Evolutionary stability & changing fitnesses

Frank Thuijsman, Philippe Uyttendaele

Avi Shmida, Gijs Schoenmakers, Ronald Westra

Biology and Game Theory, Paris, UPMC, November 4-6, 2010

Maastricht University
Department of Knowledge Engineering

Outline

1. Introduction
2. A Seasonal Model
3. Stochastic Model
4. Concluding Remarks

Evolutionary Development

Many evolutionary game theoretic models are usually based on the replicator equation:

$$\dot{x}_i = x_i \left(e_i A x - x A x \right)$$

where a population changes in view of a fitness matrix A.

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The population is assumed to be completely mixed;
what happens if it is not?

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- A seasonal model: the fitness matrix changes periodically as a function of time

Changing Fitnesses

Two approaches:

- A seasonal model: the fitness matrix changes periodically as a function of time
- A stochastic model: stochastic transitions among different fitness matrices as a function of the population distribution

Periodic Fitnesses

We examine what happens when in

$$\dot{x}_i = x_i\,(e_i Ax - xAx)$$

we use

$$A = \begin{pmatrix} 0 & 0 & 2 + \sigma cos(\rho t) \\ \alpha & 0 & \alpha \\ 2 - \sigma cos(\rho t) & 0 & 0 \end{pmatrix}$$

Population Development in the Seasonal Model

(Trajectory)

The Limit Cycle

(Rest cycle)

The Parametric Influence

$$\dot{x}_i = x_i \left(e_i A x - x A x \right)$$

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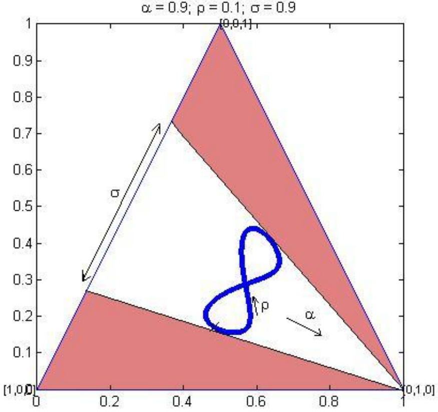

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A resource dilemma where a cake of a stochastic size has to be distributed among a number of persons.

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A resource dilemma where a cake of a stochastic size has to be distributed among a number of persons.
Sequentially, each person can claim a piece of any size.
If the sum of these pieces is no bigger than the whole cake, then everyone gets what he or she claims.
If the sum of the pieces is bigger, then no one gets anything.

A Two-Player Version

Consider a cake of size 4
where each of two players can claim to get
1 (modest), 2 (fair) or 3 (greedy),

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where each of two players can claim to get
1 (modest), 2 (fair) or 3 (greedy),
then the payoff matrix would look like

$$\begin{array}{c} \\ m \\ f \\ g \end{array} \begin{array}{c} \begin{array}{ccc} m & f & g \end{array} \\ \begin{pmatrix} 1,1 & 1,2 & 1,3 \\ 2,1 & 2,2 & 0,0 \\ 3,1 & 0,0 & 0,0 \end{pmatrix} \end{array}$$

A Population Version with 3 Cakes

$$\begin{array}{c} \\ m \\ f \\ g \end{array}\begin{array}{c} \begin{array}{ccc} m & f & g \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

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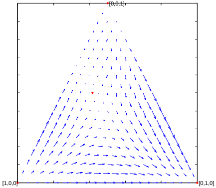

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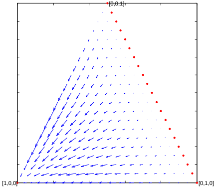

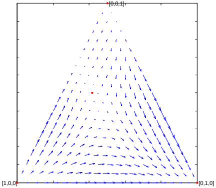

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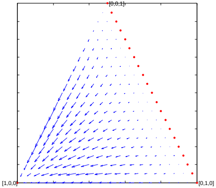

