Lec e 1: e a ca a d ec e c b e

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Se e be 2001

C a a e e a c :

fe e ce f e c a c ce e

1 An econometric problem: stochastic volatility

1.1 Time series of speculative assets

1.1.1 Daily exchange rates

Daily data against [illegible] currencies [illegible] 26 J[illegible] 1985 to 28th J[illegible] 2000.

The currencies are the Canadian, DM, FF, S[illegible] Franc, Yen and Pound. The [illegible] for each rate [illegible] de[illegible] [illegible] D[illegible].

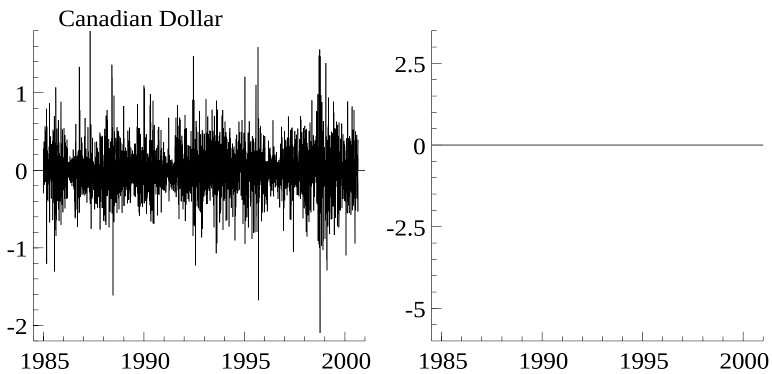
Rate [illegible] the D[illegible] [illegible] lengthened.

Ad[illegible] of the E[illegible] on 1st of January 1999 [illegible] e[illegible] the [illegible] rate between the FF and DM.

L[illegible]g-exchange rate $y^*(s)$. Let the [illegible] a[illegible] be [illegible], then [illegible] the change [illegible]

$$y_t = y^*(\quad s) - y^*((s-1)\quad), \qquad s = 1, 2, ... \tag{1}$$

N[illegible] [illegible] e[illegible] a[illegible] g[illegible] e[illegible] (e.g. [illegible] bank[illegible] account[illegible]).


Canadian Dollar
1
0
-1
-2
1985
1990
1995
2000
2.5
0
-2.5
-5
1985
1990
1995
2000

1.1.2 Equity data

I de e ec ded e d e c c e c 7 Dece b 1993 8 Oc b 2000.

DAX 30, FTSE 100, S&P 500 C e a d e N e 500.

