Corrected

Université Paris-Dauphine Année 2020-2021
L3 - Statistical modelling

MCQ 23 November 2020

This exercice requires you tick the one and unique correct answer.

Question 1 Given an iid sample $X_1, \ldots, X_n$ from a Uniform $\mathcal{U}(\theta-1, \theta+1)$ distribution, the statistic $S(X) = (X_{(1)}, X_{(n)} - X_{(1)})$ is

- [x] incomplete and sufficient
- [ ] incomplete and insufficient
- [ ] complete and sufficient
- [ ] complete and insufficient

Question 2 Let $(X_1, \ldots, X_n)$ be a random vector on $\mathbb{N}^n$ with probability mass function $p(x_1, ..., x_n) \propto 1/\binom{\theta}{n}$ if all $0 \le x_i \le \theta$'s are different and equal to zero otherwise, with $\theta \in \mathbb{N}^*$. Then the statistic $S(X_1, \ldots, X_n) = \max X_i$ is

- [ ] complete and insufficient
- [ ] incomplete and insufficient
- [x] complete and sufficient
- [ ] incomplete and sufficient

Question 3 Given an iid sample $X_1, \ldots, X_{10}$ from a Normal $\mathcal{N}(\theta, 2)$ distribution, the statistic $S(X_1, \ldots, X_{10}) = \{X_1 + X_{10}\}/2$ is

- [ ] incomplete and insufficient
- [ ] incomplete and sufficient
- [x] complete and insufficient
- [ ] complete and sufficient

Question 4 Let $X_1, \ldots, X_n$ be an iid sample from a distribution that assigns with probability $1/3$ the values $\theta - 1$, θ, or $\theta + 1$, $\theta \in \mathbb{R}$. Then $S(X_1, \ldots, X_n) = (X_{(1)}, X_{(n)})$ is

- [ ] complete and sufficient
- [x] incomplete and sufficient
- [ ] complete and insufficient
- [ ] incomplete and insufficient

Question 5 Let $X_1, \ldots, X_n$ be a random sample from a distribution with density $f(x) = \frac{1}{\theta}\exp\{-\frac{(x-\theta)}{\theta}\}\,\mathbb{I}_{(x>\theta)}$ Then $S(X_1, \ldots, X_n) = (X_{(1)}, \bar{X}_n)$ is

- [ ] complete and sufficient
- [ ] complete and insufficient
- [ ] incomplete and insufficient
- [x] incomplete and sufficient

Question 6 Given an iid sample $X_1, \ldots, X_n$ from a Uniform $\mathcal{U}(0, \theta)$ distribution, the statistic $S(X_1, \ldots, X_n) = \bar{X}_n$ is

- [ ] complete and sufficient
- [ ] incomplete and insufficient
- [ ] incomplete and sufficient
- [x] complete and insufficient

Question 7 Assume that $X_1, ..., X_n$ are iid with density $f(x;\theta) = 2\exp\{2x\}\theta^{-2}\mathbb{I}(x < \log(\theta))$. The statistic $S(X_1, \ldots, X_n) = X_{(n)}$ is

■ complete and sufficient
□ complete and insufficient
□ incomplete and sufficient
□ incomplete and insufficient

Question 8 Given an iid sample $X_1, \ldots, X_n$ from a Uniform $\mathcal{U}(0, \theta)$ distribution, the statistic $S(X_1, \ldots, X_n) = \bar{X}_n$ is

■ complete and insufficient
□ incomplete and insufficient
□ complete and sufficient
□ incomplete and sufficient

Question 9 Assume that $X_1, ..., X_n$ are iid with density $f(x;\theta) = 2\exp\{2x\}\theta^{-2}\mathbb{I}(x < \log(\theta))$. The statistic $S(X_1, \ldots, X_n) = X_{(n)}$ is

■ complete and sufficient
□ incomplete and insufficient
□ incomplete and sufficient
□ complete and insufficient

Question 10 Given an iid sample $X_1, \ldots, X_n$ from a Uniform $\mathcal{U}(\theta - 1, \theta + 1)$ distribution, the statistic $S(X) = (X_{(1)}, X_{(n)} - X_{(1)})$ is

□ complete and insufficient
□ complete and sufficient
□ incomplete and insufficient
■ incomplete and sufficient

Question 11 Given an iid sample $X_1, \ldots, X_{10}$ from a Normal $\mathcal{N}(\theta, 2)$ distribution, the statistic $S(X_1, \ldots, X_{10}) = \{X_1 + X_{10}\}/2$ is

□ incomplete and sufficient
■ complete and insufficient
□ incomplete and insufficient
□ complete and sufficient

Question 12 Let $X_1, \ldots, X_n$ be an iid sample from a distribution that assigns with probability $1/3$ the values $\theta - 1$, θ, or $\theta + 1$, $\theta \in \mathbb{R}$. Then $S(X_1, \ldots, X_n) = (X_{(1)}, X_{(n)})$ is

□ complete and insufficient
□ complete and sufficient
■ incomplete and sufficient
□ incomplete and insufficient

Question 13 Let $X_1, \ldots, X_n$ be a random sample from a distribution with density $f(x) = \frac{1}{\theta}\exp\{-\frac{(x-\theta)}{\theta}\}\,\mathbb{I}_{(x>\theta)}$ Then $S(X_1, \ldots, X_n) = (X_{(1)}, \bar{X}_n)$ is

□ complete and sufficient
□ incomplete and insufficient
□ complete and insufficient
■ incomplete and sufficient

Question 14 Let $(X_1, \ldots, X_n)$ be a random vector on $\mathbb{N}^n$ with probability mass function $p(x_1, ..., x_n) \propto 1/\binom{\theta}{n}$ if all $0 \leq x_i \leq \theta$'s are different and equal to zero otherwise, with $\theta \in \mathbb{N}^*$. Then the statistic $S(X_1, \ldots, X_n) = \max X_i$ is

- ☐ incomplete and insufficient
- ☐ incomplete and sufficient
- ■ complete and sufficient
- ☐ complete and insufficient