Université Paris-Dauphine Année 2020-2021
L3 - Statistical modelling

MCQ 16 November 2020

This exercice requires you tick the one and unique correct answer.

Question 1 Consider an iid sample $X_1, \ldots, X_n$ from a Geometric distribution $\mathcal{G}(\exp\{-\theta_0\})$, with $\mathbb{P}(X_i = 0) = e^{-\theta_0}$ and θ_0 unknown. Which of the following is correct ?

- [ ] $-n + \sum_{i=1}^n X_i {}^{e^{-\hat\theta}}/_{1-e^{-\hat\theta}}$ has expectation zero when $\hat\theta$ is solution of $\bar X_n = {}^{1-e^{-\hat\theta}}/_{e^{-\hat\theta}} = 0$
- [ ] $-n + \sum_{i=1}^n X_i {}^{e^{-\theta}}/_{1-e^{-\theta}}$ has expectation zero for all values of θ
- [ ] $-n + \sum_{i=1}^n X_i {}^{e^{-\theta_0}}/_{1-e^{-\theta_0}} = 0$
- [x] $-n + \sum_{i=1}^n X_i {}^{e^{-\theta}}/_{1-e^{-\theta}}$ has expectation zero when $\theta = \theta(X_1, \ldots, X_n)$ is maximising the likelihood

Question 2 Let $X_1, \ldots, X_n$ be an iid sample from a Normal $\mathcal{N}(1, \sigma^2)$ distribution. Denoting $\theta = \sigma^2$ as the parameter, which of the following is the correct score function ?

- [ ] $\sum_{i=1}^n X_i^2 / \theta^3 - {}^n/_{2\theta^2}$
- [x] $\sum_{i=1}^n (X_i - 1)^2 / 2\theta^2 - {}^n/_{2\theta}$
- [ ] $\sum_{i=1}^n (X_i - 1)^2 / \theta^{3/2} - {}^n/_{2\theta}$
- [ ] $n\overline{X_n}^2 / \theta^2 - {}^n/_{2\theta}$

Question 3 Take $X_1, \ldots, X_n$ as an iid sample from the zero-truncated Poisson distribution $\mathcal{P}^+(\lambda)$ distribution, meaning $\mathbb{P}(X_i = k) \propto \exp\{-\lambda\}\lambda^k/k!$ for $k = 1, 2, \ldots$. Which of the following is the correct Fisher information on λ contained in the sample ?

- [ ] $\frac{n}{1-2^{-\lambda}} \left[{}^2/_{\lambda^3} - {}^1/_{2^\lambda - 1}\right]$
- [x] $\frac{n}{1-e^{-\lambda}} \left[{}^1/_{\lambda} - {}^1/_{e^\lambda - 1}\right]$
- [ ] $\frac{n}{1-e^{-2\lambda}} \left[{}^1/_{\lambda^2} - {}^1/_{e^{2\lambda} - 1}\right]$
- [ ] $\frac{n}{1-e^{-2\lambda}} \left[{}^1/_{\lambda^2} - {}^2/_{(e^\lambda - 1)^2}\right]$

Question 4 Let $X_1, \ldots, X_n$ be an iid sample from a Normal $\mathcal{N}(1, \sigma^2)$ distribution. Denoting $\theta = e^{-\sigma^2}$ as a parameterisation of the model, which of the following is the correct Fisher information on θ contained in the sample ?

- [ ] $-{}^n/_{3\theta^2}$
- [ ] ${}^n/_3 \, {}^{\theta^{-1}}/_{\log\theta}$
- [ ] $-{}^{2n}/_{\theta^2 \log(\theta)^2}$
- [x] $-{}^n/_2 \, {}^{\theta^{-2}}/_{\log(\theta)^2}$

Question 5 Take $X_1, \ldots, X_n$ as an iid sample from the zero-truncated Poisson distribution $\mathcal{P}^+(\lambda)$ distribution, meaning $\mathbb{P}(X_i = k) \propto \exp\{-\lambda\}\lambda^k/k!$ for $k = 1, 2, \ldots$. Which of the following is the correct score function on $\theta = \lambda^{-1}$ contained in the sample ?

- [ ] $-n\theta^2 - {}^n/_{e^\theta - 1} + n\theta^{-1}\bar X_n$
- [ ] $-n - n\ln(1 - e^{-1/\theta}) + n\theta\bar X_n$
- [x] ${}^n/_{\theta^2(1-e^{-1/\theta})} - \frac{n}{\theta^2}\bar X_n$
- [ ] $-{}^n/_{\theta^2} - n\theta^{-2}\ln(1 - e^{-1/\theta}) + \frac{n}{\theta}\bar X_n$

Question 6 Consider an iid sample $X_1, \dots, X_n$ from a Geometric distribution $\mathcal{G}(\exp\{-\theta_0\})$, with $\mathbb{P}(X_i = 0) = e^{-\theta_0}$ and θ_0 unknown. Which of the following is the correct Fisher information on θ attached to the sample ?