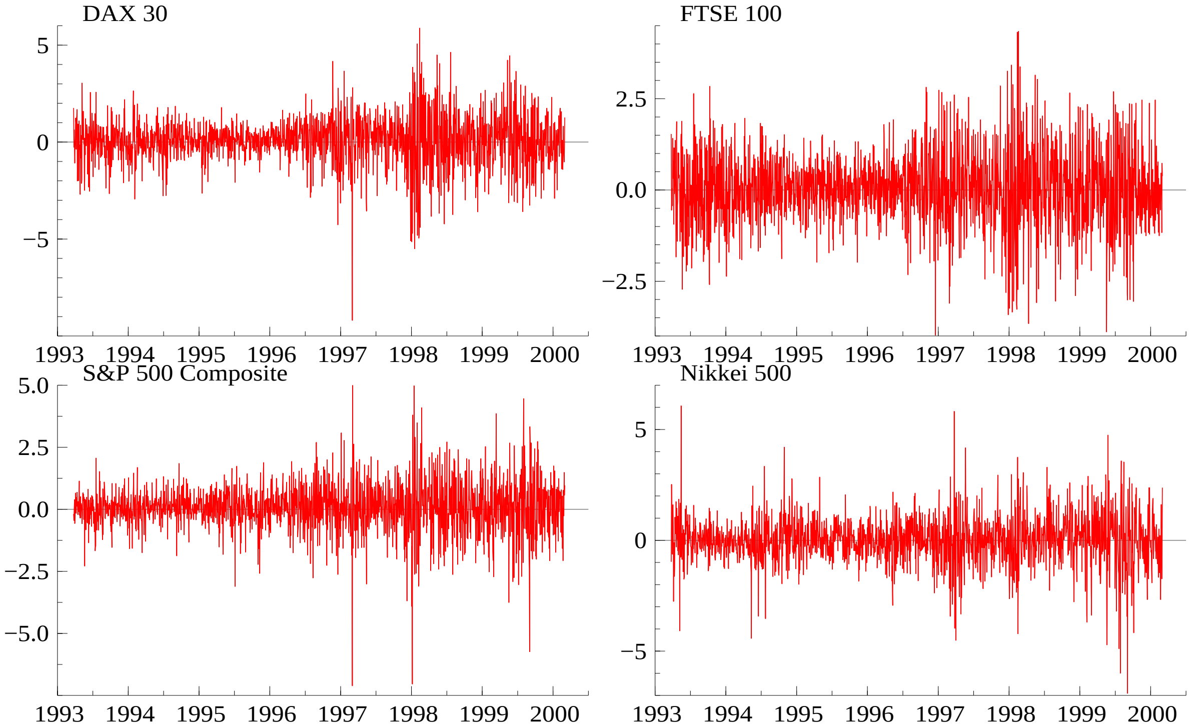
T g e e e e g ce f e de ec ded b DATASTREAM.

N a a e a cc ed.

B gge e DAX c e d b d e ce a g e da .

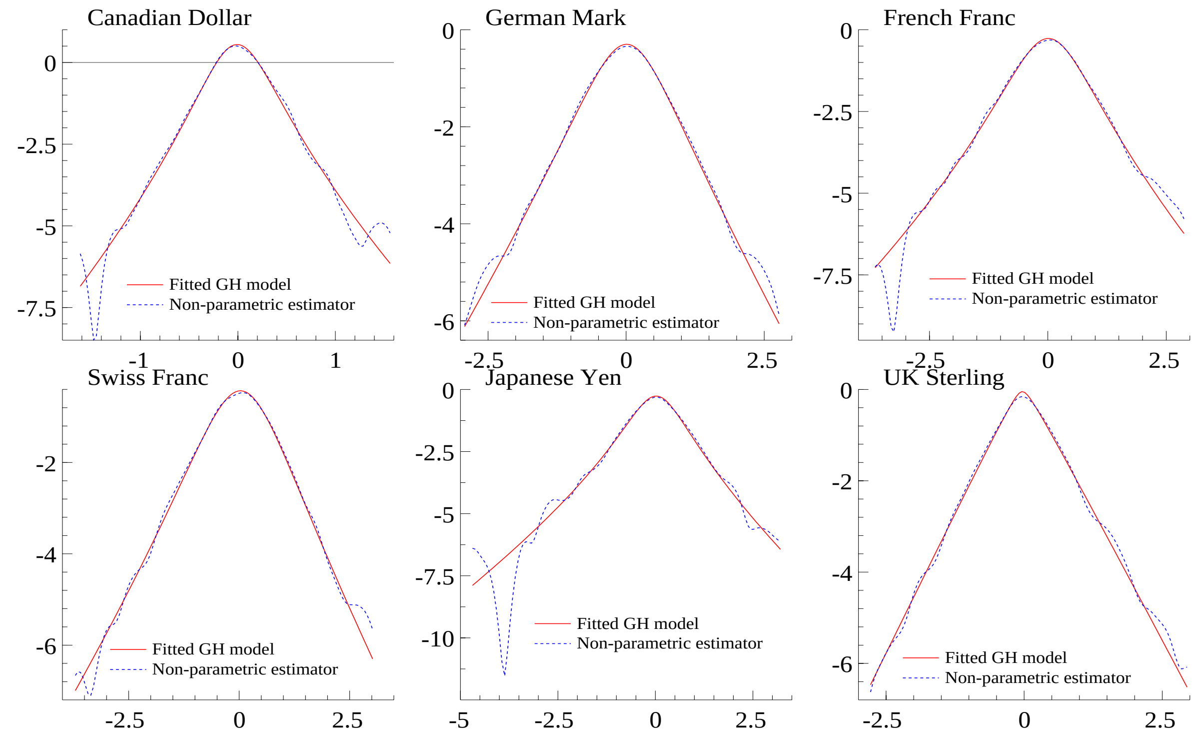
FTSE e a f e e e f d a ea a ge e e d .

