2. For any $s \in \mathbb{R}^{2k+1}$,

$$\nabla f(s) = \frac{L}{4} \begin{pmatrix} 2s_1 - s_2 - 1 \\ 2s_2 - s_1 - s_3 \\ \vdots \\ 2s_{2k} - s_{2k-1} - s_{2k+1} \\ 2s_{2k+1} - s_{2k} \end{pmatrix}.$$

For any $s, s' \in \mathbb{R}^{2k+1}$,

$$\begin{aligned} \nabla f(s) - \nabla f(s') &= \frac{L}{4} \begin{pmatrix} 2(s_1 - s_1') - (s_2 - s_2') \\ 2(s_2 - s_2') - (s_1 - s_1') - (s_3 - s_3') \\ \vdots \\ 2(s_{2k} - s_{2k}') - (s_{2k-1} - s_{2k-1}') - (s_{2k+1} - s_{2k+1}') \\ 2(s_{2k+1} - s_{2k+1}') - (s_{2k} - s_{2k}') \end{pmatrix} \\ &= \frac{L}{4} \left(2(s - s') - D_1(s - s') - D_{-1}(s - s') \right), \end{aligned}$$

where we define, for any $x \in \mathbb{R}^{2k+1}$,

$$D_1 x = \begin{pmatrix} x_2 \\ x_3 \\ \vdots \\ x_{2k+1} \\ 0 \end{pmatrix}, \quad D_{-1} x = \begin{pmatrix} 0 \\ x_1 \\ x_2 \\ \vdots \\ x_{2k} \end{pmatrix}.$$

We observe that, for any $x \in \mathbb{R}^{2k+1}$, $||D_1(x)||_2 \leq ||x||_2$ and $||D_{-1}(x)||_2 \leq ||x||_2$. As a consequence, for any $s, s' \in \mathbb{R}^{2k+1}$,

$$\begin{aligned} ||\nabla f(s) - \nabla f(s')|| &= \frac{L}{4} ||2(s - s') - D_1(s - s') - D_{-1}(s - s')||_2 \\ &\leq \frac{L}{4} \left(2||s - s'||_2 + ||D_1(s - s')||_2 + ||D_{-1}(s - s')||_2 \right) \\ &\leq \frac{L}{4} \left(2||s - s'||_2 + ||s - s'||_2 + ||s - s'||_2 \right) \\ &= L||s - s'||_2, \end{aligned}$$

which exactly says that ∇f is L-Lipschitz.