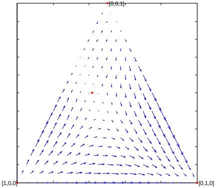

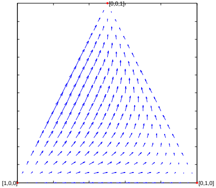

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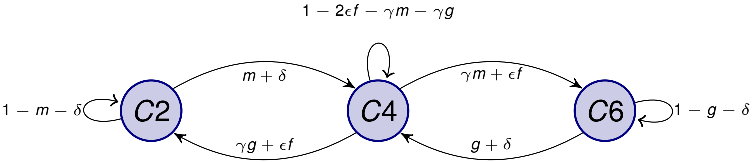
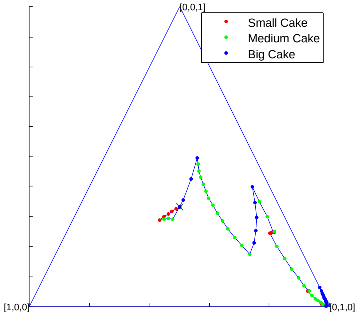


Population Development in the Stochastic Model



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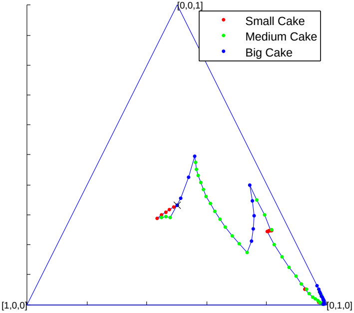

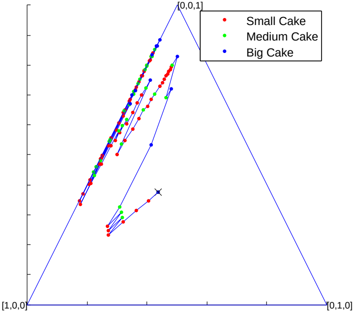

Another Population Version with 3 Cakes

Using converse proportionality among players we get:

$$\begin{array}{c} \\ m \\ f \\ g \end{array}\begin{array}{c} \begin{array}{ccc} m & f & g \end{array} \\ \begin{pmatrix} 1 & 1.3 & 1.5 \\ 0.7 & 1 & 1.2 \\ 0.5 & 0.8 & 1 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ m \\ f \\ g \end{array}\begin{array}{c} \begin{array}{ccc} m & f & g \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2.4 \\ 3 & 1.6 & 2 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ m \\ f \\ g \end{array}\begin{array}{c} \begin{array}{ccc} m & f & g \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix} \end{array}$$

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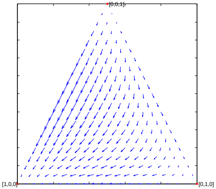

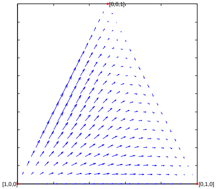

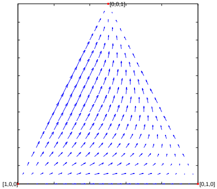

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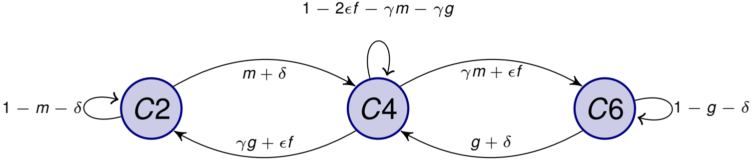

Population Development in the Stochastic Model

(Rest cycle)

Stability of Trajectories

Questions for the audience:

- How to derive theoretically the stability of a ‘limit trajectory’?

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- How to define stability in a changing environment?

Other 'Evolutionary' Work in Maastricht

- Examining local competition using replicator dynamics

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- Examining local competition using replicator dynamics
- Examining local competition with local fitness matrices

Other 'Evolutionary' Work in Maastricht

- Examining local competition using replicator dynamics
- Examining local competition with local fitness matrices
- Studying wasp ovipositioning behavior by means of an evolutionary approach

Thanks

Thank you for your attention!
Any comments are welcome!

This presentation will be available at
www.personeel.unimaas.nl/F-Thuijsman