Eac f e de e e e e ec d a f f e da a e .



Re d e c c e c , a g acc e e a e .

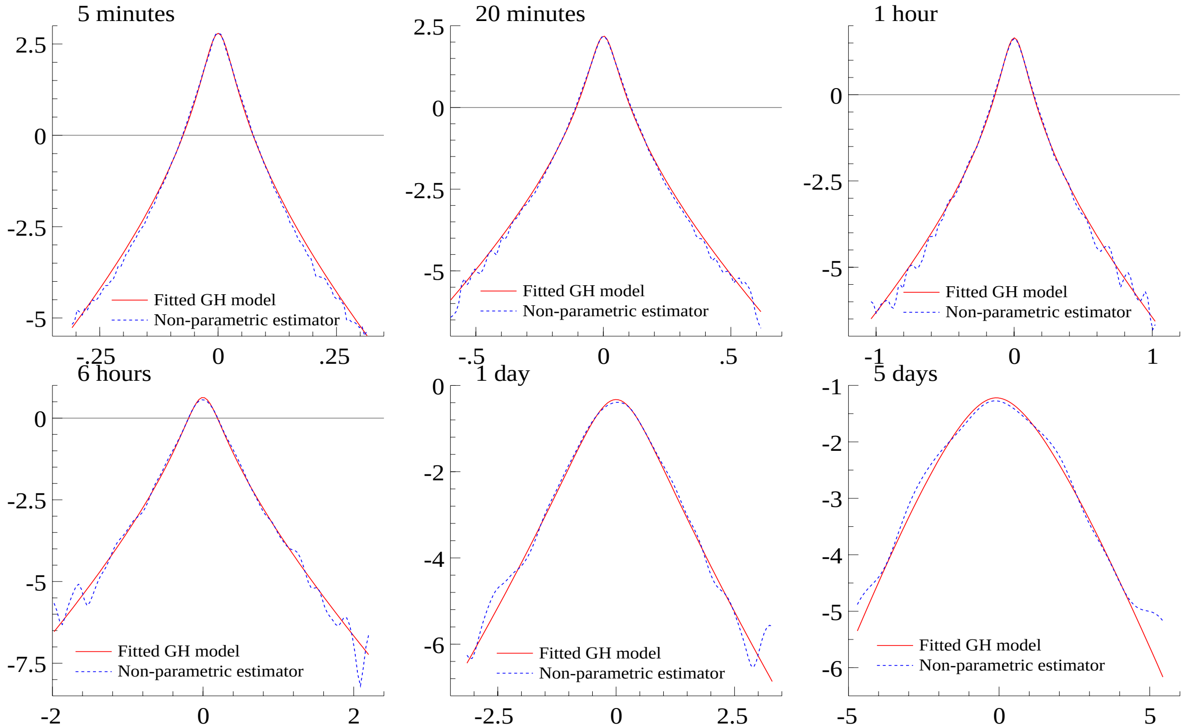
1.1.3 Stylised facts of time series



Marginal distributions of changes a e g-de e a d e a g g . Ba d d e Ga a ca e.

