Université Paris-Dauphine
L3 - Statistical modelling

Année 2020-2021

MCQ 9 November 2020

This exercice requires you to tick the unique correct answer.

Question 1
Let $(x_i)_{i=1,\dots,n}$ be an i.i.d. sample, with true distribution F given by $X_i \sim \mathcal{N}(\mu_0, 1/2)$. Assuming the statistical model $\{\mathcal{N}(\mu,1)|\mu \in \mathbb{R}\}$, what is true on the likelihood $L(\mu|x_1,\dots x_n)$ of the model ?

- [ ] it writes $\prod_i (2\pi)^{-1/2} e^{-(x_i-\mu)^2}$
- [x] $\int_{\mathbb{R}^n} L(\mu|x_1,\dots,x_n)\mathrm{d}(x_1,\dots,x_n) = 1$
- [ ] $\int_{\mathbb{R}^n} L(\mu|x_1,\dots,x_n)\mathrm{d}F(x,\dots,x_n) = 1$
- [ ] $\int_{\mathbb{R}} L(\mu|x_1,\dots,x_n)\mathrm{d}\mu = 1$

Question 2 For a sequence $x_1,\dots,x_n$ of integers between 0 and k, the log-likelihood associated with a Binomial $\mathcal{B}(k, e^\theta)$ model is

- [ ] $\ell(\theta|x_1,\dots,x_n) = \sum_i \log\left(\binom{k}{x_i}\theta^{x_i}(1-\theta)^{k-x_i}\right)$
- [ ] $\ell(\theta|x_1,\dots,x_n) = \theta^{\sum_i x_i} + \log(1-e^\theta)^{\sum_i(k-x_i)}$
- [x] $\ell(\theta|x_1,\dots,x_n) = \log(1/e^{-\theta} - 1)\sum_i x_i + nk\log(1-e^\theta)$
- [ ] $\ell(\theta|x_1,\dots,x_n) = \theta\sum_i x_i - \log(1-e^\theta)\sum_i(k-x_i)$

Question 3
For the Geometric distribution with p.m.f $f_X(x;\theta) = \theta(1-\theta)^{x-1}$ for $x = 1,2,3\dots$, $0 < \theta < 1$, and an i.i.d sample $(x_1,\dots,x_n)$, which of the following is the correct form of the log-likelihood function ?

- [x] $\ell(\theta) = N\log(\theta) + \sum_{i=1}^N (x_i-1)\log(1-\theta)$
- [ ] $\ell(\theta) = \log(N\theta) + \log((1-\theta)\sum_{i=1}^N(x_i-1))$
- [ ] $\ell(\theta) = N\log(\theta) + (1-\theta)\log(\sum_{i=1}^N(x_i-1))$
- [ ] $\ell(\theta) = \log(N)\log(\theta) + \log((1-\theta)\log(\sum_{i=1}^N(x_i-1)))$

Question 4 For a Gamma distribution with density $f(x|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}\exp(-\beta x)$, $\alpha,\beta > 0$, and an i.i.d. sample $(x_1,\dots,x_n)$, what is the log-likelihood ?

- [ ] $\ell(\alpha,\beta|x_1,\dots,x_n) = (\alpha-1)\log(\sum_{i=1}^n x_i) - \sum_{i=1}^n \beta x_i + n\alpha\log(\beta) - n\log(\Gamma(\alpha))$
- [ ] $\ell(\alpha,\beta|x_1,\dots,x_n) = (\alpha-1)\prod_{i=1}^n \log(x_i) - \prod_{i=1}^n \beta x_i + (\alpha\log(\beta))^n - \log(\Gamma(\alpha))^n$
- [x] $\ell(\alpha,\beta|x_1,\dots,x_n) = (\alpha-1)\sum_{i=1}^n \log(x_i) - \sum_{i=1}^n \beta x_i + n\alpha\log(\beta) - n\log(\Gamma(\alpha))$
- [ ] $\ell(\alpha,\beta|x_1,\dots,x_n) = \sum_{i=1}^n \log(x_i)^{\alpha-1} - \sum_{i=1}^n \beta x_i + n\log(\beta)^\alpha - n\log(\Gamma(\alpha))$

Question 5
Given n iid observations x_i from the distribution δ_0, a Dirac mass at 0, the proposed statistical model is defined by $\left\{\mathcal{N}(0,1/\rho^2)|\rho \in \mathbb{R}_*^+\right\}$. Which of the following is true about the likelihood $L(\rho|x_1,\dots,x_n)$ of the model ?

