

Given

$$X \sim \mathcal{T}(\nu, \mu, 1)$$

Student's distribution with mean μ and $\nu > 1$ degrees of freedom (and variance 1), we have

$$\mathbb{E}[|X|] = \int_0^\infty \frac{x}{[1 + (x - \mu)^2/\nu]^{(1+\nu)/2}} C(\nu) dx - \int_{-\infty}^0 \frac{x}{[1 + (x - \mu)^2/\nu]^{(1+\nu)/2}} C(\nu) dx$$

where $C(\nu)$ is the normalising constant, equal to the standard density in zero $f(0|\nu, 0, 1)$. Thus,

$$\begin{aligned} \int_0^\infty \frac{x}{[1 + (x - \mu)^2/\nu]^{(1+\nu)/2}} C(\nu) dx &= \mu \mathbb{P}_{\mu, \nu}(X > 0) - \\ &\quad \frac{\nu}{\nu - 1} \int_0^\infty \frac{d}{dx} \frac{1}{[1 + (x - \mu)^2/\nu]^{(\nu-1)/2}} C(\nu) dx \\ &= \mu \mathbb{P}_{\mu, \nu}(X > 0) + \frac{C(\nu)\nu/(\nu - 1)}{[1 + (\mu)^2/\nu]^{(\nu-1)/2}} \end{aligned}$$

and

$$\begin{aligned} \int_{-\infty}^0 \frac{x}{[1 + (x - \mu)^2/\nu]^{(1+\nu)/2}} C(\nu) dx &= \mu \mathbb{P}_{\mu, \nu}(X < 0) - \\ &\quad \frac{\nu}{\nu - 1} \int_{-\infty}^0 \frac{d}{dx} \frac{1}{[1 + (x - \mu)^2/\nu]^{(\nu-1)/2}} C(\nu) dx \\ &= \mu \mathbb{P}_{\mu, \nu}(X < 0) - \frac{C(\nu)\nu/(\nu - 1)}{[1 + (\mu)^2/\nu]^{(\nu-1)/2}} \end{aligned}$$

so

$$\mathbb{E}[|X|] = \mu(2\mathbb{P}_{\mu, \nu}(X > 0) - 1) + \frac{2C(\nu)\nu/(\nu - 1)}{[1 + (\mu)^2/\nu]^{(\nu-1)/2}}.$$

In particular, when $\mu = 0$,

$$\mathbb{E}[|X|] = \frac{2C(\nu)\nu}{\nu - 1},$$

which remains bounded when ν goes to 2, while $\mathbb{E}[|X|^2]$ and therefore $\text{var}(|X|)$ go to infinity.