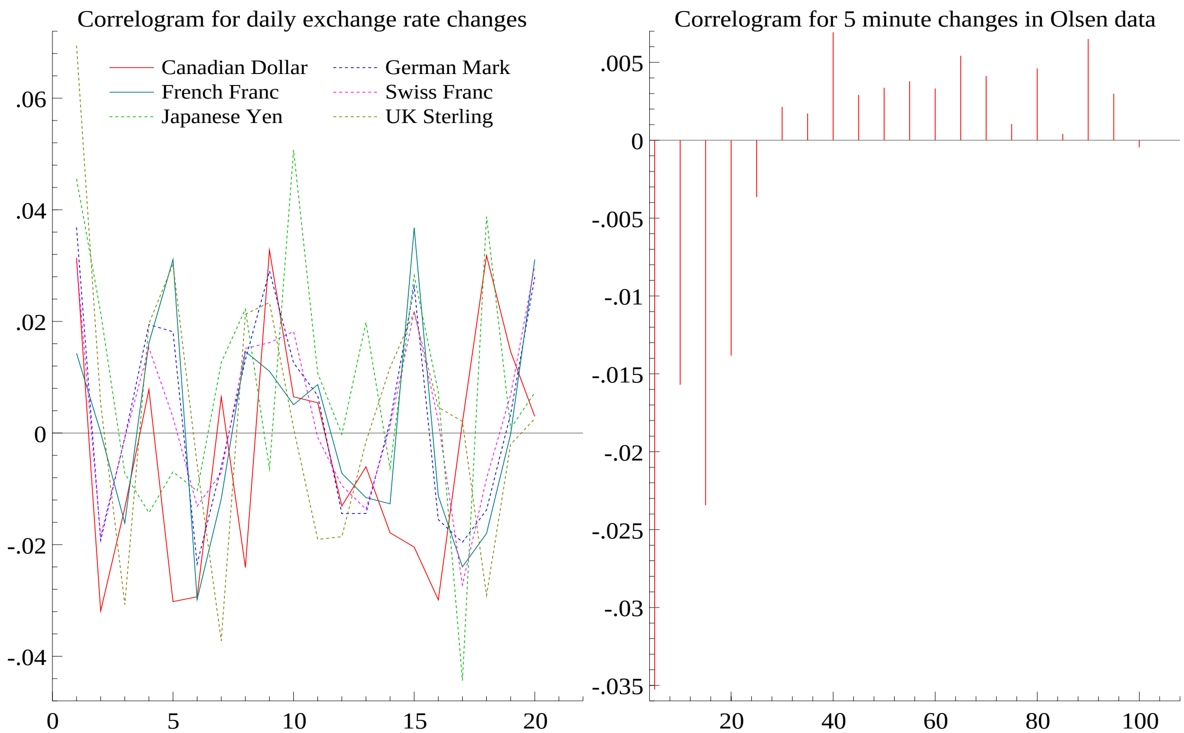
1.1.4 Temporal aggregation

5 [illegible] data [illegible] United States D[illegible] / G[illegible] De[illegible] Ma[illegible] [illegible] 1 [illegible] December 1986 [illegible] 30 [illegible] November 1996. Rec[illegible] [illegible] I[illegible] b[illegible] O[illegible] and A[illegible] [illegible] Z[illegible].

