Université Paris-Dauphine — Année 2021-2022
L3 - Statistical modelling

QCM 1

In this quizz, you have to check the correct answer(s) to the question. If none fits, do not check any box.

Question 1 Let X_i $(i = 1, 2, \dots)$ be iid random variables taking values in $\mathbb{N}^*$, such that

$$P(X = k) = Ck^{-3}, \quad \forall k \geq 1,$$

with $C > 0$ a constant and define

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i.$$

Which of the following statements are true ?

- [x] $\bar{X}_n$ converges in probability to a constant $\mu \in \mathbb{R}$.
- [ ] There exist $\mu \in \mathbb{R}$ and $\sigma^2 > 0$ such that $\sqrt{n}(\bar{X}_n - \mu)/\sigma$ converges in distribution to a standard normal distribution.
- [ ] $\bar{X}_n$ converges in $\mathbb{L}^2$ to a constant $\mu \in \mathbb{R}$.
- [x] $\bar{X}_n$ converges a.s. to a constant $\mu \in \mathbb{R}$.

Question 2 Let X be a random variable with density

$$f(x) = \frac{3}{2} x^{-5/2} \mathbb{I}_{(1,\infty)}(x)$$

The expectation $\mathbb{E}[X^k]$ only exists when

- [ ] $k \leq 2$
- [x] $k \leq 0$
- [x] $k < 3/2$
- [ ] $k < 5/2$
- [x] $k < -5/2$
- [x] $k \leq 1$

Question 3 Let X (resp. Y) be a random variable distributed from the geometric distribution with parameter p_1 (resp. p_2). We suppose that X and Y are independent. The distribution of the random variable $Z = \min\{X, Y\}$ is

- [x] the geometric distribution with parameter $p_1 + p_2 - p_1 p_2$.
- [ ] the geometric distribution with parameter $(1 - p_1)(1 - p_2)$?
- [ ] the exponential distribution with parameter $\frac{\min\{p_1, p_2\}}{p_1 + p_2}$?
- [ ] the geometric distribution with parameter $p_1 + p_2$.

Question 4
Let X_1 and X_2 be two independent random variables following an exponential distribution such that $\mathbb{E}[X_1] = \frac{1}{\lambda_1}$ and $\mathbb{E}[X_2] = \frac{1}{\lambda_2}$ with $\lambda_1, \lambda_2 > 0$. We recall that the p.d.f of an exponentially-distributed random variable with parameter λ is $f(x) = \lambda e^{-\lambda x}$.
The distribution of $\min(X_1, X_2)$ is

- ☐ The exponential distribution with parameter $\frac{\lambda_1}{\lambda_2}$
- ■ The exponential distribution with parameter $\lambda_1 + \lambda_2$
- ☐ The normal distribution $\mathcal{N}(\frac{1}{\lambda_1\lambda_2}, (\lambda_1\lambda_2)^2)$
- ☐ The uniform distribution on the interval $[\min(\lambda_1, \lambda_2), \max(\lambda_1, \lambda_2)]$

Question 5 Let $X_1, X_2, \ldots$ be real-valued, iid random variables with mean $\mu \neq 0$ and finite variance $\sigma^2 > 0$. Let $\bar{X} = (1/n)\sum_{i=1}^n X_i$ denote the empirical mean. Then $(\bar{X})^2$ converges in distribution to a random variable whose distribution is

- ☐ $\mathcal{N}(\mu^4, 16\mu^2\sigma^4)$
- ☐ $\mathcal{N}(\mu, \sigma^2)$
- ■ None of the above
- ☐ $\mathcal{N}(\mu^2, \sigma^4)$
- ☐ $\mathcal{N}(\mu^2, 4\mu^2\sigma^2)$

Question 6 Let X follow an exponential distribution with parameter $\lambda > 0$ (i.e. $\mathbb{E}(X) = \frac{1}{\lambda}$). Then $\mathbb{E}(X^2)$ is

- ☐ $\frac{1}{\lambda^4}$
- ☐ $\frac{1-\lambda}{\lambda^2}$
- ☐ $\frac{1}{\lambda^2}$
- ■ $\frac{2}{\lambda^2}$

Question 7 Let X be a random variable with density

$$f(x) = \frac{1}{2b}\exp\left(-\frac{|x-\mu|}{b}\right), \quad \text{with} \quad \mu \in \mathbb{R} \quad \text{and} \quad b > 0.$$

