CORRECTED

Université Paris-Dauphine — Année 2020-2021

L3 - Statistical modelling

QCM 1 sept 2020

Dans cet exercice il vous est demandé de donner la ou les bonnes réponses.

Question 1 Consider a sequence X_n of independent rvs such that $\mathbb{P}(X_n = 1) = 1/n = 1 - \mathbb{P}(X_n = 0)$. Which of the following statements is true about the convergence of X_n ?

- ☐ not at all
- ☐ in probability and almost surely
- ■ in probability but not almost surely
- ☐ almost surely but not in probability
- ☐ in distribution

Question 2 On $\Omega = [0, 1]$, we define $X_n = \mathbf{1}_{[0,1/2]}$ if n is even, $X_n = \mathbf{1}_{[1/2,1]}$ if n is odd. Then :

- ☐ X_n converges in L^1
- ☐ X_n converges in probability
- ☐ X_n converges a.s
- ■ X_n converges in distribution

Question 3 Let $X_1, X_2, \ldots$ be real-valued, independent and identically distributed random variables with mean $\mu \neq 0$ and finite variance $\sigma^2 > 0$. Let $\bar{X} = (1/n)\sum_{i=1}^{n} X_i$ denote the empirical mean. Then $(\bar{X})^2$ converges in distribution to a random variable which distribution is :

- ☐ $\mathcal{N}(\mu, \sigma^2)$
- ☐ $\mathcal{N}(\mu^4, 16\mu^2\sigma^4)$
- ☐ $\mathcal{N}(\mu^2, \sigma^4)$
- ■ None of the above
- ☐ $\mathcal{N}(\mu^2, 4\mu^2\sigma^2)$

Question 4 Let $X_1, X_2, \ldots$ be an independent sequence of real-valued random variables such that $X_n \sim \text{Bernoulli}(\frac{1}{n})$. Which statement about the convergence of the sequence $(X_n)_n$ is true ?

- ■ $(X_n)_n$ converges to 0 in probability, in distribution and in p-th mean for all $p > 0$ but not almost surely.
- ☐ $(X_n)_n$ converges to 0 almost surely, in p-th mean for $p = 2$ and $p = 3$ only but not in probability nor in distribution.
- ☐ $(X_n)_n$ converges to 0 in probability and in distribution but not almost surely nor in quadratic mean (p-th mean for $p = 2$).
- ☐ $(X_n)_n$ converges to 0 almost surely and in p-th mean for all $p > 0$ but not in probability nor in distribution.

Question 5 Let $f(x) = \frac{\ln(x)}{x^3+1}\mathbf{1}_{x>0}$, then

- [x] there exists K such that Kf is the density of a r.v. that admits a mean but not a variance
- [ ] there exists K such that f is a density of a r.v. that admits neither a mean nor a variance
- [ ] there exists no K such that f is a density.
- [ ] there exists K such that f is the density of a r.v. that admits a mean and a variance

Question 6 Let X be a real-valued random variable and $f : \mathbb{R} \to \mathbb{R}$ a function. With which condition on f is $f(X)$ a random variable ?

- [x] f is measurable
- [ ] f is bounded
- [ ] $f(X)$ is always a random variable
- [ ] None of the above
- [ ] $f(X)$ is never a random variable

Question 7 Let $a_n \to^{\mathcal{L}}_{n\to\infty} a \in R$ and $Z_n \to^{a.s.}_{n\to\infty} Z$, then e^{a_n}/Z_n converges :

- [ ] in probability
- [ ] almost surely
- [x] in distribution
- [ ] not at all

Question 8 Let (X_n) and (Y_n) be two sequences of random variables converging in distribution to X and Y respectively.

1. $(X_n + Y_n)$ converges in distribution to $X + Y$
2. $(X_n Y_n)$ converges in distribution to XY
3. If $Y = c$ is a constant, $(X_n Y_n)$ converges in distribution to cX

What statements are true ?

- [ ] 1. and 3.
- [ ] None of all
- [ ] all
- [ ] 2. and 3.
- [x] 3.