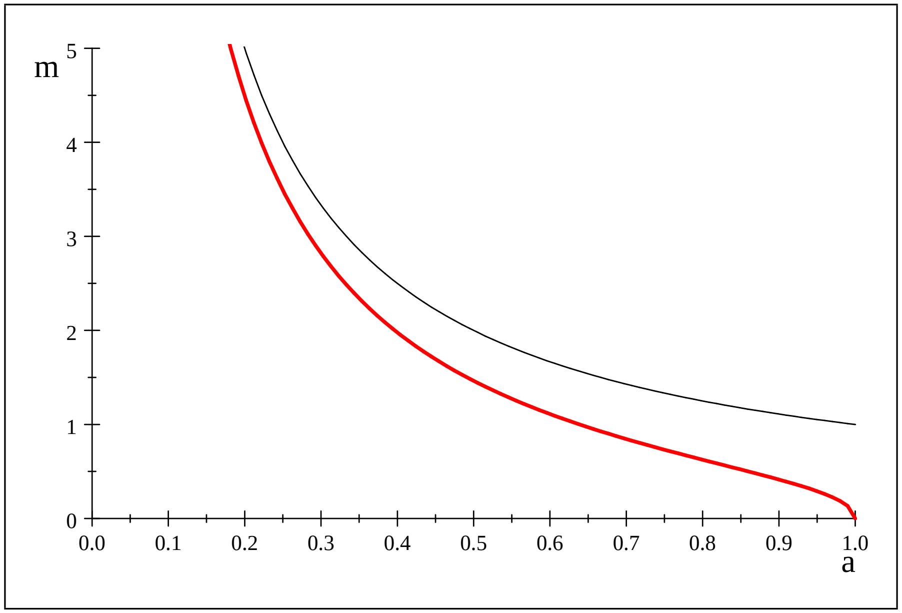
$$s(t) = at, \text{ with maximum search time } \tau = 1/a.$$

$$T(a,m) = \frac{2m - am^2}{-2a + 2a^2 m + 2}, \text{ depends on } m \text{ for any } a$$



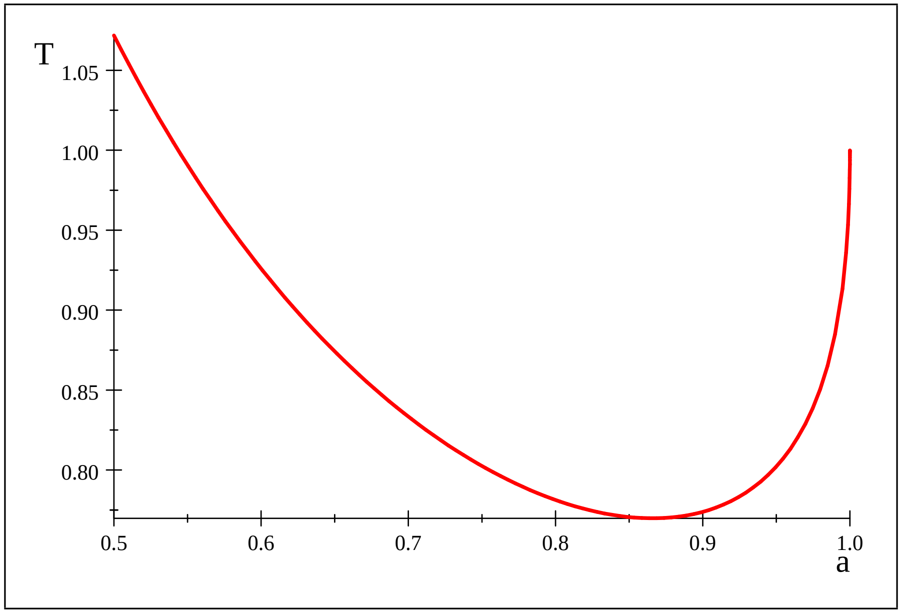
$T(a,m)$, $a = 1$ (red), .866 (purple) 1/2 (blue), 1/3 (green)