The cdf of X in realisation t is given by

- ☐ $\frac{1}{2}\exp\left(-\frac{t-\mu}{b}\right)$
- ■ $\mathbf{I}\{t \geq \mu\} + \frac{1}{2}\exp\left(-\frac{t-\mu}{b}\right)(\mathbf{I}\{t < \mu\} - \mathbf{I}\{t \geq \mu\})$
- ☐ $1 - \frac{1}{2}\exp\left(-\frac{t-\mu}{b}\right)$
- ☐ $\mathbf{I}\{t < \mu\} + \frac{1}{2}\exp\left(-\frac{t-\mu}{b}\right)(\mathbf{I}\{t \geq \mu\} - \mathbf{I}\{t < \mu\})$
- ☐ $\frac{1}{2}\exp\left(-\frac{|t-\mu|}{b}\right)$

Question 8 Let $f(x) = \frac{|\ln(x)|}{x^3+1}\mathbf{1}_{x>0}$, then

- ■ there exists K such that Kf is the density of a r.v. that admits a mean but not a variance
- ☐ there exists K such that Kf is the density of a r.v. that admits a mean and a variance
- ☐ there exists no K such that Kf is a density.
- ☐ there exists K such that Kf is a density of a r.v. that admits neither a mean nor a variance

Question 9 Take $X, Y \overset{\sim}{\underset{\sim}{\text{iid}}} \mathcal{N}(1,1)$. The expectation $\mathbb{E}[X/Y]$ is equal to

- ☐ 1
- ☐ 2
- ☐ $+\infty$
- ☐ 0
- ■ it does not exist

Question 10 Let X be a normally distributed $\mathcal{N}(0, \sigma^2)$, $\sigma > 0$ random variable and let $Y = aX^2 + bX + c$. For which values of (a, b, c) are X and Y uncorrelated ?

- ☐ $a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}$
- ☐ $a = 0, b = 1, c = 1$
- ■ $a \in \mathbb{R}, b = 0, c \in \mathbb{R}$
- ☐ X and Y are always correlated

Question 11 To produce a random variable X with density

$$f(x) = \sin(x)\mathbf{I}_{[0,\pi/2]}(x)$$

one can rely on the following transform of a uniform $U \sim U_{[0,1]}$

- ☐ $X = \cos(U) + \pi/2$
- ☐ $X = \arcsin(U)$
- ■ $X = \arccos(1 - U)$
- ☐ $X = \cos(U)$
- ☐ $X = \sin(U)$

Question 12 Let $X_1, X_2, \ldots$ be real-valued, iid random variables with mean $\mu \neq 0$ and finite variance $\sigma^2 > 0$. Let $\bar{X} = (1/n)\sum_{i=1}^n X_i$ denote the empirical mean. Then $(\bar{X})^2$ converges in distribution to a random variable whose distribution is

- ☐ $\mathcal{N}(\mu^4, 16\mu^2\sigma^4)$
- ☐ $\mathcal{N}(\mu^2, \sigma^4)$
- ☐ $\mathcal{N}(\mu^2, 4\mu^2\sigma^2)$
- ☐ $\mathcal{N}(\mu, \sigma^2)$
- ■ None of the above

Question 13
Let X_1 and X_2 be two independent random variables following an exponential distribution such that $\mathbb{E}[X_1] = \frac{1}{\lambda_1}$ and $\mathbb{E}[X_2] = \frac{1}{\lambda_2}$ with $\lambda_1, \lambda_2 > 0$. We recall that the p.d.f of an exponentially-distributed random variable with parameter λ is $f(x) = \lambda e^{-\lambda x}$.
The distribution of $\min(X_1, X_2)$ is

- ☐ The exponential distribution with parameter $\frac{\lambda_1}{\lambda_2}$
- ☐ The uniform distribution on the interval $[\min(\lambda_1, \lambda_2), \max(\lambda_1, \lambda_2)]$
- ☐ The normal distribution $\mathcal{N}(\frac{1}{\lambda_1\lambda_2}, (\lambda_1\lambda_2)^2)$
- ■ The exponential distribution with parameter $\lambda_1 + \lambda_2$

Question 14 Take $X, Y \overset{\text{iid}}{\sim} \mathcal{N}(1,1)$. The expectation $\mathbb{E}[X/Y]$ is equal to