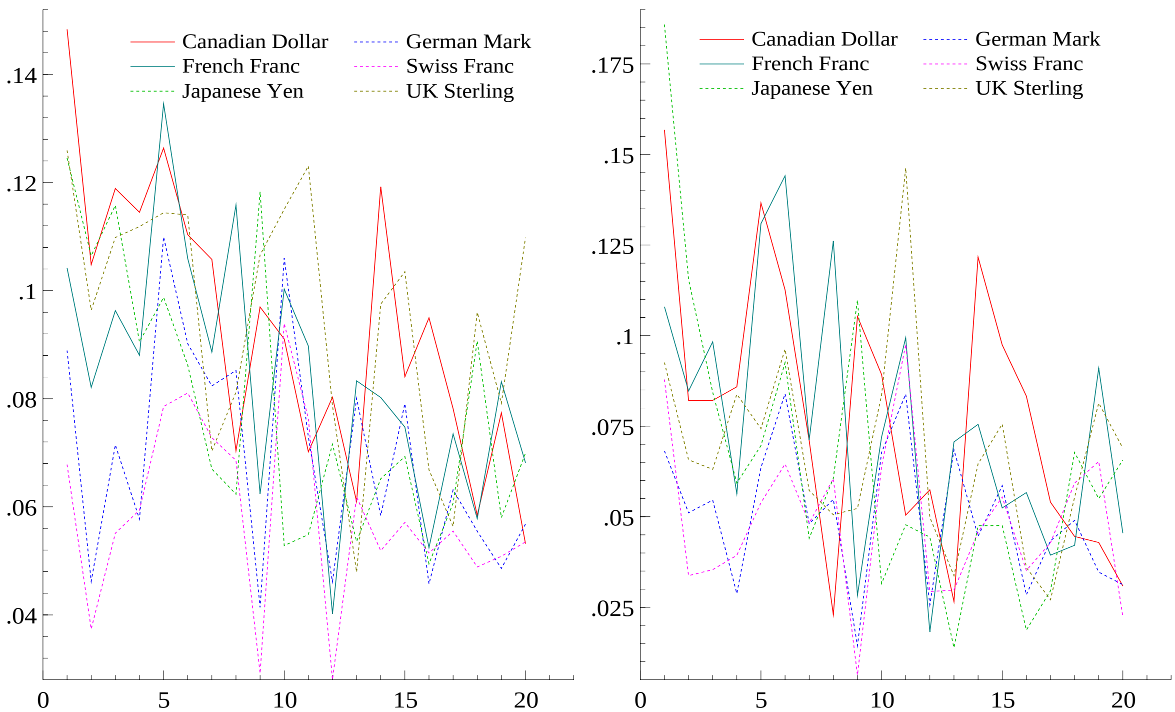

5 minutes
20 minutes
1 hour
6 hours
1 day
5 days
Fitted GH model
Non-parametric estimator

	Mean	Variance	Skewness	Kurtosis
5 minutes	-0.0000256	0.001847	0.146	44.2
20 minutes	-0.000102	0.006803	0.0628	27.6
1 hour	-0.000307	0.01929	0.263	21.3
6 hours	-0.00184	0.1162	0.0959	9.47
1 day	-0.00738	0.4903	0.00328	5.27
1 week	-0.0369	2.427	0.144	3.77

1.2 Serial dependence in changes in prices



S a e f c a ge a e a [c e a ed



S ed fac

- L e a c e a e e
- I a (g ag) a c e a a g ab e a d a e
- Ma g a a fa a
- Agg ega a Ga a
- S e ega e e a c a d d a c

1.3 Interests

- [illegible]

- [illegible]

- [illegible]

2 Models

2.1 Discrete time SV model

S e de e g- a SV de (Ta (1982))

$$
\begin{aligned}
y_t &= \beta e^{h_t/2}\varepsilon_t \,,\; t \geq 1 \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \sigma_\eta \eta_t \,,\; t \geq 2 \\
h_1 &\sim \mathcal{N}\left(\mu, \frac{\sigma^2}{1-\phi^2}\right) \quad \text{and} \quad \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim NID\left(0, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right)
\end{aligned} \tag{2}
$$

S e c de

- Ta (1994)
- G e , Ha e , a d Re a (1996)
- S e a d (1996)
- β, μ, ϕ, σ_η a d ρ (e $\beta = 1$).

$$
\begin{aligned}
y_t &= \beta e^{h_t/2}\varepsilon_t\,,\ t \geq 1 \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \sigma_\eta \eta_t\,,\ t \geq 2 \\
h_1 &\sim \mathcal{N}\left(\mu, \frac{\sigma^2}{1-\phi^2}\right).
\end{aligned}
\tag{3}
$$

If $\mid \gamma_1 \mid < 1$ [illegible]:

$$
\mu_h = E(h_t) = \frac{\gamma_0}{1-\gamma_1}, \qquad \sigma_h^2 = \mathrm{Va[illegible]}(h_t) = \frac{\sigma_\eta^2}{1-\gamma_1^2}.
$$

A ε_t a a [illegible], y_t be [illegible] h_t [illegible].