- ■ $\nicefrac{n}{1-e^{-\theta}}$
- □ $\nicefrac{ne^{-\theta}}{(1-e^{-\theta})^2}$
- □ $\nicefrac{1-e^{-\theta}}{n}$
- □ $\nicefrac{n}{e^{\theta}-1}$

Question 7 Take $X_1, \dots, X_n$ as an iid sample from the zero-truncated Poisson distribution $\mathcal{P}^+(\lambda)$ distribution, meaning $\mathbb{P}(X_i = k) \propto \exp\{-\lambda\}\lambda^k/k!$ for $k = 1, 2, \dots$. Which of the following is the correct Fisher information on λ contained in the sample ?

- □ $\frac{n}{1-e^{-2\lambda}}\left[\nicefrac{1}{\lambda^2} - \nicefrac{2}{(e^{\lambda}-1)^2}\right]$
- □ $\frac{n}{1-e^{-2\lambda}}\left[\nicefrac{1}{\lambda^2} - \nicefrac{1}{e^{2\lambda}-1}\right]$
- □ $\frac{n}{1-2^{-\lambda}}\left[\nicefrac{2}{\lambda^3} - \nicefrac{1}{2^{\lambda}-1}\right]$
- ■ $\frac{n}{1-e^{-\lambda}}\left[\nicefrac{1}{\lambda} - \nicefrac{1}{e^{\lambda}-1}\right]$

Question 8 Let $X_1, \dots, X_n$ be an iid sample from a Normal $\mathcal{N}(1, \sigma^2)$ distribution. Denoting $\theta = \sigma^2$ as the parameter, which of the following is the correct score function ?

- □ $\nicefrac{n\overline{X_n}^2}{\theta^2} - \nicefrac{n}{2\theta}$
- □ $\sum_{i=1}^n \nicefrac{X_i^2}{\theta^3} - \nicefrac{n}{2\theta^2}$
- □ $\sum_{i=1}^n \nicefrac{(X_i-1)^2}{\theta^{3/2}} - \nicefrac{n}{2\theta}$
- ■ $\sum_{i=1}^n \nicefrac{(X_i-1)^2}{2\theta^2} - \nicefrac{n}{2\theta}$

Question 9 Consider an iid sample $X_1, \dots, X_n$ from a Geometric distribution $\mathcal{G}(\exp\{-\theta_0\})$, with $\mathbb{P}(X_i = 0) = e^{-\theta_0}$ and θ_0 unknown. Which of the following is the correct Fisher information on θ attached to the sample ?

- □ $\nicefrac{n}{e^{\theta}-1}$
- □ $\nicefrac{1-e^{-\theta}}{n}$
- ■ $\nicefrac{n}{1-e^{-\theta}}$
- □ $\nicefrac{ne^{-\theta}}{(1-e^{-\theta})^2}$

Question 10 Take $X_1, \dots, X_n$ as an iid sample from the zero-truncated Poisson distribution $\mathcal{P}^+(\lambda)$ distribution, meaning $\mathbb{P}(X_i = k) \propto \exp\{-\lambda\}\lambda^k/k!$ for $k = 1, 2, \dots$. Which of the following is the correct score function on $\theta = \lambda^{-1}$ contained in the sample ?

- ■ $\nicefrac{n}{\theta^2(1-e^{-1/\theta})} - \frac{n}{\theta^2}\bar{X}_n$
- □ $-n - n\ln(1 - e^{-1/\theta}) + n\theta\bar{X}_n$
- □ $-n\theta^2 - \nicefrac{n}{e^{\theta}-1} + n\theta^{-1}\bar{X}_n$
- □ $-\nicefrac{n}{\theta^2} - n\theta^{-2}\ln(1 - e^{-1/\theta}) + \frac{n}{\theta}\bar{X}_n$

Question 11 Consider an iid sample $X_1, \dots, X_n$ from a Geometric distribution $\mathcal{G}(\exp\{-\theta_0\})$, with $\mathbb{P}(X_i = 0) = e^{-\theta_0}$ and θ_0 unknown. Which of the following is correct ?

- □ $-n + \sum_{i=1}^n X_i \nicefrac{e^{-\hat\theta}}{1-e^{-\hat\theta}}$ has expectation zero when $\hat\theta$ is solution of $\bar{X}_n = \nicefrac{1-e^{-\hat\theta}}{e^{-\hat\theta}} = 0$
- ■ $-n + \sum_{i=1}^n X_i \nicefrac{e^{-\theta}}{1-e^{-\theta}}$ has expectation zero when $\theta = \theta(X_1, \dots, X_n)$ is maximising the likelihood
- □ $-n + \sum_{i=1}^n X_i \nicefrac{e^{-\theta}}{1-e^{-\theta}}$ has expectation zero for all values of θ
- □ $-n + \sum_{i=1}^n X_i \nicefrac{e^{-\theta_0}}{1-e^{-\theta_0}} = 0$

Question 12 Let $X_1, \ldots, X_n$ be an iid sample from a Normal $\mathcal{N}(1, \sigma^2)$ distribution. Denoting $\theta = e^{-\sigma^2}$ as a parameterisation of the model, which of the following is the correct Fisher information on θ contained in the sample ?

- [ ] $n/3\,\theta^{-1}/\log\theta$
- [ ] $-2n/\theta^2\log(\theta)^2$
- [x] $-n/2\,\theta^{-2}/\log(\theta)^2$
- [ ] $-n/3\theta^2$