How Long to Wait against steady-state Predator, speed a



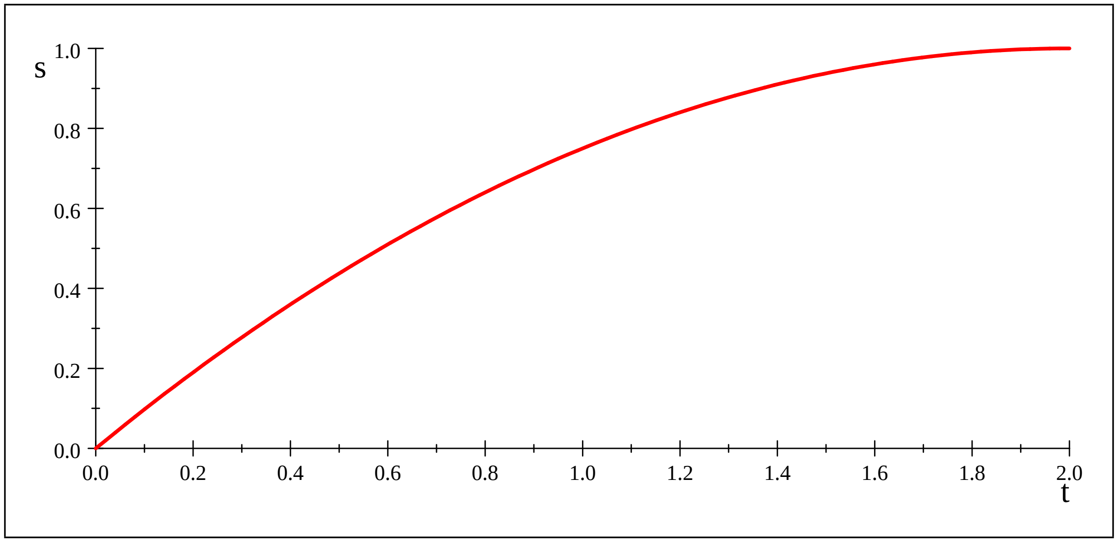
maximizing $\bar{m}\left(a\right) = \frac{1}{a^2}\left(a + \sqrt{-a^2+1} - 1\right)$ (red), $1/a$ (black)

Optimal Steady State Predator speed $\bar{a} = \frac{\sqrt{3}}{2} \simeq 0.866$



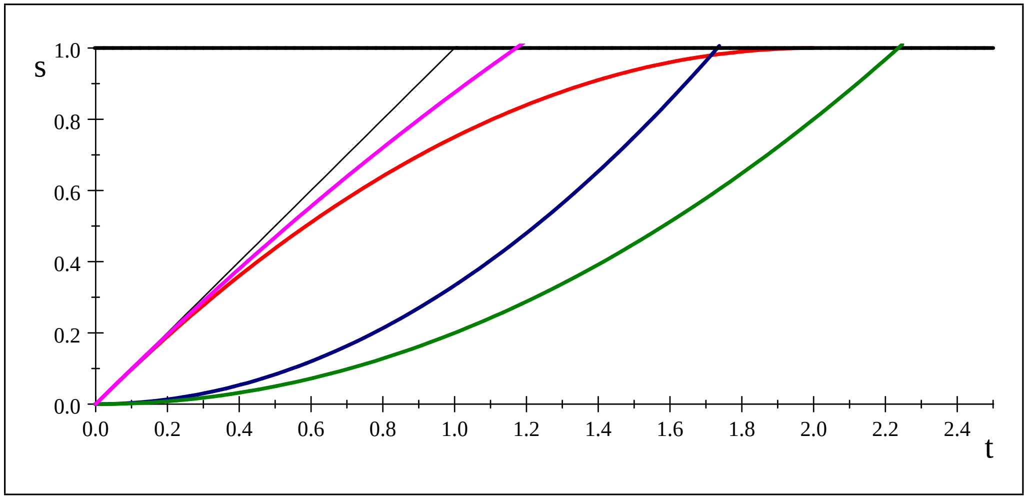
$\bar{T}(a)$, min at $a = .86$ of .77

Time Varying Solution: a quadratic function



Optimal fraction $\bar{s}\,(t) = \frac{-1}{4} t^2 + t$ of space searched by time t. $\hat{T}\,(\bar{s}, m) = 2/3$, for any m.

Start mainly searching ($s' = 1$), end by mainly ambushing ($s' = 0$).



ren curve $\bar{s}$, minimax

- against purple: move right away

- against blue: move when same slope as $\bar{s}$ (time 1)

- against green: don't move

Take fraction searched s as state variable

$$s' = \frac{ds}{dt} = \sqrt{1-s}.$$

Square Root Law of Predation Search

The optimal probability of searching (as opposed to ambushing) equals the square root of the fraction of the search region in which the prey is known to be located.

Prey Strategy

At any time when a fraction s of the region has been searched, move in a small time interval of length Δt with probability Δt times

$$\frac{3}{2\sqrt{1-s}}.$$