U[illegible]g g-[illegible] a d [illegible] b [illegible]

$$
E(y_t^4)/(\sigma_{y^2}^2)^2 = 3\,\mathrm{e}[illegible](\sigma_h^2) \geq 3.
$$

$$
\begin{aligned}
y_t &= \beta e^{h_t/2}\varepsilon_t\,,\ t \geq 1 \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \sigma_\eta \eta_t\,,\ t \geq 2 \\
h_1 &\sim \mathcal{N}\left(\mu, \frac{\sigma^2}{1-\phi^2}\right).
\end{aligned} \tag{4}
$$

- A ε_t d, y_t a MD a d WN f $| \gamma_1 |< 1$.
- A h_t a Ga a AR(1),

$$
\begin{aligned}
\mathrm{C}\quad(y_t^2, y_{t-s}^2) &= \mathrm{e}\quad\{2\mu_h + \sigma_h^2(1+\gamma_1^s)\} - \{E(y_t^2)\}^2 \\
&= \mathrm{e}\quad(2\mu_h + \sigma_h^2)\{\mathrm{e}\quad(\sigma_h^2\gamma_1^s) - 1\}
\end{aligned}
$$

a d

-

$$
\rho_{y_t^2}(s) = \mathrm{C}\quad(y_t^2 y_{t-s}^2)/\mathrm{Va}\ (y_t^2) = \frac{\mathrm{e}\quad(\sigma_h^2\gamma_1^s) - 1}{3\,\mathrm{e}\quad(\sigma_h^2) - 1} \simeq \frac{\mathrm{e}\quad(\sigma_h^2) - 1}{3\,\mathrm{e}\quad(\sigma_h^2) - 1}\gamma_1^s. \tag{5}
$$

T e a c e a f c f a ARMA(1,1) ce . T e SV de be a e a a e a e GARCH(1,1) de .

2.1.1 Superposition model

$$y_t = \beta e^{h_t/2} \varepsilon_t \, , \; t \geq 1$$

$$h_t = \sum_{j=1}^{M} h_{t,j},$$

where $h_{t,j}$ are independent Gaussian autoregressive processes. e.g. $M = 2$ and

$$\begin{aligned} h_{t+1,1} &= \mu + \phi_1 (h_{t,1} - \mu_1) + \sigma_{\eta,1} \eta_{t,1} \, , \\ h_{t+1,2} &= \phi_2 h_{t,2} + \sigma_{\eta,2} \eta_{t,2} \end{aligned}$$

where $\phi_1 > \phi_2$.

2.2 Discretely observed diffusions

SDE [illegible]

$$dy(t) = a\left\{y(t), t, \theta\right\} dt + b\left\{y(t), t; \theta\right\} dw(t), \quad (6)$$

[illegible] $a\{y(t), t, \theta\}$ [illegible] $b\{y(t), t, \theta\}$ [illegible]

Be[illegible] e.g. [illegible]

Ob[illegible]?

L[illegible]

- [illegible] (1993) & G[illegible], M[illegible], [illegible] (1993).

- [illegible] Ga[illegible] Ta[illegible] (1996) [illegible] Ga[illegible] L[illegible] (1997).

$$dy(t) = a\{y(t), t, \theta\}\, dt + b\{y(t), t; \theta\}\, dw(t),$$

- e - a a e c a ace f A -Sa a (1996a), A -Sa a (1996b) a d J a g a d K g (1997).
- e a g f c , ee Ke a d S e e (1999), S e e (1997), F e -Z (1989), Ha e a d Sc e a (1996)
- e e d ba ed e d f Pede e (1995).