- [ ] 0
- [x] it does not exist
- [ ] $+\infty$
- [ ] 1
- [ ] 2

Question 15 Let X be a random variable with density

$$f(x) = \frac{3}{2} x^{-5/2} \mathbb{I}_{(1,\infty)}(x)$$

The expectation $\mathbb{E}[X^k]$ only exists when

- [x] $k \leq 0$
- [x] $k \leq 1$
- [ ] $k < 5/2$
- [ ] $k \leq 2$
- [x] $k < 3/2$
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Question 16 Let X follow an exponential distribution with parameter $\lambda > 0$ (i.e. $\mathbb{E}(X) = \frac{1}{\lambda}$). Then $\mathbb{E}(X^2)$ is

- [ ] $\frac{1}{\lambda^2}$
- [x] $\frac{2}{\lambda^2}$
- [ ] $\frac{1-\lambda}{\lambda^2}$
- [ ] $\frac{1}{\lambda^4}$

Question 17 To produce a random variable X with density

$$f(x) = \sin(x) \mathbf{I}_{[0,\pi/2]}(x)$$

one can rely on the following transform of a uniform $U \sim U_{[0,1]}$

- [ ] $X = \arcsin(U)$
- [ ] $X = \cos(U) + \pi/2$
- [ ] $X = \cos(U)$
- [ ] $X = \sin(U)$
- [x] $X = \arccos(1-U)$

Question 18 Let X be a normally distributed $\mathcal{N}(0, \sigma^2)$, $\sigma > 0$ random variable and let $Y = aX^2 + bX + c$. For which values of (a, b, c) are X and Y uncorrelated ?

- [ ] $a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}$
- [ ] $a = 0, b = 1, c = 1$
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Question 19 Let X (resp. Y) be a random variable distributed from the geometric distribution with parameter p_1 (resp. p_2). We suppose that X and Y are independent. The distribution of the random variable $Z = \min\{X, Y\}$ is

- [ ] the exponential distribution with parameter $\frac{\min\{p_1, p_2\}}{p_1+p_2}$?
- [x] the geometric distribution with parameter $p_1 + p_2 - p_1 p_2$.
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- [ ] the geometric distribution with parameter $p_1 + p_2$.

Question 20 Let $f(x) = \frac{|\ln(x)|}{x^3+1}\mathbf{1}_{x>0}$, then

- [ ] there exists no K such that Kf is a density.
- [ ] there exists K such that Kf is the density of a r.v. that admits a mean and a variance
- [x] there exists K such that Kf is the density of a r.v. that admits a mean but not a variance
- [ ] there exists K such that Kf is a density of a r.v. that admits neither a mean nor a variance

Question 21 Let X be a random variable with density

$$f(x) = \frac{1}{2b} \exp\left(-\frac{|x - \mu|}{b}\right), \quad \text{with} \quad \mu \in \mathbb{R} \quad \text{and} \quad b > 0.$$

The cdf of X in realisation t is given by

- [ ] $\frac{1}{2} \exp\left(-\frac{t-\mu}{b}\right)$
- [ ] $\frac{1}{2} \exp\left(-\frac{|t-\mu|}{b}\right)$
- [ ] $1 - \frac{1}{2} \exp\left(-\frac{t-\mu}{b}\right)$
- [x] $\mathbf{I}\{t \geq \mu\} + \frac{1}{2} \exp\left(-\frac{t-\mu}{b}\right) (\mathbf{I}\{t < \mu\} - \mathbf{I}\{t \geq \mu\})$
- [ ] $\mathbf{I}\{t < \mu\} + \frac{1}{2} \exp\left(-\frac{t-\mu}{b}\right) (\mathbf{I}\{t \geq \mu\} - \mathbf{I}\{t < \mu\})$

Question 22 Let X_i $(i = 1, 2, \dots)$ be iid random variables taking values in $\mathbb{N}^*$, such that

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with $C > 0$ a constant and define

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Which of the following statements are true ?

- [ ] There exist $\mu \in \mathbb{R}$ and $\sigma^2 > 0$ such that $\sqrt{n}(\bar{X}_n - \mu)/\sigma$ converges in distribution to a standard normal distribution.
- [x] $\bar{X}_n$ converges a.s. to a constant $\mu \in \mathbb{R}$.
- [ ] $\bar{X}_n$ converges in $\mathbb{L}^2$ to a constant $\mu \in \mathbb{R}$.
- [x] $\bar{X}_n$ converges in probability to a constant $\mu \in \mathbb{R}$.