- [ ] $L(\rho|x_1,\dots,x_n) \overset{\rho\to\infty}{\longrightarrow} 0$
- [ ] it is not defined
- [ ] $L(0|x_1,\dots,x_n) = 1$
- [x] $L(\rho|x_1,\dots,x_n)$ has no maximum

Question 6
Given n iid observations x_i from the distribution δ_0, a Dirac mass at 0, the proposed statistical model is defined by $\{\mathcal{N}(0, 1/\rho^2)|\rho \in \mathbb{R}_*^+\}$. Which of the following is true about the likelihood $L(\rho|x_1, \ldots, x_n)$ of the model ?

- [ ] $L(\rho|x_1, \ldots, x_n) \overset{\rho\to\infty}{\longrightarrow} 0$
- [x] $L(\rho|x_1, \ldots, x_n)$ has no maximum
- [ ] it is not defined
- [ ] $L(0|x_1, \ldots, x_n) = 1$

Question 7 For a sequence $x_1, \ldots, x_n$ of integers between 0 and k, the log-likelihood associated with a Binomial $\mathcal{B}(k, e^\theta)$ model is

- [ ] $\ell(\theta|x_1, \ldots, x_n) = \theta^{\sum_i x_i} + \log(1 - e^\theta)^{\sum_i (k - x_i)}$
- [ ] $\ell(\theta|x_1, \ldots, x_n) = \theta \sum_i x_i - \log(1 - e^\theta) \sum_i (k - x_i)$
- [x] $\ell(\theta|x_1, \ldots, x_n) = \log(1/e^{-\theta} - 1) \sum_i x_i + nk \log(1 - e^\theta)$
- [ ] $\ell(\theta|x_1, \ldots, x_n) = \sum_i \log \left(\binom{k}{x_i} \theta^{x_i} (1 - \theta)^{k - x_i} \right)$

Question 8
For the Geometric distribution with p.m.f $f_X(x; \theta) = \theta(1-\theta)^{x-1}$ for $x = 1, 2, 3 \ldots, 0 < \theta < 1$, and an i.i.d sample $(x_1, \ldots, x_n)$, which of the following is the correct form of the log-likelihood function ?

- [ ] $\ell(\theta) = N \log(\theta) + (1 - \theta) \log(\sum_{i=1}^N (x_i - 1))$
- [ ] $\ell(\theta) = \log(N) \log(\theta) + \log((1 - \theta) \log(\sum_{i=1}^N (x_i - 1)))$
- [ ] $\ell(\theta) = \log(N\theta) + \log((1 - \theta) \sum_{i=1}^N (x_i - 1))$
- [x] $\ell(\theta) = N \log(\theta) + \sum_{i=1}^N (x_i - 1) \log(1 - \theta)$

Question 9
Let $(x_i)_{i=1,\ldots,n}$ be an i.i.d. sample, with true distribution F given by $X_i \sim \mathcal{N}(\mu_0, 1/2)$. Assuming the statistical model $\{\mathcal{N}(\mu, 1)|\mu \in \mathbb{R}\}$, what is true on the likelihood $L(\mu|x_1, \ldots x_n)$ of the model ?

- [ ] it writes $\prod_i (2\pi)^{-1/2} e^{-(x_i - \mu)^2}$
- [x] $\int_{\mathbb{R}^n} L(\mu|x_1, \ldots, x_n) \mathrm{d}(x_1, \ldots, x_n) = 1$
- [ ] $\int_{\mathbb{R}^n} L(\mu|x_1, \ldots, x_n) \mathrm{d}F(x, \ldots, x_n) = 1$
- [ ] $\int_{\mathbb{R}} L(\mu|x_1, \ldots, x_n) \mathrm{d}\mu = 1$

Question 10 For a Gamma distribution with density $f(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\beta x)$, $\alpha, \beta > 0$, and an i.i.d. sample $(x_1, \ldots, x_n)$, what is the log-likelihood ?

- [ ] $\ell(\alpha, \beta|x_1, \ldots, x_n) = \sum_{i=1}^n \log(x_i)^{\alpha-1} - \sum_{i=1}^n \beta x_i + n \log(\beta)^\alpha - n \log(\Gamma(\alpha))$
- [x] $\ell(\alpha, \beta|x_1, \ldots, x_n) = (\alpha - 1) \sum_{i=1}^n \log(x_i) - \sum_{i=1}^n \beta x_i + n\alpha \log(\beta) - n \log(\Gamma(\alpha))$
- [ ] $\ell(\alpha, \beta|x_1, \ldots, x_n) = (\alpha - 1) \log(\sum_{i=1}^n x_i) - \sum_{i=1}^n \beta x_i + n\alpha \log(\beta) - n \log(\Gamma(\alpha))$
- [ ] $\ell(\alpha, \beta|x_1, \ldots, x_n) = (\alpha - 1) \prod_{i=1}^n \log(x_i) - \prod_{i=1}^n \beta x_i + (\alpha \log(\beta))^n - \log(\Gamma(\alpha))^n$