2.3 The illustion of data

[illegible]

[illegible]

[illegible]

2.4 Continuous time SV models

Te e g- ce $y^*(s)$ f e e SDE

$$\mathrm{d}y^*(s) = \left\{\mu + \beta\sigma^2(s)\right\}\mathrm{d}s + \sigma(s)\mathrm{d}w(s), \tag{7}$$

e $\sigma^2(s)$, e *instantaneous spot volatility*, g g be a ed (a e) a e ca a e eg ab e a e a , e be g c a ca de- e de f e a da d B a $w(s)$. O a a f e f e g > 0 e a e de ed a

$$y_t = y^*(\ t) - y^*((t-1)\ ), \qquad t = 1, 2, ... \tag{8}$$

c e a a e e de f σ^2, f a

$$y_t|\sigma_t^2 \sim N(\mu\ + \beta\sigma_t^2, \sigma_t^2).$$

e

$$\sigma_t^2 = \sigma^{2*}(t\ ) - \sigma^{2*}\left\{(t-1)\ \right\}, \quad \text{and} \quad \sigma^{2*}(s) = \int_0^s \sigma^2(u)\mathrm{d}u.$$

2.5 Spot volatility model

[illegible]. The first is the constant elasticity of variance (CEV) process which solves the SDE

$$\mathrm{d}\sigma^2(s) = -\lambda\left\{\sigma^2(s) - \xi\right\}\mathrm{d}s + \omega\sigma(s)^\eta \mathrm{d}b(\lambda s), \qquad \eta \in [1,2],$$

where $b(s)$ is standard Brownian motion uncorrelated with $w(s)$. Of course the special case of $\eta = 1$ delivers the square root process, while when $\eta = 2$ we have Nelson's GARCH diffusion. These models have been heavily favoured by Meddahi and Renault (2000) in this context. The second process is the non-Gaussian Ornstein-Uhlenbeck, or OU for short, process which solves the SDE

$$\mathrm{d}\sigma^2(s) = -\lambda\sigma^2(s)\mathrm{d}s + \mathrm{d}z(\lambda s), \tag{9}$$

where $z(s)$ is a Lévy process with non-negative increments. These models have been developed in this context by Barndorff-Nielsen and Shephard (2001).

3 Statistical models

Ma f e ab e de a e ec a ca e f -Ga a , - ea a e ace de .

Le e e a e α_t a d b a a y_t. S a e a e Ma a d e b - a a e c d a de e de g e c e a e.

M de ec ed g

$$f(y_t|\alpha_t) \quad \text{a d} \quad f(\alpha_{t+1}|\alpha_t).$$

U f a e c e a d a d e de Ga a , ea c e.

- N ca eg a e K aga a (1987) (g d e)
- MCMC Ca P , a d S (1992), Ca a d K (1994), F -Sc a (1994), S e a d (1994), S e a d a d P (1997) e c (g)
- Pa c e G d , Sa d, a d S (1993), P a d S e a d (1999), D ce , de F e a , a d G d (2001) (e d).

4 Classes of state space models: MCMC design

- Univariate case: Ma[illegible] and [illegible] mixtures. Ca[illegible], P[illegible], and S[illegible] (1992)

- Conditionally Gaussian: $y|s$ [illegible] a Gaussian SSF. Carter and Kohn (1994) (s Markov and discrete), Shephard (1994) (s [illegible] Markov).

- Non-Gaussian measurement SSF: i.e. $\alpha_{t+1}|\alpha_t$ Gaussian but $f(y_t|\alpha_t)$ non-Gaussian. Shephard and Pitt (1997).

5 Outline of lectures

1. MCMC e d g f a e ace de

2. SV fe e ce

   (a) U a a e: MCMC & a c e e

   (b) M a a e: MCMC & a c e e

3. I fe e ce f d ba ed de

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