Université Paris-Dauphine

L3 - Statistical modelling

### QCM 11 oct 2020

Dans cet exercice il vous est demandé de donner l'unique bonne réponse.

If x is a continuous sample of size n=12 and bootstrap is used to study the Question 1 variability of the mean of the sample, what is the number of values taken by the bootstrapped realisations



When given a sample x of size n from F and considering the median med(X)Question 2 as the quantity of interest, a bootstrap approximation of a 95% interval of variability of the empirical median is given by



quantile(median(matrix(sample(x,n\*m,rep=TRUE),m)),c(.025,.975)) median(apply(matrix(sample(x,n\*m,rep=TRUE),m),1,sum),prob=.95) quantile(apply(matrix(sample(x,n\*m,rep=TRUE),m),1,median),c(.02,.97)) quantile(matrix(sample(median(x),n\*m,rep=TRUE),m)),c(.035,.985))

### Question 3

We want to compute the variance of the empirical quantile of order p, we proceed by bootstraping, which of the following returns this estimation, for x a vector of size 200, a sample of observations.



var(sapply(1:100,function(y)quantile(sample(x,length(x),replace=T),p))) quantile(sapply(1:100,function(y)var(sample(x,length(x)),replace=T)),p) var(sapply(1:100,function(x)quantile(sample(x,length(x)),p)))

Question 4 For a sample  $\mathbf{x}$  of size *n* the standard deviation of the sample median can be estimated by bootstrap as

| 5 |  |
|---|--|
| n |  |
| 5 |  |
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sd(matrix(sample(quantile(x,prob=.5),n\*m,rep=TRUE),m)) nedian(apply(matrix(sample(sd(x),n\*m,rep=TRUE),m),1,mean)) sd(apply(matrix(sample(x,n\*m,rep=TRUE),m),1,median)) median(apply(matrix(sample(x,n\*m,rep=TRUE),m),1,sd))

## Question 5

We observe the realisation of a discrete random variable with values in  $1, \ldots, n$ , from the  $Y_i$ , iid realisations, we create the dataset **x** as a vector of size *n* defined by  $x_i = \sum_i \mathbf{1}_{Y_i=i}$ , that is the number of times the value j has been drawn. We want to estimate the bias of the estimator  $\hat{p} = x_1 (\sum x_i)^{-1}$  of the P(Y = 1). The bias of this estimator :

is 0.

| can be estimated by bootstrap with mean(sapply(1:100,function(y){x[1]/length(x)}) - x[1]/length(x)).

can be estimated by bootstrap with mean(sapply(1:100,function(y){z=sample(x,replace=T); return(sum(z=1)/sum(z)))) - x[1]/length(x).

Question 6 If x is a continuous sample of size n=13 and bootstrap is used to study the variability of the median of the sample, what is the number of values taken by the bootstrapped realisations



## Question 7

We want to compute the variance of the empirical quantile of order p, we proceed by bootstraping, which of the following returns this estimation, for  $\mathbf{x}$  a vector of size 200, a sample of observations.



```
var(sapply(1:100,function(x)quantile(sample(x,length(x)),p)))
quantile(sapply(1:100,function(y)var(sample(x,length(x)),replace=T)),p)
var(sapply(1:100,function(y)quantile(sample(x,length(x),replace=T),p)))
```

**Question 8** When given a sample x of size n from F and considering the median med(X) as the quantity of interest, a bootstrap approximation of a 95% interval of variability of the empirical median is given by



```
median(apply(matrix(sample(x,n*m,rep=TRUE),m),1,sum),prob=.95)
quantile(median(matrix(sample(x,n*m,rep=TRUE),m)),c(.025,.975))
quantile(apply(matrix(sample(x,n*m,rep=TRUE),m),1,median),c(.02,.97))
quantile(matrix(sample(median(x),n*m,rep=TRUE),m)),c(.035,.985))
```

Question 9 If x is a continuous sample of size n=13 and bootstrap is used to study the variability of the median of the sample, what is the number of values taken by the bootstrapped realisations



# Question 10

We observe the realisation of a discrete random variable with values in  $1, \ldots, n$ , from the  $Y_i$ , iid realisations, we create the dataset **x** as a vector of size *n* defined by  $x_j = \sum_i \mathbf{1}_{Y_i=j}$ , that is the number of times the value *j* has been drawn. We want to estimate the bias of the estimator  $\hat{p} = x_1 (\sum x_i)^{-1}$  of the P(Y = 1). The bias of this estimator :

is 0.

can be estimated by bootstrap with mean(sapply(1:100,function(y){x[1]/length(x)})
- x[1]/length(x)).

can be estimated by bootstrap with mean(sapply(1:100,function(y){z=sample(x,replace=T); return(sum(z==1)/sum(z))})) - x[1]/length(x).

#### Corrected



**Question 12** For a sample  $\mathbf{x}$  of size n the standard deviation of the sample median can be estimated by bootstrap as

median(apply(matrix(sample(sd(x),n\*m,rep=TRUE),m),1,mean))
sd(apply(matrix(sample(x,n\*m,rep=TRUE),m),1,median))
median(apply(matrix(sample(x,n\*m,rep=TRUE),m),1,sd))
sd(matrix(sample(quantile(x,prob=.5),n\*m,rep=TRUE),m))