CORRECTED

Université Paris-Dauphine — Année 2020-2021

L3 - Statistical modelling

QCM 05 oct. 2020

Dans cet exercice il vous est demandé de donner la ou les bonnes réponses.

Question 1
Let $f(x|\mu)$ be the density of a Cauchy distribution with location parameter μ i.e $f(x|\mu) = \frac{1}{\pi[1+(x-\mu)^2]}$. Which of the following is true ?

- [x] The Cauchy distribution is not an exponential family.
- [ ] The Cauchy distribution is an exponential family with natural parameter $\tau(\mu) = \mu$ and natural parameter space $\Theta = \mathbb{R}$.
- [ ] The Cauchy distribution is an exponential family with natural parameter $\tau(\mu) = \mu^2$ and natural parameter space $\Theta = \mathbb{R}^+$.
- [ ] The Cauchy distribution is an exponential family with natural parameter $\tau(\mu) = \mu$ and natural parameter space $\Theta = \mathbb{R}^+$.
- [ ] The Cauchy distribution is an exponential family with natural parameter $\tau(\mu) = \mu^2$ and natural parameter space $\Theta = \mathbb{R}$.

Question 2 Which of the following family is not an exponential family :

- [x] The binomial distribution, with n and p unknown
- [ ] The Laplace distribution with μ known
- [ ] The beta distribution
- [ ] The multinomial distribution with n known

Question 3
Let $X \sim \mathcal{P}(X)$, with $P(X=n) = \exp(-\lambda)\lambda^n/n!$, and let $\phi : \mathbf{N} \to E \subset \mathbf{Q}$ be a bijective function, then what is the true statement about the distribution of $Y = \phi(X)$:

- [ ] is not in the exponential family
- [ ] is in the exponential family with sufficient statistic y
- [ ] is in the exponential family with sufficient statistic $\log(y)$
- [x] is in the exponential family with sufficient statistic $\phi^{-1}(y)$
- [ ] is in the exponential family with sufficient statistic $\phi(y)$

Question 4 Which family is not exponential ?

- [ ] $f(x|\theta) \propto \mathbf{1}_{\{0\le x\le\phi\}}\frac{\theta}{1+x^2}$ for $\theta \in (0,\infty)$ and ϕ known
- [x] $f(x|\theta) \propto \mathbf{1}_{\{-\theta\le x\le\theta\}}\exp(-x^2/\theta)$ for $\theta \in (0,\infty)$
- [ ] $f(x|\theta) \propto \frac{\theta}{2}\exp(-\theta x^2)(\log(1+x)^{\sqrt{\theta}})$ for $\theta \in (0,\infty)$
- [ ] $f(x|\theta) \propto e^{-x}x^{2\log(1+\theta^2)}$ for $\theta \in \mathbb{R}$

Question 5
Let $f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)}x^{-\alpha-1}\exp(-\beta/x)$, with α and β unknown, what is **false** about this density :

- [ ] a natural parameter is $(-\beta, -\alpha-1)$
- [ ] a natural parameter is (α, β)
- [x] a sufficient statistic is $(x, \log(x))$
- [ ] a sufficient statistic is $(1/x, \log(x))$

Question 6
Let $f(x|\mu,\sigma)$ be the density of the Laplace distribution i.e $f(x|\mu,\sigma) = \frac{1}{2\sigma}\exp(-|x-\mu|/\sigma)$, $\sigma > 0$. Which of the following is true ?

- [ ] The Laplace distribution with known μ is not an exponential family
- [ ] The Laplace distribution is an exponential family with natural parameter $\tau(\mu,\sigma) = (-\mu,\sigma^2)$.
- [ ] The Laplace distribution is an exponential family with natural parameter $\tau(\mu,\sigma) = (-\mu/\sigma,\sigma^2)$.
- [x] The Laplace distribution with known μ is an exponential family with natural parameter $\tau(\sigma) = -\frac{1}{\sigma}$.

Question 7 Which family is not exponential ?

- [ ] $f(x|\theta) \propto \mathbf{1}_{\{0\le x\le\phi\}}\frac{\theta}{1+x^2}$ for $\theta \in (0,\infty)$ and ϕ known
- [x] $f(x|\theta) \propto \mathbf{1}_{\{-\theta\le x\le\theta\}}\exp(-x^2/\theta)$ for $\theta \in (0,\infty)$
- [ ] $f(x|\theta) \propto e^{-x}x^{2\log(1+\theta^2)}$ for $\theta \in \mathbb{R}$
- [ ] $f(x|\theta) \propto \frac{\theta}{2}\exp(-\theta x^2)(\log(1+x)^{\sqrt{\theta}})$ for $\theta \in (0,\infty)$

Question 8 Which of the following family is not an exponential family :

- [ ] The beta distribution
- [x] The binomial distribution, with n and p unknown
- [ ] The multinomial distribution with n known
- [ ] The Laplace distribution with μ known

Question 9
Let $X \sim \mathcal{P}(X)$, with $P(X=n) = \exp(-\lambda)\lambda^n/n!$, and let $\phi : \mathbf{N} \to E \subset \mathbf{Q}$ be a bijective function, then what is the true statement about the distribution of $Y = \phi(X)$:

- [ ] is in the exponential family with sufficient statistic $\log(y)$
- [ ] is not in the exponential family
- [ ] is in the exponential family with sufficient statistic y
- [ ] is in the exponential family with sufficient statistic $\phi(y)$
- [x] is in the exponential family with sufficient statistic $\phi^{-1}(y)$

Question 10
Let $f(x|\mu)$ be the density of a Cauchy distribution with location parameter μ i.e $f(x|\mu) = \frac{1}{\pi[1+(x-\mu)^2]}$. Which of the following is true ?

- [ ] The Cauchy distribution is an exponential family with natural parameter $\tau(\mu) = \mu$ and natural parameter space $\Theta = \mathbb{R}^+$.
- [ ] The Cauchy distribution is an exponential family with natural parameter $\tau(\mu) = \mu^2$ and natural parameter space $\Theta = \mathbb{R}^+$.
- [ ] The Cauchy distribution is an exponential family with natural parameter $\tau(\mu) = \mu$ and natural parameter space $\Theta = \mathbb{R}$.
- [ ] The Cauchy distribution is an exponential family with natural parameter $\tau(\mu) = \mu^2$ and natural parameter space $\Theta = \mathbb{R}$.
- [x] The Cauchy distribution is not an exponential family.

Question 11

Let $f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} \exp(-\beta/x)$, with α and β unknown, what is **false** about this density :

- [ ] a natural parameter is (α, β)
- [ ] a sufficient statistic is $(1/x, \log(x))$
- [x] a sufficient statistic is $(x, \log(x))$
- [ ] a natural parameter is $(-\beta, -\alpha - 1)$

Question 12

Let $f(x|\mu, \sigma)$ be the density of the Laplace distribution i.e $f(x|\mu, \sigma) = \frac{1}{2\sigma} \exp(-|x - \mu|/\sigma)$, $\sigma > 0$. Which of the following is true ?

- [ ] The Laplace distribution is an exponential family with natural parameter $\tau(\mu, \sigma) = (-\mu/\sigma, \sigma^2)$.
- [ ] The Laplace distribution with known μ is not an exponential family
- [x] The Laplace distribution with known μ is an exponential family with natural parameter $\tau(\sigma) = -\frac{1}{\sigma}$.
- [ ] The Laplace distribution is an exponential family with natural parameter $\tau(\mu, \sigma) = (-\mu, \sigma^2)$.